

# ECE 2110

# **Electrical Engineering Laboratory I**

A Companion Course to ECE 2020 - Electrical Circuits I

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# **Course Information**

#### 1 Introduction

This course is intended to enhance the learning experience of the student in topics encountered in ECE 2020. In this lab, students are expected to get hands-on experience in using the basic measuring devices used in electrical engineering and in interpreting the results of measurement operations in terms of the concepts introduced in the first electrical circuits course. How the student performs in the lab depends on his/her preparation, participation, and teamwork. Each team member must participate in all aspects of the lab to insure a thorough understanding of the equipment and concepts. The student, lab teaching assistant, and faculty coordinator all have certain responsibilities toward successful completion of the lab's goals and objectives.

#### 1.1 Student Responsibilities

The student is expected to be prepared for each lab. Lab preparation includes reading the lab experiment and related textbook material. If you have questions or problems with the preparation, contact your Laboratory Teaching Assistant (LTA), but in a timely manner. Do not wait until an hour or two before the lab and then expect the LTA to be immediately available.

Active participation by each student in lab activities is expected. The student is expected to ask the LTA any questions they may have. *Do not make costly mistakes because you did not ask a question before proceeding*.

A large portion of the student's grade is determined in the comprehensive final exam, resulting in a requirement of understanding the concepts and procedure of each lab experiment for the successful completion of the lab class. The student should remain alert and use common sense while performing a lab experiment. They are also responsible for keeping a professional and accurate record of the lab experiments in the lab manual wherever tables are provided. Students should report any errors in the lab manual to the teaching assistant.

#### 1.2 Laboratory Teaching Assistant Responsibilities

The LTA shall be completely familiar with each lab prior to class. The LTA shall provide the students with a syllabus and safety review during the first class. The syllabus shall include the LTA's office hours, telephone number, and the name of the faculty coordinator. The LTA is responsible for ensuring that all the necessary equipment and/or preparations for the lab are available and in working condition. Lab experiments should be checked in advance to make sure everything is in working order. The LTA should fully answer any questions posed by the students and supervise the students performing the lab experiments. The LTA is expected to grade the lab notebooks and reports in a fair and timely manner. The reports should be returned to the students in the next lab period following submission. The LTA should report any errors in the lab manual to the faculty coordinator.

#### 1.3 Faculty Coordinator Responsibilities

The faculty coordinator should ensure that the laboratory is properly equipped, i.e., that the teaching assistants receive any equipment necessary to perform the experiments. The coordinator is responsible for supervising the teaching assistants and resolving any questions or problems that are identified by the teaching assistants or the students. The coordinator may supervise the format of the final exam for the lab. They are also responsible for making any necessary corrections to this manual and ensuring that it is continually updated and available.

#### 1.4 Lab Policy and Grading

The student should understand the following policy:

**ATTENDANCE**: Attendance is mandatory and any absence must be for a valid excuse and must be documented. If the instructor is more than 15 minutes late, students may consider lab for the day cancelled.

LAB RECORDS: The student must:

- 1. Keep all work in preparation of and obtained during lab; and
- 2. Prepare a lab report on experiments selected by the LTA.

**GRADING POLICY**: The final grade of this course is determined using the criterion detailed in the syllabus:

In-class work will be determined by the LTA, who, at their discretion using information and observations of the students performing the lab. The final exam will contain a written part and a practical (physical operations) part.

**PRE-REQUISITES AND CO-REQUISITES**: The lab course is to be taken during the same semester as ECE 2020, but receives a separate grade. If ECE 2020 is dropped, then ECE 2110 must be dropped as well. Students are required to have completed both MTHSC 108 and PHYS 122 with a

C or better grade in each. Students are also assumed to have completed a programming class and be familiar with the use of a computer-based word processor.

Note that the instructor reserves the right to alter any part of this information at their discretion. Any changes will be announced in class and distributed in writing to the students prior to the changes taking effect.

#### 1.5 Course Goals and Objectives

The Electrical Circuits Laboratory I is designed to provide the student with the knowledge to use basic measuring instruments and techniques with proficiency. These techniques are designed to complement the concepts introduced in ECE 2020. In addition, the student should learn how to record experimental results effectively and present these results in a written report. More explicitly, the class objectives are:

- 1. To gain proficiency in the use of common measuring instruments.
- 2. To enhance understanding of basic electric circuit analysis concepts including:
  - Independent sources.
  - Passive circuit components (resistors, capacitors, inductors, and switches).
  - Ohm's law, Kirchhoff's voltage law, and Kirchhoff's current law.
  - Power and energy relations.
  - Thvenin's theorem and Norton's theorem.
  - Superposition.
- 3. To develop communication skills through:
  - Maintenance of succinct but complete laboratory notebooks as permanent, written descriptions of procedures, results, and analyses.
  - Verbal interchanges with the laboratory instructor and other students.
  - Preparation of succinct but complete laboratory reports.

4. To compare theoretical predictions with experimental results and to determine the source of any apparent differences.

#### 2 Use of Laboratory Instruments

One of the major goals of this lab is to familiarize the student with the proper equipment and techniques for making electrical measurements. Some understanding of the lab instruments is necessary to avoid personal or equipment damage. By understanding the device's purpose and following a few simple rules, costly mistakes can be avoided.

#### 2.1 Ammeters and Voltmeters

The most common measurements are those of voltages and currents. Throughout this manual, the ammeter and voltmeter are represented as shown in Figure 1.



Figure 1: Ammeter and Voltmeter

Ammeters are used to measure the flow of electrical current in a circuit. Theoretically, measuring devices should not affect the circuit being studied. Thus, for ammeters, it is important that their internal resistance be very small (ideally near zero) so they will not restrict/reduce the flow of current. However, if the ammeter is connected across a voltage difference, it will conduct a large current and damage the ammeter. Therefore, ammeters must always be connected in series in a circuit, never in parallel with a circuit element. High currents may also damage the needle on an analog ammeter. The high currents cause the needle to move too quickly, hitting the pin at the end of the scale. Always set the ammeter to the highest scale possible, then adjust downward to the appropriate level.

Voltmeters are used to measure the potential difference between two points. Since the voltmeter should not affect the circuit, the voltmeters have very high (ideally infinite) impedance. Thus, the voltmeter should not draw any current, and not affect the circuit. Given these requirements, voltmeters must be connected in parallel in a circuit, never in series with a circuit element.

In general, all devices have physical limits. These limits are specified by the device manufacturer and are referred to as the device rating. The ratings are usually expressed in terms of voltage limits, current limits, or power limits. It is up to the engineer to make sure that in device operation, these ratings (limit values) are not exceeded. The following rules provide a guideline for instrument protection.

#### 2.2 Instrument Protection Rules

- 1. Set instrument scales to the highest range before turning on the power/source.
- 2. Be sure instrument grounds are connected properly. Avoid accidental grounding of "hot" leads, i.e., those that are above ground potential.

- 3. Check polarity markings and connections of instruments carefully before connecting power.4)Never connect an ammeter across a voltage source. Only connect ammeters in series with loads.
- 4. Do not exceed the voltage and current ratings of instruments or other circuit elements. This particularly applies to wattmeters since the current or voltage rating may be exceeded with the needle still on the scale.
- 5. Be sure the fuse and circuit breakers are of suitable value. When connecting electrical elements to make up a network in the laboratory, it is easy to lose track of various points in the network and accidentally connect a wire to the wrong place. A procedure to follow that helps to avoid this is to connect the main series part of the network first, then go back and add the elements in parallel. As an element is added, place a small check by it on your circuit diagram. Then go back and verify all connections before turning on the power. One day someone's life may depend upon your making sure that all has been done correctly.

#### 3 Data Recording and Reports

#### 3.1 The Laboratory Notebook

Students must record their experimental values in the provided tables in this laboratory manual and reproduce them in the lab reports. Reports are integral to recording the methodology and results of an experiment. In engineering practice, the laboratory notebook serves as an invaluable reference to the technique used in the lab and is essential when trying to duplicate a result or write a report. Therefore, it is important to learn to keep accurate data. Make plots of data and sketches when these are appropriate in the recording and analysis of observations. Note that the data collected will be an accurate and permanent record of the data obtained during the experiment and the analysis of the results. You will need this record when you are ready to prepare a lab report.

#### 3.2 The Lab Report

Reports are the primary means of communicating your experience and conclusions to other professionals. In this course you will use the lab report to inform your LTA about what you did and what you have learned from the experience. Engineering results are meaningless unless they can be communicated to others. You will be directed by your LTA to prepare a lab report on a few selected lab experiments during the semester. Your assignment might be different from your lab partner's assignment.

Your laboratory report should be clear and concise. *The lab report shall be typed on a word processor.* As a guide, use the format on the next page. Use tables, diagrams, sketches, and plots, as necessary to show what you did, what was observed, and what conclusions you can draw from this. Even though you will work with one or more lab partners, your report will be the result of your individual effort in order to provide you with practice in technical communication.

#### Formatting and Style

- The lab report shall be typed in a word processor.
- All page margins must be 1.25 inches. All content (including text, figures, tables, etc.) must fit within the margins.
- Body text should be double-spaced.
- Basic text should be in 12-point size in a commonly used text font.
- Set your main text justified (with even left/right margins).
- The first line of each paragraph should have a left indent.
- All the tables should have titles and should be numbered. Tables should be labelled numerically as Table 1, Table 2, etc. Table captions appear **above** the table. The column headings should be labeled with the units specified.

- Graphs should be presented as figures. All the figures should have titles and should be numbered. Figure captions appear **below** the figure. Graphs should have labeled axes and clearly show the scales and units of the axes.
- All the figures and tables must be centered on the page.
- All the figures and tables in your report must be referenced in your discussion. References to figures in the main body of the text are usually written in abbreviated form (e.g. 'see Fig. 1").
- Use MS-Word equation (under Insert  $\rightarrow$  Equation menu), MathType, or a similar tool to type formulas.
- If you need to copy a schematic or figure from the lab manual to your report, use *Copy and Paste* function or take a screenshot by using *Snipping Tool* in MS-Windows.
- Do not place screenshots of your lab notebook in the report! Diagrams, tables, calculations, etc. must be generated using the existing tools in the word processor.

#### Order of Lab Report Components

**COVER PAGE -** Cover page must include lab name and number, your name, your lab partner's name, and the date the lab was performed.

**OBJECTIVE** - Clearly state the experiment objective in your own words.

**EQUIPMENT USED** - Indicate which equipment was used in performing the experiment.

#### FOR EACH PART OF THE LAB:

- Write the lab's part number and title in bold font.
- Firstly, describe the problem that you studied in this part, give an introduction of the theory, and explain why you did this experiment. Do not lift the text from the lab manual; use your own words.
- Secondly, describe the experimental setup and procedures. Do not follow the lab manual in listing out individual pieces of equipment and assembly instructions. That is not relevant information in a lab report! Instead, describe the circuit as a whole (preferably with diagram), and explain how it works. Your description should take the form of a narrative, and include information not present in the manual, such as descriptions of what happened during intermediate steps of the experiment.
- Thirdly, explain your findings. This is the most important part of your report, because here, you show that you understand the experiment beyond the simple level of completing it. Explain (compare expected results with those obtained). Analyse (analyze experimental error). Interpret (explain your results in terms of theoretical issues and relate to your experimental objectives). This part includes tables, graphs, and sample calculations. When showing calculations, it is usual to show the general equation, and one worked example. All the results should be presented even

if there is any inconsistency with the theory. It should be possible to understand what is going on by just reading through the text paragraphs, without looking at the figures. Every figure/table must be referenced and discussed somewhere in the text.

• Finally, provide a summary of what was learned from this part of the laboratory experiment. If the results seem unexpected or unreliable, discuss them and give possible explanations.

**CONCLUSIONS** - The conclusion section should provide a take-home message summing up what has been learned from the experiment:

- Briefly restate the purpose of the experiment (the question it was seeking to answer)
- Identify the main findings (answer to the research question)
- Note the main limitations that are relevant to the interpretation of the results
- Summarise what the experiment has contributed to your understanding of the problem.

**PROBING FURTHER QUESTIONS -** Questions pertaining to this lab must be answered at the end of laboratory report.

# Lab 1 - Orientation

### Introduction

In the first lab period, the students should become familiar with the location of equipment and components in the lab, the course requirements, and the teaching instructor. Students should also make sure that they have all of the co-requisites and pre-requisites for the course at this time.

## Objective

To familiarize the students with the lab facilities, equipment, standard operating procedures, lab safety, and the course requirements.

### PreLab

Read the Introduction and Appendix A, in this manual. Download and install the "WaveForms" software on your personal computer, available here.

## **Equipment Needed**

ECE 2110 lab manual.

## Procedure

- 1. During the first laboratory period, the instructor will provide the students with a general idea of what is expected from them in this course. Each student will receive a copy of the syllabus, stating the instructor's contact information. In addition, the instructor will review the safety concepts of the course.
- 2. During this period, the instructor will briefly review the equipment which will be used throughout the semester. The location of instruments, equipment, and components (e.g. resistors, capacitors, connecting wiring) will be indicated. The guidelines for instrument use will be reviewed.

# **Probing Further**

During the next period, the instructor may ask questions or give a quiz to determine if you have read the introductory material. As a professional engineer, it will be your responsibility to prepare yourself to do your job correctly. Learn as much as you can "up front". You will find that as a practicing professional if you wait until the last minute, you might have to pay a very painful price emotionally, financially, and professionally.

# Lab 2 - DC Measurements

### Introduction

Voltage and current values may be used to determine the power consumed (or provided) by an electrical circuit. Electric power consumption is a very important factor in all electrical applications, ranging from portable computers to megawatt industrial complexes. Thus, an understanding of power and how it is measured is vital to all engineers.

### Objective

By the end of this lab, the student should know how to make DC measurements of voltages and currents to determine power dissipation/delivery for circuit elements, branches, and various combinations of elements and branches.

## PreLab

Read the introductory material in the ECE 2020 textbook describing the passive sign convention for circuit elements. Also, review the lab manual section *Use of Laboratory Instruments*.

**Prior to coming to lab class**, calculate the values of voltage, current, and power absorbed or delivered for each circuit element in Figure 2.1 and complete Table 2.1 (i.e. **Part 0** of the Procedure).

## Equipment

Analog Discovery 2 instrument . Digital Multimeter. Resistors:  $75\Omega$ ,  $180\Omega \ge 2$ .

# Procedure

**Part (0)** For the circuit given in Figure 2.1, calculate the voltages across and currents through each circuit element. Using these values, determine the power absorbed or delivered by each circuit element. Include your calculations in your laboratory notebook. Record all of your calculated results in Table 2.1 for later comparison with your experimental values.



Figure 2.1: DC Resistive Network

Table $2.1$ :	Calculated	Values
---------------	------------	--------

	Voltage (V)	Current (mA)	Power (mW)
$75\Omega$			
$180\Omega$			
$180\Omega$			
$V_{source}$	$5\mathrm{V}$		

**Part (1)** Before setting up the circuit, use the digital multimeter (DMM) to measure actual value of each resistor by setting DMM to ohmmetter mode ( $\Omega$ ). Record your measurements in Table 2.2

Table 2.2: Measured Value of Each Resistor

	Nominal Value	Measured Value
$R_1$	$75\Omega$	
$R_2$	$180\Omega$	
$R_3$	$180\Omega$	

**Part (2)** Set up the circuit in Figure 2.1. Open the *WaveForm* software and adjust the output of the DC power supply under the *Supplies* menu to +5V. Turn on the DC power supply by enabling the "Master". Set the DMM to measure DC Volts ( $\bar{V}$ ) and connect it across "V+" and "GND" ports. Record the output voltage in Table 2.3. Now, measure the voltage across each individual circuit element. Before making each measurement, use the connection scheme shown in Figure 2.2 to verify that your voltmeter connection is correct (voltmeter must be connected in parallel with the component). Record your measurements in Table 2.3.



Figure 2.2: Connection Scheme for Circuit Voltage Measurements

**Part (3)** For the circuit in Figure 2.1, measure the current through each circuit element using the DMM (set to measure DC Amps  $\overline{A}$  and move the cables to A and COM ports). Before each measurement, the circuit will have to be turned off and rewired to insert the ammeter in series with the component under test. Use the connection scheme shown in Figure 2.3 to verify that your ammeter connection method is correct (the ammeter must be placed in series with the element whose current is to be measured). Record the measurements in Table 2.3 and calculate the percentage difference.



Figure 2.3: Connection Scheme for Circuit Current Measurements

Table 2.3: Measured and Calculated Values of Voltage and Current

	$V_s$ (V)	$V_1$ (V)	$V_2$ (V)	$V_3$ (V)	$I_1 (mA)$	$I_2 (mA)$	$I_3 (mA)$
Measured Value							
Calculated Value (Part 0)	5V						
Percentage Difference							

**Part (4)** Using your measurements from Parts 2 and 3, calculate the power absorbed or delivered by each circuit element. Record these values in Table 2.4. Compare them with the values obtained through circuit analysis in Part 0. Calculate the percent difference from the theoretical values.

Table 2.4: Absorbed/Delivered Power

_		Theoretical Power (mW)	Actual Power (mW)	Percentage Difference
-	$R_1$			
	$R_2$			
	$R_3$			
	$V_{source}$			

## **Probing Further Questions**

Q1. What is the relationship between the power values obtained from your measured values of voltage and current and those calculated theoretically in Part 0? What do you think are sources of error? Explain.

**Q2.** How does the sum of power absorbed by the resistances in the circuit compare to the amount delivered by the source?

# Lab 3 - Workstation Characteristics

### Introduction

Every engineer relies on equipment to drive and measure an electrical system under study. These devices are rarely ideal, and have their own internal characteristics which must be considered. The internal characteristics of various devices often have a significant effect on circuit operation. This may be accounted for in circuit design and analysis to more adequately predict actual operation in the lab. Students and engineers should understand the internal characteristics of the equipment they are using. This experiment will explore some of those characteristics for the devices used in this course.

### Objective

By the end of this lab, the student should know how to determine the internal resistance of meters and sources. The student should understand how the internal resistance of these instruments affects the measurements.

## PreLab

Read the section Use of Lab instruments and Appendix B, Fundamentals of Electrical Measurement, in this manual. Additionally, answer the exercise questions in Appendix B.

# Equipment

Analog Discovery 2 instrument and Digital Multimeter. Resistance substitution box.

# Procedure

**Note:** The various range scales of voltmeters, ammeters, and similar measurement instruments are achieved by changing the internal resistances of the instruments. Therefore, when measuring the internal resistance of an instrument, it is important to set the instruments range to one value and not change

it until the measurements are completed. Therefore, choose a scale that will allow making the desired measurements.

#### 1. Measure DC Resistance of the DMM Voltmeter

An ideal voltmeter has infinite resistance: it is an open circuit. Although it is impossible to make a physical voltmeter with infinite resistance, a well-designed voltmeter exhibits a very large internal input resistance. In some experiments, it is important to take into account the finite, non-ideal, internal resistance. In this exercise you will determine the internal resistance of the DMM voltmeter.

A. Set up the circuit as shown in Figure 3.1, using the Fluke DMM for the voltmeter and the Analog Discovery 2 for the power supply.



Figure 3.1: Circuit Diagram to Measure the Internal Resistance of the Voltmeter

- B. Set the resistance of R-Box to  $0\Omega$  by shorting its terminals (use wire or a banana plug). Set the "Supply V+" voltage to +5.00 Volts. Switch the DMM to DC Voltage mode ( $\bar{V}$ ) and turn on the power supply. Record the voltage indicated by the meter in Table 3.1.
- C. Remove the short across R-Box and increase its resistance to  $1M\Omega$ . The voltmeter reads the voltage across its internal resistance. Record the voltage in Table 3.1 and stop the power supply.
- D. Measure the actual resistance of R-Box with DMM ohmmeter ( $\Omega$ ) after removing it from the circuit, rather than relying on the indicated settings of the box. Record the measured value.
- E. Since the circuit has only a single branch, the same current (i) flows through the R-Box and voltmeter. Calculate the voltage drop across R-Box, then the current i, and finally find  $R_{Vi}$ .

	Value
V <sub>source</sub> (Part B)	
$V_{meter}$ (Part C)	
Measured $R$ (Part D)	
Calculated $R_{Vi}$ (Part E)	

Table 3.1: Measured and Calculated Values

#### 2. Measure DC Resistance of the DMM Ammeter

Ideally an ammeter would have no resistance  $(0 \ \Omega)$  so that the circuit in which it has been placed is not disturbed. However, as with the voltmeter, no ammeter can ever be ideal, and therefore all ammeters have some small internal resistance. In this exercise you will determine the internal resistance of the DMM ammeter. Record the measured values with maximum available precision.

- A. Set the "Supply V+" voltage to +4.00 Volts. Turn on the power supply by enabling the "Master" and record the actual voltage indicated by the meter in Table 3.2. Stop the power supply.
- B. Set up the circuit as shown in Figure 3.2, using the Fluke DMM for the ammeter and the Analog Discovery 2 for the DC power supply. To use the DMM as an ammeter, move the DMM cables to A and COM and switch the DMM to measure **DC Amps**  $(\bar{A})$ .



Figure 3.2: Circuit Diagram to Measure the Internal Resistance of the Ammeter

- C. Set the resistance of R-Box to  $60\Omega$  and turn on the power supply. Record the current indicated by the ammeter in Table 3.2 and stop the power supply.
- D. Measure the actual resistance of R-Box with DMM ohmmeter ( $\Omega$ ) after removing it from the circuit, rather than relying on the indicated settings of the box. For this, remember to use "V $\Omega$ " and "COM" ports. Record the measured value in Table 3.2.
- E. The total resistance in the circuit is  $R_{total} = R \cdot Box + R_{Ai}$ . By using the known quantities Vs,  $R \cdot Box$ , i and applying the Ohm's law we can solve for the unknown quantity  $R_{Ai}$ . Please note that this is an approximate value due to the DMM measurement resolution and rounding errors.

	Value
$V_{source}$ (Part A)	
Measured $i$ (Part C)	
Measured $R$ (Part D)	
Calculated $R_{Ai}$ (Part E)	

Table 3.2: Measured and Calculated Values

#### 3. Measure Output Resistance of the Power Supply V+

Any device which produces a voltage output has a limit to the current it can provide. This can be represented as a resistance in series with the ideal voltage output of the device. In this step you will measure the internal resistance of the Analog Discovery's V+ power supply.

- A. Before constructing the circuit, set the "Supply V+" voltage to +0.5 Volts and use DMM voltmeter to measure the output voltage of DC power supply. Record the measured value in Table 3.3. To use the DMM as an voltmeter, move the DMM cables to "VΩ" and "COM" and switch the DMM to measure **DC** Volts ( $\bar{V}$ ).
- B. Set up the circuit as shown in Figure 3.3, using the Fluke DMM for the voltmeter and the Analog Discovery 2 for the power supply.



Figure 3.3: Circuit Diagram to Measure the Internal Resistance of the DC Supply

- C. Set the resistor R to  $1\Omega$  and turn on the power supply by enabling the "Master". Do not go below  $1\Omega$  since it may cause a large amount of current flow in the circuit and damage your equipment. Record the measured voltage in Table 3.3.
- D. Measure the actual resistance of R-Box with DMM ohmmeter ( $\Omega$ ) after removing it from the circuit, rather than relying on the indicated settings of the box. Record the measured value in Table 3.3.
- E. From these readings, calculate current i by using  $V_{meter}$  and measured value of R-Box, find voltage drop across  $R_{VPS}$ , and use the Ohm's law to calculate  $R_{VPS}$ , the equivalent internal resistance of the voltage source.

Table 3.3: Meas	ured and Ca	alculated Values
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#### 4. Measure Output Resistance of the Function Generator

Repeat Part 3 using the Function Generator (FGEN) instead of the DC power source.

A. Before constructing the circuit, set the FGEN to output a sine wave with peak amplitude of 5V (which is equal to 3.53V RMS /  $10V_{P-P}$ ) and frequency of 100 Hz. To do this, use the *Wavegen* menu inside the *WaveForms* application and W1 port on the board. Switch the DMM to measure AC Volts ( $\tilde{V}$ ) and record the RMS voltage in Table 3.4. Keep in mind that the AC voltmeter displays the RMS value of voltage, where for a sinusoidal waveform:

$$V_{RMS} = \frac{V_{Peak}}{\sqrt{2}} = \frac{V_{P-P}}{2\sqrt{2}}$$
(3.1)

B. Construct the circuit shown in Figure 3.4.



Figure 3.4: Circuit Diagram to Measure the Internal Resistance of the FGEN

- C. Set the resistor R to  $1k\Omega$  and RUN the FGEN. Continue to decrease the resistance R until the voltmeter reading drops to about one half of the RMS value in Part A. Record the RMS voltage in Table 3.4. Stop the FGEN.
- D. Measure the actual resistance of R-Box with DMM ohmmeter ( $\Omega$ ) after removing it from the circuit, rather than relying on the indicated settings of the box. Record the measured value in Table 3.4.
- E. Repeat the same procedure as in Part 3-E to find the  $R_{FGEN}$  value. However, the internal resistance of FGEN may slightly change by varying the output voltage or load impedance. This is due to the non-linear property of these power sources.

	Value
$V_{Source-RMS}$ (Part A)	
$V_{Half-RMS}$ (Part C)	
Measured R (Part D)	
$R_{FGEN}$ (Part E)	

Table 3.4: Measured and Calculated Values

## **Probing Further Questions**

**Q1.** Given your values for the internal resistances of the DC source and the function generator, what do you think is the maximum amount of current that can be supplied by each source?

**Q2.** How does the voltmeters resistance affect measurements? Consider the circuit shown in Figure 3.5, and assume that the power supply is ideal.



Figure 3.5: Circuit Diagram

For resistance values  $R = 1k\Omega$ ,  $100k\Omega$ , and  $20M\Omega$ , calculate the voltage that would be displayed on an ideal voltmeter (i.e., if it had infinite resistance). Repeat the calculation using the real internal resistance you measured in Part 1. Show your work in your lab book and display your results in a table such as shown here:



$R(\Omega)$	$V_{ideal}$	$V_{real}$
1k		
100k		
20M		

Briefly discuss the significance of these results.

# Lab 4 - Introduction to LTspice

### Introduction

The widespread availability of computers has enabled the development of software which quite accurately models the behavior of electrical circuits. The most widely used program is SPICE, developed by the University of California-Berkeley in the mid-1970's and updated several times since then. Amongst the commercially available versions of SPICE is LTspice from Beige Bag Software. LTspice is loaded on the ECE 2110 laboratory computers, and a Lite version is available for free from the company website (check the syllabus for download link). As an introduction, you will learn to use LTspice circuit simulation software with relatively simple circuits. Later, in the introductory electronics courses ECE 320 and ECE 321, you will learn to use LTspice to simulate the operation of more advanced components such as diodes, transistors, and even a few integrated circuits.

## Objective

This lab should give the student a basic understanding of how to use LTspice to simulate circuit operating conditions. After this lab, the student should be able to use LTspice to solve or check basic circuit problems.

#### PreLab

Prior to coming to lab class, calculate the voltages and currents for each resistor shown in the circuit of Figure 4.1 (i.e. do **Part 0** of the Procedure). It is strongly recommended to watch some LTspice tutorial videos on YouTube before the lab meeting.

# Equipment

A computer with LTspice loaded and ready to use.

# Procedure

**Part (0)** Before coming to lab, determine the voltages across and currents through each resistor in the circuit of Figure 4.1. Include your calculations in your laboratory notebook. Record all of your calculated results in Table 4.1 for later comparison with LTspice.



Figure 4.1: Resistive Network for Hand Analysis and LTspice Simulation

Table 4.1: Hand Calculated Values for Figure 4.1

	$V_{1k\Omega}$	$\mathbf{V}_{2k\Omega}$	$V_{3k\Omega}$	$\mathbf{V}_{4k\Omega}$	$V_{5k\Omega}$	$\mathbf{V}_{6k\Omega}$	$I_1 (mA)$	$I_2 (mA)$	$I_3 (mA)$
Hand Calculated Value									

**Part (1)** The lab TA will provide an introduction to the LT Spice software environment. During this instruction session, you will learn how to:

- Open the software and create a new project.
- Place circuit components in your project workspace.
- Connect circuit components together.
- Set up circuit measurements.
- Change simulation execution settings.
- Run simulations.
- Display or export the simulation output.

One important principal to keep in mind is that if you want LTspice to display a graph or a table of something, you must include a suitable measurement device (e.g., voltmeter or ammeter) for that value. Anything without a meter wont be graphed.

**Part (2)** Use LTspice to solve for all of the resistor voltages and currents for the circuit of Figure 4.1 and record the results in Table 4.2. Compare your simulation measurements with the results of your calculations in Part 0.

 Table 4.2:
 LTspice Results for Figure 4.1

	$V_{1k\Omega}$	$\mathbf{V}_{2k\Omega}$	$\mathrm{V}_{3k\Omega}$	$V_{4k\Omega}$	$V_{5k\Omega}$	$V_{6k\Omega}$	$I_1 (mA)$	$I_2 (mA)$	$I_3 (mA)$
LTspice Result									
Value from Part $(0)$									

**Part (3)** Use LT Spice to simulate the circuit used in Part (0) of the Laboratory 2 (Figure 2.1). Determine the voltages and currents for each circuit component and complete Table 4.3 using the simulation and hand calculated values (Table 2.1).

Table 4.3: Comparison of the Voltage and Current Values

	$75\Omega$	$180\Omega$	$180\Omega$
Voltage from Lab 2 (V)			
Voltage from LTspice (V)			
Current from Lab 2 (mA)			
Current from LTspice (mA)			

**Part (4)** Variability is inherent in any process, including the manufacture of resistors and capacitors. Such components are usually marked with a nominal (target) value and a tolerance within which the actual value might fall. Consider the circuit of Figure 4.2, the same circuit you simulated in Part (3) and measured in Laboratory 2.



Figure 4.2: LTspice Representation of the Circuit of Figure 2.1

If the resistors you used in Laboratory 2 had a tolerance of 10%, their actual values might have been anywhere in the following ranges:

```
\begin{array}{l} 67.5\Omega \leq R_1 \leq 82.5\Omega \\ 162\Omega \leq R_2 \leq 198\Omega \\ 162\Omega \leq R_3 \leq 198\Omega \end{array}
```

To understand how such variation might affect your measurements of current and voltage in an actual circuit, use LTspice to calculate the currents and voltages if the resistors were at some of their extreme values, as indicated in the table below:

$R_1(\Omega)$	$R_2(\Omega)$	$R_3(\Omega)$	$I_1 (mA)$	$I_2 (mA)$	$I_3 (mA)$	$V_1$ (V)	$V_{2,3}$ (V)
82.5	198	198					
67.5	162	162					
82.5	162	198					
67.5	198	162					

Table 4.4: Impact of Variation on the Current and Voltage

Simulate the circuit for all the given values in Table 4.4. Compare the voltages and currents of the first row (82.5 $\Omega$ , 198 $\Omega$ , and 198 $\Omega$ ) to the voltages and currents found in the simulation of Procedure 3 and explain your observations.

For an understanding how the variability of a process is characterized by measurement of the mean, the variance, and the standard deviation of a parameter, read Appendix C, *Fundamentals of Statistical Analysis* in this manual.

## **Probing Further Questions**

**Q1.** How do the LTspice simulation results in Part 1 compare to your calculations in Part 0? Can you account for any differences?

**Q2.** How do the LTspice simulation results of Part 3 compare to your measurements in Laboratory 2? Can you account for any differences?

# Lab 5 - Network Theorems I

#### Introduction

An understanding of the basic laws of electrical voltages and currents is essential to electrical engineering. Circuit analysis is dependent upon knowing the nature of the laws governing voltage and current characteristics. This lab studies Kirchhoff's Voltage Law, Kirchhoff's Current Law, voltage division, current division, and equivalent resistance.

## Objective

By the end of this lab, the student should understand KVL, KCL, voltage division, current division, and equivalent resistance combinations.

## PreLab

Read the material in the textbook that describes Kirchhoff's Voltage Law, Kirchhoff's Current Law, voltage division, current division, and equivalent resistance combinations. Be able to perform circuit calculations using these principles. Before coming to class, **analyze each circuit (all parts) and determine the theoretical values** that should be obtained during the lab. Verify your calculations by performing LTspice simulations for each circuit and record your values in the Tables.

## Equipment

Analog Discovery 2 instrument. Digital Multimeter. Resistors:  $75\Omega$ ,  $150\Omega$ ,  $330\Omega$ ,  $470\Omega$ ,  $820\Omega$ ,  $1k\Omega$ ,  $1.5k\Omega$ ,  $2.2k\Omega$ , and resistance substitution box.

### Procedure

**Part (1)** Before coming to the lab, determine the voltages across and currents through each resistor in the circuit of Figure 5.1 and record all of your calculated results in Table 5.1.



Figure 5.1: Determining Parallel Equivalent Resistance

In the lab, adjust the output of the DC power supply to 5V and verify with the digital multimeter. Set up the circuit as shown in Figure 5.1. Measure and record only the **total current** into the circuit and **voltages** across each resistor.

Table 5.1: Measured and Calculated Values for Figure 5.1

	$V_{source}$	$V_{330\Omega}$	$\mathbf{V}_{1.5k\Omega}$	$\mathbf{V}_{2.2k\Omega}$	$I_1 (mA)$	$I_2 (mA)$	$I_3 (mA)$	I <sub>total</sub> (mA)
Calculated Value	5V							
Measured Value					-	-	-	

Next, using the measured total current and source voltage, determine the equivalent resistance of the parallel components in the circuit using ohm's law ( $V = I \ge R$ ). Replace the parallel resistors with a resistance substitution box set to the equivalent resistance and measure the current as before. Compare the experimentally determined equivalent resistance to the theoretical value.

 $I_{total}$  for equivalent circuit =

**Part (2)** Before coming to the lab, determine and record the voltages across each resistor in the **voltage divider circuit** below for the given range of R in Table 5.2.



Figure 5.2: Effect of R on the Component Voltages

In the lab, adjust the output of the DC power supply to 5V and verify with the digital multimeter. Begin with  $R = 510\Omega$  and measure and record the voltage across each resistor in Table 5.2. Repeat with  $R = 1k\Omega$ ,  $2k\Omega$ ,  $3k\Omega$ ,  $4.3k\Omega$ , and  $5.1k\Omega$ . Compare the measured voltages to those calculated using the voltage divider relation.

R-Box	Calc	ulated	Measured		
	$V_{820\Omega}$	$V_{R-Box}$	$V_{820\Omega}$	$V_{R-Box}$	
510 $\Omega$					
$1.0~\mathrm{k}\Omega$					
$2.0~\mathrm{k}\Omega$					
$3.0~\mathrm{k}\Omega$					
$4.3~\mathrm{k}\Omega$					
$5.1~\mathrm{k}\Omega$					

Table 5.2: Measured and Calculated Voltages for Figure 5.2

**Part (3)** Before coming to the lab, determine and record the currents through each resistor in the current division circuit below for the given range of R in Table 5.3. Since the resolution of Fluke DMM Ammeter is 1mA, instead of directly measuring the current in branches, measure the voltage across each resistor and use the Ohm's law to find currents. In order to find precise current values use measured value of resistors (use DMM ohmmeter).



Figure 5.3: Sum of Currents at a Node

In the lab, adjust the output of the DC power supply to 5V and verify with the digital multimeter. Begin with  $R = 510\Omega$  and measure and record the current through each resistor in Table 5.3. Repeat with  $R = 1k\Omega$ ,  $2k\Omega$ ,  $3k\Omega$ ,  $4.3k\Omega$ , and  $5.1k\Omega$ . To save time, measure one current (e.g.  $i_1$ ) for different values of R first then reconstruct the circuit to measure another one.

R-Box		Cal	culated			Me	asured	
It-Dox	$i_1 (mA)$	$i_2 (mA)$	$i_3 (mA)$	$i_2+i_3 (\mathrm{mA})$	$i_1 (mA)$	$i_2 (mA)$	$i_3$ (mA)	$i_2+i_3 (\mathrm{mA})$
510 $\Omega$								
$1.0~\mathrm{k}\Omega$								
$2.0~\mathrm{k}\Omega$								
$3.0~\mathrm{k}\Omega$								
$4.3~\mathrm{k}\Omega$								
$5.1~\mathrm{k}\Omega$								

Table 5.3: Measured and Calculated Currents for Figure 5.3

Compare the measured currents to those calculated using the current divider relation. Determine whether or not each set of measurements agrees with Kirchhoff's Current Law.

**Part (4)** Before coming to the lab, determine the voltages across each resistor in the circuit of Figure 5.4 and record all of your calculated results in Table 5.4.



Figure 5.4: Sum of Voltages around a Loop

Adjust the output of the DC power supply to 5V and verify with the digital multimeter. Setup the circuit as shown in Figure 5.4. Measure the voltage across each component. Compare the measured voltages to those calculated using the voltage divider relation. Determine whether or not your measurements agree with Kirchhoff's Voltage Law.

Table 5.4: Measured and Calculated Voltages for Figure 5.4

	$V_{source}$	$330\Omega$	$1 \mathrm{k} \Omega$	$470\Omega$	$V_{330} + V_{1k} + V_{470}$
Calculated Voltage (V)	5				
Measured Voltage (V)					

**Part (5)** Before coming to the lab, determine the voltages across each resistor in the circuit of Figure 5.5 and record all of your calculated results in Table 5.5.



Figure 5.5: Sum of Voltages around Different Loops

Adjust the output of the DC power supply to 5V and verify with the digital multimeter. Set up the circuit as shown in Figure 5.5. Measure the voltage across each component in loop 1 and record your measurements in Table 5.5. Repeat for loop 2 and loop 3. Compare your measured values with the terms in the KVL equation written for each loop.

Table 5.5: Measured and Calculated Voltages for Figure 5.5

	$V_{src}$	$V_{330\Omega}$	$\mathbf{V}_{1k\Omega}$	$V_{470\Omega}$	$V_{2.2k\Omega}$	Loop 1 $-V_{src}+V_{330}+V_{1k}$	Loop 2 $V_{470}+V_{2,2k}-V_{1k}$	Loop 3 - $V_{src}$ + $V_{330}$ + $V_{470}$ + $V_{2.2k}$
						370 300 11		370 - 350 - 410 - 2.24
Calculated	5							
Measured								

Determine whether or not your measurements agree with Kirchhoff's Voltage Law. Explain the reasons for any discrepancies found.

### **Probing Further Questions**

**Q1.** In part 2, what would the value of R have to be so that the voltage across R is 4/5 of the source voltage? Your answer should be quantitative (i.e. a number).

**Q2.** In part 3, what would the value of  $R_3$  have to be so that the current through  $R_3$  is 10 times the current through  $R_2$ ? Your answer should be quantitative.

# Lab 6 - Network Theorems II

## Introduction

This lab focuses on the Thévenin equivalent and maximum power transfer theorems. Complex circuits are often replaced with their Thévenin equivalent to simplify analysis. For example, in the analysis of large industrial power systems the Thévenin equivalent is used in short circuit studies. Maximum power transfer is also an important concept which allows the designer to determine an optimal design when power is a constraint.

# Objective

By the end of this lab, the student should be able to verify Thévenin's equivalence theorem and the concept of maximum power transfer.

## PreLab

Read the material in the textbook that describes Thévenin's equivalence theorem and maximum power transfer. Prior to coming to the lab, complete **Part 1** of the Procedure.

# Equipment

Analog Discovery 2 instrument. Digital Multimeter. Resistors:  $75\Omega$ ,  $150\Omega$ ,  $220\Omega$ ,  $470\Omega$ , and resistance substitution box.

# Procedure

**Part (1)** Determine the Thévenin equivalent circuit of Figure 6.1 from nodes A and B, and record all parameters in Table 6.1.



Figure 6.1: Determining the Thévenin Equivalent Circuit

Table 6.1: Calculated Thévenin Equivalent Parameters of Figure 6.1

	Calculated Value
$V_{th}$ (V)	
$\mathbf{R}_{th}$ ( $\Omega$ )	
$I_{sc}$ (mA)	

**Part (2)** Set up the circuit as shown in Figure 6.1. Adjust the output of the DC power supply to 5V and verify with the digital multimeter. Measure the open circuit voltage  $(V_{th})$  between nodes A and B (i.e. connect the voltmeter between nodes A and B). Measure the short circuit current  $(I_{sc})$  between nodes A and B (i.e. connect the ammeter between nodes A and B). Using these measurements, determine the  $R_{th}$  by using ohm's law (V = I x R) and complete Table 6.2.

Table 6.2: Measured Thévenin Equivalent Parameters of Figure 6.1

	Measured Value
$V_{th}$ (V)	
$I_{sc} (mA)$	
$\mathbf{R}_{th}$ ( $\Omega$ )	

**Part (3)** Set up the newly determined Thévenin equivalent circuit as shown in Figure 6.2 (i.e. use the values from Part 2) and verify that this circuit has the same open circuit voltage and short circuit current as the previous circuit by performing the same procedures as in Part 2. Record all your measurements in Table 6.3 and compare with the values obtained from Parts 2 and 3. Save this circuit for Part 4.


Figure 6.2: Thévenin Equivalent Circuit of Figure 6.1

Table 6.3: Measured Thévenin Equivalent Parameters of Figure 6.2

	Newly Measured Value	Value from Part 2	Value from Part 1
$V_{th}$ (V)			
$I_{sc}$ (mA)			
$\mathbf{R}_{th}$ ( $\Omega$ )			

**Part (4)** This part of the lab is to illustrate maximum power transfer. Use the Thévenin equivalent circuit developed in Part 3. As shown in Figure 6.3, connect a resistance substitution box R between nodes A and B.



Figure 6.3: Circuit for Maximum Power Transfer

Measure the voltage across R-Box if  $R = 1\Omega$  and calculate the power dissipated by this resistor  $(P=V^2/R)$ . Repeat with  $R = 50\Omega$ ,  $100\Omega$ ,  $145\Omega$ ,  $180\Omega$ ,  $220\Omega$ ,  $250\Omega$ ,  $300\Omega$ , and  $320\Omega$ . Record all your measurements in Table 6.4.

R-Box	$V_{R-Box}$ (V)	$P_{R-Box}$ (mW)
$1\Omega$		
$50\Omega$		
$100\Omega$		
$145\Omega$		
$180\Omega$		
$220\Omega$		
$300\Omega$		
$400\Omega$		
$500\Omega$		

Table 6.4: Voltage and Power for Figure 6.3

Use the data in Table 6.4 and MS-Excel to plot power (Y-axis) versus resistance (X-axis). At which value is the power at maximum? Sketch the plot in the blank space below.

### **Probing Further Questions**

Q1. Use LTspice to determine the Thévenin equivalent for the circuit in Part 1. First, enter the circuit shown in Figure 6.1 using node B as the reference or "ground" node. The voltage at node A is then the open circuit voltage. To measure the short-circuit current between points A and B, place an ammeter between the points. Determine the Thévenin equivalent and compare to your experimentally obtained equivalent circuit in Part 1. Record your LTspice file and the data obtained from the simulation in your laboratory notebook by pasting in the printouts. Highlight the open-circuit voltage value and short-circuit current value obtained from the simulation.

**Q2.** Use LTspice to simulate the circuit of Part 4. Start with the value of  $R = R_{max-power}$  that you determined experimentally to give maximum power transfer and find, from the LTspice simulation, the power delivered to this resistance. Then repeat with 20 $\Omega$  increments through 100 $\Omega$  where,

$$R_{max-pwr} - 100 \le R \le R_{max-pwr} + 100$$

Compare the values of power obtained by simulation with those you obtained experimentally. Record your LTspice file and the data obtained from them in your laboratory notebook.

# Lab 7 - The Oscilloscope

### Introduction

The digital oscilloscope allows the engineer to examine time varying waveforms in order to determine the magnitude, frequency, phase angle, and other waveform characteristics which depend upon the interaction of circuit elements with the sources driving them.

### Objective

By the end of the lab the student should be familiar with the controls of a digital oscilloscope and be able to use the instrument to observe periodic waveforms.

### PreLab

Review "Oscilloscope" section in the "WaveForms" reference manual available at DIGILENT website. Be familiar with the following: voltage scaling (Volts/division), time base (seconds/division), triggering, and measurement probes.

## Equipment

Analog Discovery 2 instrument. Resistors:  $820\Omega$  and  $330\Omega$ .

## Procedure

**Part (0)** If you are using the Digilent Discovery BNC Adapter: The BNC Adapters ground is internally connected to the Analog Discovery power supply ground. In this case, the oscilloscope can only measure voltages across components that are connected to ground terminal. Avoid grounding errors!

Part (1) Basic Setup.

A. Connect Channel 1 of the oscilloscope (1+ and 1- pins) across the waveform generator terminal (W1 and Ground). The pinout diagram of Analog Discover 2 is shown in Figure 7.1.



Figure 7.1: Analog Discovery 2 Pinout Diagram

B. Open the WaveForms software. Use the settings under the "Wavegen" menu to output a 100Hz sine wave with amplitude = 1.5 V-Peak and DC offset = 0V. "RUN" the FGEN.

Channel 1		
Run	🗹 Enable	Simple
Type:	${\rm sine}$	<b>▼</b> 🚯
Frequency:	100 Hz	~
Amplitude:	1.5 V	~
Offset:	0 V	~
Symmetry:	50 %	~
Phase:	0 °	~

Figure 7.2: Wavegen Settings

C. Open the "Scope" window. Enable Channel 1 and uncheck the "Noise" box under this channel's settings.

Channel 1					
Color					
Offset as divisions					
Division 🔹					
1X ~					
v ~					
DC 👻					
Average 📼					
Export					
Channel 1					
Label: C1					

Figure 7.3: Channel 1 Settings

D. Right-click on the oscilloscope display and disable the "Edge Histogram". Now "RUN" the the oscilloscope. Compare the oscilloscope output with the expected waveform based on the function generator's output settings (compare from t = 0 ms). Save the oscilloscope output screen as an image by using the "Export" option under the "File" menu.



Figure 7.4: Oscilloscope Screen

Part (2) Voltage Scaling (Range).

A. The volts/division controls the vertical scale. Run the function generator to output the same sine wave. Run the Oscilloscope and count the number of peak-to-peak divisions the waveform spans

on the oscilloscope's vertical scale (Y-axis). Multiply this number by the value shown in front of the "Range" setting to determine  $V_{P-P}$  (span from higher peak to lower peak). Does your measurement match the FGEN's output voltage? Save the oscilloscope output screen.

B. Change the "Range" value to 320 mV/div and repeat Part A. Compare your measurement with the expected value.

🗹 Time		0	
Position:	0 s	$\sim$	
Base:	1 ms/div	$\sim$	
	Ļ		
🐈 Add Channel 🛛 🔻			
Channel :	1	8	
Offset:	0 V	$\sim$	
Offset: Range:	0 V 500 mV/div	~	

Figure 7.5: Voltage Scale Manipulation

Part (3) Time Scaling (Base).

- A. The time/division controls the horizontal scale. The amount of time each horizontal division shows is commensurately changed when you adjust the time/division. Run the function generator to output the same sine wave as in Part 1-B. Run the Oscilloscope and count the number of horizontal divisions from one peak to the next (one cycle) of your oscillating signal. Multiply this number by the value shown in front of the "Base" setting to find the signal's period. Calculate the signals frequency with this equation: Frequency = 1/Period. Is this number similar to the FGEN's frequency setting? Save the oscilloscope output screen.
- B. Change the "Base" value to 10 ms/div and repeat Part A. Compare your measurements.

🗹 Time		0		
Position:	0 s	$\sim$		
Base:	1 ms/div	~		
	Ţ			
🐈 Add Channel 🛛 🔻				
🗹 Channel 1	🗹 Channel 1 🛛 🔞			
Offset:	0 V	$\sim$		
Offset: Range:	0 V 500 mV/div	~		

Figure 7.6: Time Scale Manipulation

#### Part (4) Cursor Function.

A. Run the function generator to output the same sine wave as in Part 1-B. Run the Oscilloscope to display a single screen update of between 1-3 cycles of the output function. Drag the cursors to appropriate points on the waveform to measure the period of the sine wave. Then adjust the cursors to measure the peak-to-peak voltage of the sine wave. Compare these measurements to those expected based on the function generator's output settings. Save the oscilloscope output screen.



Figure 7.7: Cursor Function

Part (5) Measurement Functions.

- A. The "Measurements" view shows the measured value for a list of selected measurements. To activate this window click on "View", and "Measurements". On the measurement window, pressing the "Add Defined Measurement" button opens the "Add Measurement" window. On the left side is the channel list, and on the right side is a tree view containing the measurement types in groups. Pressing the Add button here (or double-clicking an item) adds it to the measurement list. The first column in the list shows the channel, the second shows the type, and the third shows the measurement result. Add the following measurements: AC RMS, Peak-to-Peak, Period, and Frequency.
- B. Run the FGEN and then oscilloscope to display a single screen update of between 1-3 cycles. Compare values from the "Measurement" view to those expected based on the function generator.



Figure 7.8: Measurement View

Part (6) Reversing the Polarity.

- A. Use the bread board to connect Channel 1 of the oscilloscope (1+ and 1- pins) across the waveform generator terminal (W1 and Ground). Now connect Channel 2 of the oscilloscope (2+ and 2- pins) across the same waveform generator terminal but in **reverse polarity** (2+ to GND, and 2- to W1).
- B. Make sure both Channel 1 and 2 boxes are checked and the "Noise" option is not active. Setup the function generator to output the same sine wave as in Part 1-B. Run both FGEN and the oscilloscope.
- C. You should be seeing two waveforms on the oscilloscope output screen. Change the settings to display only 1 cycle. Repeat your voltage and period measurements as in Part 5 for both signals. Do you see any value for frequency and period in the Measurement section? Why? Now, change the appropriate settings to see a value. Save the oscilloscope output screen.

**Part (7)** Connect the voltage divider circuit shown in Figure 7.9. Set the function generator to output a 1kHz sine wave with amplitude = 2V-Peak and DC offset = 0. Display the function generator output on Channel 1 of the oscilloscope and the voltage across the  $330\Omega$  resistor on Channel 2. You may need to adjust the oscilloscope's Range and Base settings to obtain a good display of both waveforms. Measure the period of both waveforms using the cursor function. Save the oscilloscope output screen and record your settings for Volts/div and seconds/div. Compare your voltage measurements with theoretical

calculations based on the voltage divider equation. Compare your waveform period measurement with the theoretical value obtained from the input frequency.



Figure 7.9: Voltage Divider Circuit

## **Probing Further Questions**

**Q1.** In part 6, what effect did reversing the polarity of the output voltage measurement have on the oscilloscope display and the oscilloscope voltage measurements?

**Q2.** How is the accuracy of your measurements affected by adjustment of the volts/division control? Seconds/division control?

# Lab 8 - RC and RL Circuits

### Introduction

This lab deals with RC and RL circuits. RC and RL circuits are used in many configurations for a large variety of design purposes. In addition, real components can be modeled in a given frequency range by a combination of R and C or R and L, as appropriate. For example, a wirewound resistor can be modeled as a "pure" resistor, R, at DC or very low frequencies, but as the frequency of operation increases, the inductive effects of the winding must be taken into account. This lab illustrates some of the basic features of the transient response of circuits in which resistance and capacitance or resistance and inductance are both present.

## Objective

By the end of this lab, the student should know how to measure the time constants of RC and RL circuits.

## PreLab

Review the material in the textbook on RC and RL circuits. Before coming to the lab, determine the theoretical time constants of the all circuits used in the lab. Be sure to account for the internal resistance of the function generator in your calculations (use the value from Laboratory 3).

## Equipment

Analog Discovery 2 instrument. Digital Multimeter. Resistance, Capacitance, and Inductance - substitution boxes.

### Procedure

Part (1) Prior to coming to the lab, calculate theoretical time constant of the RC circuit in Figure 8.1 for the given range of C (include  $R_{FGEN}$  in your calculations). Record your values in Table 8.1.



Figure 8.1: RC Circuit

Set up the RC circuit as shown and connect Channel 1 of the oscilloscope across the resistor and Channel 2 across the capacitor. Set the function generator to give a square wave output with amplitude of 500mV, offset = 500mV, and frequency of 50Hz. Measure the voltage across each component and record them in Table 8.1. If you are using the BNC Adapter for Analog Discovery, be aware of grounding errors!

Table 8.1: Measured and Calculated Values of the RC Circuit

R	С	Theoretical Time Constant $(\tau)$	Measured $V_R$ (mV)	Measured $V_C$ (mV)
$10 \mathrm{k}\Omega$	$0.01 \mu F$			
$10 \mathrm{k}\Omega$	$0.047 \mu F$			
$10 \mathrm{k}\Omega$	$0.1 \mu F$			

To determine the actual time constant of the circuit, first use the oscilloscope cursors function to find when the voltage across **capacitor** reaches **0.632** times its final value (peak) and record your measurement in Table 8.2. Now, find the time when the **resistor's voltage** reaches **0.368** times its initial peak value (the voltage decays here). Save the oscilloscope plots as an image file for your future reference. For each circuit the frequency of the waveform generator may have to be changed to achieve the flat sections at top and bottom of the waveforms. Compare your measured values of the RC circuit time constant with the theoretical values in Table 8.1.

Table 8.2: Measuring Time Constant of RC Circuit

R	C	Time for $V_R$ to Reach 0.368	Time for $V_C$ to Reach 0.632
$10 \mathrm{k}\Omega$	$0.01 \mu F$		
$10 \mathrm{k}\Omega$	$0.047\mu F$		
$10 \mathrm{k}\Omega$	$0.1 \mu F$		

Part (2) Prior to coming to the lab, calculate theoretical time constant of the RL circuit in Figure 8.2 for the given range of L (include  $R_{FGEN}$  in your calculations). Record your values in Table 8.3.



Figure 8.2: RL Circuit

Set up the RL circuit as shown and connect Channel 1 of the oscilloscope across the resistor and Channel 2 across the inductor. Set the function generator to give a square wave output with amplitude equal to 500mV and adjust the frequency of the function generator (start with 50Hz) so that the waveform shown has definite flat sections at the top and bottom. Measure the voltage across each component and record them in Table 8.3. If you are using the BNC Adapter for Analog Discovery, be aware of grounding errors!

Table 8.3: Measured and Calculated Values of the RL Circuit

R	L	Theoretical Time Constant $(\tau)$	Measured $V_R$ (mV)	Measured $V_L$ (mV)
$2\mathrm{k}\Omega$	200mH			
$2\mathrm{k}\Omega$	$400 \mathrm{mH}$			
$2\mathrm{k}\Omega$	$600 \mathrm{mH}$			
$2\mathrm{k}\Omega$	$800 \mathrm{mH}$			

To determine the actual time constant of the circuit, first use the oscilloscope cursors function to find when the voltage across **resistor** reaches **0.632** times its final value (peak) and record your measurement in Table 8.4. Now, find the time when the **inductor's voltage** reaches **0.368** times its initial peak value (the voltage decays here). Save the oscilloscope plots as an image file for your future reference. For each circuit the frequency of the waveform generator may have to be changed to achieve the flat sections at top and bottom of the waveforms. Compare your measured values of the RL circuit time constant with the theoretical values in Table 8.3.

Table 8.4: Measuring Time Constant of RL Circuit

R	L	Time for $V_R$ to Reach 0.632	Time for $V_L$ to Reach 0.368
$2k\Omega$	200mH		
$2k\Omega$	$400 \mathrm{mH}$		
$2k\Omega$	$600 \mathrm{mH}$		
$2k\Omega$	$800 \mathrm{mH}$		

### **Probing Further Questions**

**Q1.** Use LTspice to simulate the RC circuit response for the circuit of Figure 8.1. Repeat for the RL circuit response for the circuit of Figure 8.2. For both, use LTspices Transient Sweep simulation to obtain waveforms and compare these waveforms to those you obtained from your experiments.

Q2. Explain the significance of the 0.632 and 0.368 multipliers and why they are used in the lab.

## Lab 9 - Series RLC Circuits

#### Introduction

This lab illustrates some of the properties of series RLC circuits. The differential equation for this circuits has the following characteristic equation:

$$S^2 + 2\alpha S + \omega_0^2 = 0 \tag{9.1}$$

where  $\alpha$  is the Neper frequency or attenuation,  $\omega_0$  is the resonance frequency, and  $S_{1,2}$  are the roots of the characteristic equation (circuit response). These parameters are defined by,

$$\alpha = \frac{R}{2L} \quad \text{and} \quad \omega_0 = \frac{1}{\sqrt{LC}}$$
(9.2)

 $\alpha > \omega_0$ 

 $\alpha < \omega_0$ 

and,

$$S_1, S_2 = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2} = -\frac{R}{2L} \pm \sqrt{\frac{R^2}{4L^2} - \frac{1}{LC}}$$
(9.3)

Depending on the component values, series RLC circuits are overdamped, critically damped, or underdamped. The conditions for the three cases are as follows:

Overdamped (distinct real roots):

Critically damped (repeated real root):  $\alpha = \omega_0$ 

Underdamped (complex-conjugate roots):

For the component values used in this experiment,  $\alpha < \omega_0$ , so the circuit is underdamped. Thus, the roots of the characteristic equation are complex. If we define,

$$\omega_d = \sqrt{\omega_0^2 - \alpha^2} = \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}} = \frac{2\pi}{T}$$
(9.4)

where T is the time period, then the circuits current may be written:

$$i(t) = e^{-\alpha t} (A \cos \omega_d t + B \sin \omega_d t)$$
(9.5)

## Objective

By the end of this lab, the student should be able to relate the nature of the physical response of a series RLC circuit to the parameter values  $\alpha$  and  $\omega_0$  determined by the component values.

## PreLab

Review the material in the textbook on the RLC circuit response. Review the concepts of overdamped, underdamped, and critically damped response. **Before coming to the lab**, complete Part 0 of the Procedure.

## Equipment

Analog Discovery 2 instrument. Digital Multimeter. Resistance, Capacitance, and Inductance - substitution boxes.

## Procedure

**Part (0)** Prior to coming to the lab, calculate the theoretical parameter values of  $S_1$ ,  $S_2$ ,  $\alpha$ ,  $\omega_d$  and T (time period) for the RLC circuit shown in Figure 9.1 and record them in Table 9.1.



Figure 9.1: Series RLC Circuit

Parameter	Calculated Value
α	
$\omega_0$	
$\omega_d$	
$\mathrm{S}_1$	
$S_2$	
Т	

Table 9.1: Theoretical Parameter Values of Figure 9.1

**Part (1)** In Figure 9.1,  $R_{tot}$  is the total resistance of the circuit. In practice, part of that resistance is due to the internal resistance of the function generator and part is supplied by the resistance substitution box:

$$R_{tot} = R_{FGEN} + R_s$$

where  $R_{FGEN}$  is the internal resistance of the function generator, and  $R_s$  is the resistance set on the resistance substitution box. Using this equation and the value of  $R_{FGEN}$  from Laboratory 3, you can easily calculate the value of  $R_s$  to keep the total resistance of the circuit ( $R_{tot}$ ) to 200 $\Omega$ .

$$R_s = 200 - R_{FGEN} =$$

A. After determining the value of  $R_s$  and before connecting the components, verify the values of L and C using the DMM according to the values shown in Figure 9.2. Set up the circuit and set the function generator to output a square wave with amplitude = 1V, DC offset = 0V, and frequency in the range of 10Hz to 100Hz.



Figure 9.2: Series RLC Circuit

- B. Connect CH1 of the oscilloscope across the Function Generator and CH2 to display the voltage across the inductor. Adjust the frequency of the square wave, if necessary, so that the damped sinusoidal waveform decreases to a negligible value (i.e., dampens out) before the next square-wave pulse occurs. Something in the range of 10 Hz to 100 Hz probably will be adequate.
- C. Draw an accurate representation of the transient sinusoidal waveform (the damped oscillations) in your laboratory notebook. Save the oscilloscope plot as an image in your USB flash drive or export the data as a CSV file and use another software to plot the waveform.

D. Expand the time scale of the oscilloscope to show 3 to 6 peaks of the damped sinusoidal oscillation (change the BASE setting). Using the CURSORS function of the oscilloscope, measure the period of the damped sinusoidal waveform. Compare this value to T obtained in Part 0.

Table	9.2:	Time	Period
-------	------	------	--------

	Value (ms)
Calculated T (Part 0)	
Measured T (Part D)	

**Part (2)** Measure the peak height of the first full peak of the damped sinusoidal waveform and the peak height at the next peak (+1 cycle). Measure the time difference  $\Delta t$  between these two peaks, and the peak voltage of these cycles.

$$\begin{array}{l} \Delta t = \\ V_{peak1} = \\ V_{peak2} = \end{array}$$

A. Determine the Neper frequency (damping coefficient),  $\alpha$ , using your measurements and the equation 9.6. Compare your measured  $\alpha$  to the value you calculated in Part 0.

$$V_{peak2} = V_{peak1}.e^{-\alpha\Delta t} \tag{9.6}$$

Table 9.3: Damping Coefficient  $(\alpha)$ 

ValueCalculated  $\alpha$  (Part 0)Experimental  $\alpha$  (Part A)

B. Use the value of experimental  $\alpha$  from Part A to calculate the total resistance  $R_{tot}$  in the circuit by using,

$$\alpha = \frac{R_{tot}}{2L} \to R_{tot} =$$

C. Calculate the function generators internal resistance using,

$$R_{FGEN} = R_{tot} - R_s =$$

D. Now knowing  $R_{FGEN}$ , adjust  $R_s$  so that  $R_{tot} = 200\Omega$ . Again measure the damping coefficient,  $\alpha$ , and compare the new measured value to your calculated value from Part 0.

Table 9.4: Damping Coefficient (e	$\alpha$	)
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ValueCalculated  $\alpha$  (Part 0)Experimental  $\alpha$  (Part A)Experimental  $\alpha$  (Part D)

**Part (3)** Now connect CH2 of the oscilloscope to measure the voltage across the capacitor. In your laboratory notebook, sketch the waveform of the voltage across the capacitor. Save the plot on your USB Flash Drive as well.

Notice that if you were to perform the calculation for  $\alpha$  using these data, you would need to subtract from the peak heights the 1-volt offset provided by the pulse from the function generator. Doing so, you could again calculate  $\alpha$  as in Part 2.

**Critical damping:** For the values of L and C used in this circuit, calculate the value of total series resistance  $R_{tot}$  that gives critical damping. Change the resistance  $R_s$  so that  $R_{tot}$  equals this value. Again observe the voltage drop across the capacitor and sketch the resulting waveform in your laboratory notebook.

**Overdamped:** Change  $R_s$  so that  $R_{tot}$  is 10 times the value you calculated for critical damping. Observe and record the capacitor voltage response.

### **Probing Further Questions**

**Q1.** Describe the differences between the underdamped, overdamped, and critically damped responses in the capacitor voltage,  $V_C$ . What are the potential advantages or disadvantages of an electrical system behaving in each mode of operation? What is the functional form of the transient response (be specific) if the circuit is critically damped?.

**Q2.** Explain why the inductor voltage,  $V_L$ , is greater than the amplitude of the incoming square wave at the time of a square wave transition (initial condition).

## Lab 10 - Statistical Analysis

### Introduction

The student is already aware of some error introduced by assuming ideal meters in the measurement process. Another uncertainty lies in the use of the circuit components themselves. While a component is designed to have a particular value (its "nominal" value), which is marked on the outside covering (the "case" or "encapsulation"), random fluctuations in materials and production processes will result in some range of values for the manufactured devices. Thus, components are usually specified by a nominal value and a range, called the tolerance, in which the actual value is expected to lie. For example, resistors are specified as being within a stated tolerance of the given (nominal) value. This tolerance can be as small as 1% for 'precision' resistors to as large as 20%. The tolerance may be indicated by a color band or by a percentage value printed on the body of the resistor. It is important to know how to identify the tolerance of the resistors used in a particular circuit and to understand what the specified nominal value and tolerance for a component means statistically.

## Objective

By the end of this lab, the student should know how to apply statistical methods to obtain the best estimate of the true value of a circuit component.

### PreLab

Read Appendix C, *Fundamentals of Statistical Analysis*. Become familiar with the concepts of mean, standard deviation, and variance and the formulas used for calculating these quantities.

## Equipment

Digital Multimeter. Resistors:  $2200\Omega \ge 5 \ge 1000$  x 5 x Number of Teams

## Procedure

**Part(1)** The lab instructor will provide each team with a group of five similar resistors. Each group of resistors should be labeled GROUP A, GROUP B, GROUP C, etc. For each group of resistors, measure the resistance of each individual resistor using the digital multimeter and complete Table 10.1. These resistor groups must be passed in turn from lab team to lab team.

	$R_1(\Omega)$	$R_2(\Omega)$	$R_3(\Omega)$	$R_4(\Omega)$	$R_5(\Omega)$	Mean	VAR	SD
GROUP A (2200 $\Omega$ )								
GROUP B (2200 $\Omega)$								
GROUP C (2200 $\Omega)$								
GROUP D (2200 $\Omega$ )								
TOTAL (All Resistors)	-	-	-	-	-			

Table 10.1: Measuring the Value of Individual Resistors in Each Group

To find the distribution of resistors, calculate the mean, the standard deviation, and the variance for the resistors in each group by using MS-Excel. For the "TOTAL" row, calculate the mean, standard deviation, and variance for all of the resistance values that you measured. Confirm whether the given resistors' tolerance by the manufacturer matches your calculations.

**Part(2)** Make another table with rows labeled "TEAM 1", "TEAM 2", etc. and record the values of mean, standard deviation, and variance for each resistor group determined by other teams (Part 1) in Tables 10.2 and 10.3. Why do the values differ?

	GR	OUP A	1	GROUP B		GROUP C		GROUP D		)		
	Mean	VAR	SD	Mean	VAR	SD	Mean	VAR	SD	Mean	VAR	SD
TEAM 1												
TEAM 2												
TEAM 3												
TEAM 4												

Table 10.2: Values from other Lab Teams

Now use the "TOTAL" values from each team to calculate the total mean, standard deviation, and variance. Record your data in Table 10.3.

	TOTAL						
	Mean VAR SE						
TEAM 1		-	-				
TEAM $2$		-	-				
TEAM 3		-	-				
TEAM 4		-	-				
TOTAL							

Table 10.3: Total Mean, SD and Variance

**Part(3)** Using the information in Parts 1 and 2, what would be the best estimate of the resistance of each group? What would be the best estimate of the resistance for all groups of resistors combined? Why?

## **Probing Further Questions**

Q1. Is it reasonable to assume the resistors have a normal distribution? Why?

**Q2.** How does the mean value of resistance for the total, obtained by your lab team, compare to the nominal value of resistance marked on the resistor's body?

**Q3.** How does the standard deviation obtained by your team compare to the tolerance marked on the resistor case?

## Lab 11 - Design Lab

### Introduction

In this lab you will design and build a circuit to meet certain criteria. In calling for a circuit design, two fundamental questions must be answered: What function is the circuit to perform? How well is the circuit expected to perform this function? The detailed answers to these questions are usually called the *functional requirement* and the *specifications*, respectively.

### Objective

The objective of this laboratory exercise is to introduce you to the nature of the engineering design process, i.e., the process of selecting combinations of components to perform a given function with a given degree of precision.

### PreLab

Review the material in your circuits textbook on voltage dividers. Before you come to lab, you should have your circuit design completed, with the design procedure and the resulting circuit recorded in your laboratory notebook. This is not a team exercise. Each individual will be required to have a design completed in the notebook prior to coming to lab class.

## Equipment

You specify.

## Procedure

**Part(0)** A voltage divider is a simple circuit which turns a large voltage into a smaller one. Using just two series resistors and an input voltage, we can create an output voltage that is a fraction of the input. You are to design a voltage divider to maintain a certain voltage across a variable load resistance,  $R_{Load}$ , which may vary anywhere between 1000 $\Omega$  to 1500 $\Omega$ . The supply voltage source is to be 5.00V.



Figure 11.1: Voltage Divider Circuit

**Part(1)** Determine the values of  $R_1$  and  $R_2$  to obtain no more than +5% variation about 3.00V ( $3V \le V_{out} \le 3.15V$ ) as the value of  $R_{Load}$  ranges from 1000 $\Omega$  to 1500 $\Omega$ . Use **LTspice** to verify your design for lower and upper extremes of  $R_{Load}$ .

Table 11.1: Determined Values for Part (1)

**Part(2)** Find values of  $R_1$  and  $R_2$  according to the specified conditions in Table 11.2 such that no more than  $\pm 7\%$  variation about 3.00V (2.79V  $\leq V_{out} \leq 3.21$ V) is obtained as the value of  $R_{Load}$  ranges from 1000 $\Omega$  to 1500 $\Omega$ . Additionally, calculate the **power delivered by the source** for selected  $R_1$  and  $R_2$  values when  $\mathbf{R}_{Load} = \mathbf{1500}\Omega$  (find  $i_1$  and use P = VI). Use **LTspice** to verify your design.

Table 11.2: Determined Values for Part (2)
--

Condition	$R_1(\Omega)$	$R_2(\Omega)$	$V_{out}$ @R <sub>1000Ω</sub>	$V_{out}$ @R <sub>1500Ω</sub>	$P_{source}(mW)$
$1\Omega \le R_1, R_2 \le 100\Omega$					
$1\Omega \leq R_1 \leq 100\Omega$ , $100\Omega \leq R_2 \leq 500\Omega$					
$1\Omega \leq R_1 \leq 250\Omega$ , $500\Omega \leq R_2 \leq 1k\Omega$					
$300\Omega \le R_1 \le 400\Omega$ , $500\Omega \le R_2 \le 1.5k\Omega$					
$450\Omega \le R_1 \le 1k\Omega$ , $800\Omega \le R_2 \le 2k\Omega$					

**Part(3)** Repeat Part 2 of this lab but  $R_1$  and  $R_2$  are to be selected from the Standard Resistor Values table. Use **LTspice** to verify your design.

Table 11.3: Determined Values for Part (3)

Condition	$R_1(\Omega)$	$R_2(\Omega)$	$V_{out}@R_{1000\Omega}$	$V_{out}@R_{1500\Omega}$	$P_{source}(mW)$
$1\Omega \le R_1, R_2 \le 100\Omega$					
$1\Omega \leq R_1 \leq 100\Omega$ , $100\Omega \leq R_2 \leq 500\Omega$					
$1\Omega \leq R_1 \leq 250\Omega$ , $500\Omega \leq R_2 \leq 1k\Omega$					
$300\Omega \leq R_1 \leq 400\Omega$ , $500\Omega \leq R_2 \leq 1.5k\Omega$					
$450\Omega \le R_1 \le 1k\Omega$ , $800\Omega \le R_2 \le 2k\Omega$					

Table 11.4: Standard Resistor Values

1.0	10	100	1.0K	10K	100K	1.0M
1.1	11	110	1.1K	11K	110K	1.1M
1.2	12	120	1.2K	12K	120K	1.2M
1.3	13	130	1.3K	13K	130K	1.3M
1.5	15	150	1.5K	15K	150K	1.5M
1.6	16	160	1.6K	16K	160K	1.6M
1.8	18	180	1.8K	18K	180K	1.8M
2.0	20	200	2.0K	20K	200K	2.0M
2.2	22	220	2.2K	22K	220K	2.2M
2.4	24	240	2.4K	24K	240K	2.4M
2.7	27	270	2.7K	27K	270K	2.7M
3.0	30	300	3.0K	30K	300K	3.0M
3.3	33	330	3.3K	33K	330K	3.3M
3.6	36	360	3.6K	36K	360K	3.6M
3.9	39	390	3.9K	39K	390K	3.9M
4.3	43	430	4.3K	43K	430K	4.3M
4.7	47	470	4.7K	47K	470K	4.7M
5.1	51	510	5.1K	51K	510K	5.1M
5.6	56	560	5.6K	56K	560K	5.6M
6.2	62	620	6.2K	62K	620K	6.2M
6.8	68	680	6.8K	68K	680K	6.8M
7.5	75	750	7.5K	75 <b>K</b>	750K	7.5M
8.2	82	820	8.2K	82K	820K	8.2M
9.1	91	910	9.1K	91K	910K	9.1M

**Part(4)** You are to select  $R_1$  and  $R_2$  from Table 11.3 to build your voltage divider circuit. Which combination will you choose? Why? If the power rating of these resistor is 1/4 Watt, could they handle the current that goes through them for extreme values of  $R_{load}$ ?

## **Probing Further Questions**

**Q1.** We assumed that  $R_1$  and  $R_2$  are ideal (i.e. 0% tolerance) in all parts of this laboratory. However, if only 5% tolerance resistors are available, what difference would various combinations make in the performance if you use the  $R_1$  and  $R_2$  values from Part 4? The extreme values of  $R_1$  and  $R_2$  in this case are:

$$0.95R_1 \le R_1 \le 1.05R_1$$
  
 $0.95R_2 \le R_2 \le 1.05R_2$ 

You need to check  $V_{out}$  for four different combinations of extreme values. You may use LTspice for this purpose.

## Lab 12 - Final Exam

### Introduction

This has been your first engineering laboratory. Although you have been provided with a "cookbook" (this manual), hopefully you have not just blindly followed instructions in order to get a good grade. If you have, you have cheated only yourself. The theory which you have learned from your textbook and lectures is a way of looking at reality and thinking about how to organize experience, i.e., dealing with real, physical things. It is only through combining practical experience with theory that you can begin to develop the necessary analytical skills to aid you in "taking things apart and putting them together in new ways" that is the essence of the practice of real engineering.

#### Objective

This examination is designed to help you and your lab TA determine how much you have developed your knowledge, skills, and self confidence.

### PreLab

Review your lab manual. Think carefully of the procedures which you have followed and what you have learned from them. How do you calculate KVL, KCL, Power, etc.? How do you measure voltage, current, resistance, frequency, etc.? How accurate are your measurements? When your measurements do not agree with those of your peers, what is the cause? How do your measuring instruments affect the circuit in which you are trying to make measurements?

## Equipment

- A Single Page Formula Sheet
- Calculator
- AD2 Device and Bread Board
- Laptop

## Procedure

The final exam will consist of a written exam and a practical exam. The practical exam involves performing a lab procedure to obtain a desired result. The written exam will cover the application of theory to understand circuit behavior, based upon your laboratory experiences in this course.

## **Probing Further Questions**

**Q1.** This has been your first in a series of four (4) laboratory courses. What have you done to develop your expertise and skills? What will you do differently in the next course? What have you learned from your experience?

# Appendix A - Safety

Electricity, when improperly used, is very dangerous to people and to equipment. This is especially true in an industrial environment where large amounts of power is used, and where high voltages are present [1]; in environments where people are especially susceptible to electric shock such as maintenance of a high voltage system (while in operation) or in hospitals where electrical equipment is used to test or control physiological functions [2, 3]; and in an experimental or teaching laboratory where inexperienced personnel may use electrical equipment in experimental or nonstandard configuration.

Engineers play a vital role in eliminating or alleviating the danger in all three types of environments mentioned above. For conditions where standard equipment is used in standard configurations, governmental agencies and insurance underwriters impose strict laws and regulations on the operation and use of electrical equipment including switchgear, power lines, safety devices, etc. As a result, corporations and other organizations in turn impose strict rules and methods of operation on their employees and contractors. Engineers who are involved in using electrical equipment, in supervising others who use it, and in designing such systems, have a great responsibility to learn safety rules and practices, to observe them, and to see that a safe environment is maintained for those they supervise. In any working environment there is always pressure to "get the job done" and take short cuts. The engineer, as one who is capable of recognizing hazardous conditions, is in a responsible position both as an engineer and as a supervisor or manager and must maintain conditions to protect personnel and avoid damage to equipment.

Because of their non-standard activities, experimental laboratories are exempt from many of these rules and regulations. This puts more responsibility on the engineer in this environment to know and enforce the safest working procedures.

The knowledge and habit-forming experience to work safely around electrical equipment and the ability to design safe electrical equipment begins with the first student laboratory experience and continues through life. This includes learning the types of electrical injuries and damage, how they can be prevented, the physiology of electrical injuries, and steps to take when accidents.

### **Physiology of Electrical Injuries**

There are three main types of electrical injuries: electrical shock, electrical burns, and falls caused by electrical shock. A fourth type, 'sunburned' eyes from looking at electric arcs, such as arc-welding, is very painful and may cause loss of work time but is usually of a temporary nature. Other injuries may be indirectly caused by electrical accidents, e.g., burns from exploding oil-immersed switch gear or transformers.

Although electric shock is normally associated with high-voltage AC contact, under some circumstances death can occur from voltages from substantially less than the nominal 120 Volts AC found in residential systems. Electric shock is caused by an electric current passing through a part of the human body. The human body normally has a high resistance to electric currents so that a high voltage is usually required to cause lethal currents. This resistance is almost all in the skin, but when the skin is wet its resistance is much lower. When a person is hot and sweaty or is standing in water, contact with 120 Volts or less is likely to cause a fatal shock.

Electric shock is not a single phenomenon but is a disturbance of the nerves that is caused by electric current. A current through a part of the body such as the arm or leg will cause pain and muscle contraction. If a victim receives an electric shock from grasping a live conductor, a current of greater than 15 to 30 mA through the arm will cause muscle contractions so severe that the victim cannot let go. Similar currents through leg muscles may cause sudden contractions causing the victim to jump or fall, resulting in possible injuries or death. It is also possible for a prolonged period of contact of more than a minute or so to cause chest muscles to be contracted, preventing breathing and resulting in suffocation or brain damage from lack of oxygen.

The predominant cause of death by electric shock is generally attributed to ventricular fibrillation, which is an uncontrolled twitching or beating of the heart that produces no pumping action and therefore no blood circulation. Unless corrective action is taken, death follows quickly from lack of oxygen to the brain. While the amount of current that will cause fibrillation depends on several variables, 0.5 to 5A through the body will normally cause the very small current through the heart that causes fibrillation in most people. Larger currents than this through the heart causes contraction or clamping of the heart muscle and resulting death unless corrective action is taken.

Burns from electric currents may be caused by electric currents flowing in or near parts of the body. Such burns, if caused by high frequency sources, are deeper than burns from other sources and take longer to heal but otherwise are not different from normal heat burns.

#### Source of Electric Shock

Since electric shock is caused by an electric current through a part of the body, it is prevented by not allowing the body to become part of any electric circuit. From this viewpoint, electric circuits may be classified as either grounded or ungrounded.

Grounded circuits are safer for most conditions, since they result in known voltages at other points in the circuit and provide easier and better protection against faulty conditions in the circuit. The disadvantage is that a person standing on a non-insulated floor can receive a shock by touching only one conductor.

Almost all electric power generation, transmission, and distribution systems are grounded to protect people and equipment against fall conditions caused by windstorms, lightning, etc. Residential, commercial, and industrial systems such as lighting and heating are always grounded for greater safety. Communication, computer, and similar systems are grounded for safety reasons and to prevent or reduce noise, crosstalk, static, etc. Many electronic equipment or instruments are grounded for safety and noise prevention, also. Common examples are DC power supplies, oscilloscopes, oscillators, and analog and digital multimeters.

Ungrounded circuits are used in systems where isolation from other systems is necessary, where low voltages and low power are used, and in other instances where obtaining a ground connection is difficult or impractical. In the ungrounded circuit, contact with two points in the circuit that are at different potentials is required to produce an electrical shock. The difficulty is that with no known ground, a hidden fault can occur, causing some unknown point to be grounded. Thus, touching a supposedly safe conductor while standing on the ground would result in an electric shock.

### Prevention of Shock in the Laboratory

Prevention of electric shock to individuals and damage to equipment in the laboratory can be done by strict adherence to several common-sense rules summarized below:

- 1. When hooking up a circuit, connect to the power source last, while power is off.
- 2. Before making changes in a circuit, turn off or disconnect the power first, if possible.
- 3. Never work alone where the potential of electric shock exists.
- 4. When changing an energized connection, use only one hand. Never touch two points in the circuit that are at different potentials.
- 5. Know that the circuit and connections are correct before applying power to the circuit.
- 6. Avoid touching capacitors that may have a residual charge. The stored energy can cause a severe shock even after a long period of time.
- 7. Insulate yourself from ground by standing on an insulating mat where available.

### After Accident Action

Since accidents do happen despite all efforts to prevent them, plans for appropriate reaction to an accident can save time and lives. Such a plan should include immediate availability of first aid material suitable for minor injuries or for injuries that are likely because of the nature of the work. Knowledge of how to obtain trained assistance such as Emergency Medical Services (EMS) should be readily available for everyone.

Treating victims for electrical shock includes four basic steps that should be taken immediately. Step two requires qualification in CPR and step three requires knowledge of mouth-to-mouth resuscitation. Everyone who works around voltages that can cause dangerous electrical shock should take advantage of the many opportunities available to become qualified in CPR and artificial respiration.

## Immediate Steps After Electric Shock

- 1. Shut off all power and remove victim from the electric circuit. If the power cannot be shut off immediately, use an insulator of some sort, such as a wooden pole, to remove victim from the circuit. Attempts to pull the victim from the circuit with your hands will almost always result in your joining the victim in the electric shock.
- 2. If you are qualified in CPR, check for ventricular fibrillation or cardiac arrest. If either is detected, external cardiac massage should be started at once. Whether you are qualified in CPR or not, notify EMS and the ECE Department at once, using the telephone numbers listed below.
- 3. Check for respiratory failure and take appropriate action. This may have resulted from physical paralysis of respiratory muscles or from a head injury. Sometimes many hours pass before normal respiration returns. Artificial respiration should be continued until trained EMS assistance arrives.
- 4. Check for and treat other injuries such as fractures from a fall or burns from current entry and exit sites. Investigations are always after accidents. As an engineer you will be involved as a part of the investigating team or in providing information to an investigator. Information obtained and notes written immediately after the emergency will aid this investigation and assist in preventing future accidents of a similar nature.

## **Emergency Numbers**

Fire / EMS: 911 or (864) 656-2222 Student Health Center: (864) 656-2233 ECE Department Office: (864) 656-5650

## Appendix A References

[1] W.F. Cooper, *Electrical Safety Engineering*, Newnes-Butterworth, London-Boston, 1978.

[2] W.H. Buschsbaum and B. Goldsmith, *Electrical Safety in the Hospital, Medical Economics Company*, Oradel, NJ, 1975.

[3] J.G. Wester, editor, *Medical Instrumentation Application and Design*, Houghton Mifflin Company, Boston, 1978.

# Appendix B - Fundamentals of Electrical Measurements

Electrical engineers make and use a wide variety of measurements of electrical circuit variables, such as voltage, current, frequency, power, and energy, as well as of electrical circuit parameters such as resistance, capacitance, and inductance. Even though many instruments which can be used to make such measurements are commercially available, the proper use and interpretation of these measurements depend on a fundamental understanding of how these measuring instruments work, their capabilities, and their limitations

This appendix provides a brief overview of the fundamentals of electrical measurements. There are many textbooks on the subject. See References [B-1] through [B-6] of this appendix, for example. As your exposure to various types of electrical measuring instruments increases in this and subsequent laboratory courses, you will find some of these books, or similar books that you can find, useful in developing your understanding and measurement skills. In addition, many commercial instrument manufacturers publish handbooks and applications notes that provide more information on specific measurement techniques.

### Measurement of Current and Voltage

The basic electrical circuit variables of current and voltage are measured with **ammeters** (for current) and **voltmeters** (for voltage). These instruments may use either **analog** (or continuous) or **digital** (or numerical) indicators (or "readouts").

#### **Analog Meter Instruments**

The analog instruments were those developed early in the history of electrical science and technology. Most are based on the d'Arsonval galvanometer movement. A brief description of this meter movement and its use in ammeters and voltmeters is given in the textbook ELECTRIC CIRCUITS, Fifth Edition by J.W. Nilsson [7]. More information is available in References [1, 2]. In the d'Arsonval galvanometer, current through a coil of fine wire develops a magnetic field which opposes that of a permanent magnet to rotate a needle across a scale which is calibrated (marked off) in units of the measured variable.

#### **Current Measurement**

The basic meter movement can be modeled as a resistance,  $R_M$ , which represents the resistance of the fine wire coil, in series with an ideal (zero resistance) meter movement, M, as shown in Figure 14.1. The important parameters for this meter are the full-scale current,  $I_{MFS}$ , and  $R_M$ . The full-scale current is the value of current that rotates the indicating needle through its maximum arc. Note that a larger current may damage the meter movement permanently, rendering it useless.



Figure 14.1: Electrical Circuit Model for d'Arsonval Meter Movement

The voltage drop across the meter movement will be,

$$V_M = IR_M \tag{14.1}$$

and the maximum rated voltage drop for the meter will be,

$$V_{MFS} = I_{MFS} R_M \tag{14.2}$$

When it is necessary to measure currents larger than  $I_{MFS}$ , a resistor with a value of resistance,  $R_{sh}$ , smaller than that of the meter,  $R_M$ , is placed in parallel with the meter movement. The parallel resistor is usually called the shunt resistor. Figure 14.2 shows a circuit diagram for this arrangement.



Figure 14.2: Electrical Circuit Model for d'Arsonval Meter Movement with a Shunt Resistance

From Figure 14.2, it is obvious that,

$$I = I_M + I_{sh} = \frac{V_M}{R_M} + \frac{V_M}{R_{sh}}$$
(14.3)

and that the full scale current  $I_{FS}$  for this metering arrangement will be,

$$I_{FS} = I_{MFS} (1 + \frac{R_M}{R_{sh}})$$
(14.4)

Below, in Figure 14.3 is the circuit diagram symbol for an ammeter compared to the equivalent circuit model for the analog ammeter which uses a d'Arsonval galvanometer.



Figure 14.3: Electrical Circuit Model for Ammeter using a d'Arsonval Meter Movement with a Shunt Resistance and the Circuit Diagram Symbol for the Ammeter

**Example 14.1:** Consider a d'Arsonval meter movement with the following characteristics:

$$I_{MFS} = 50 \ \mu A$$
  

$$R_M = 5 \ k\Omega$$
  

$$V_{MFS} = 250 \ mV$$

Find the value of shunt resistance that will enable the measurement of up to 1mA of current.

From Equation 14.4,

$$1 + \frac{R_M}{R_{sh}} = \frac{I_{FS}}{I_{MFS}} = \frac{1mA}{50\mu A} = 20$$

therefore,

$$R_{sh} = \frac{5000}{19} = 263.16\Omega$$

**Exercise 14.1:** What value of  $R_{sh}$  is required to make a meter that would measure 100mA full scale?
#### Voltage Measurement

A d'Arsonval galvanometer movement can also be used to make a voltmeter. Instead of placing a resistor in shunt, a resistor is placed is series with the meter movement. Figure 14.4 shows the equivalent circuit for such an arrangement, connected in series with a voltage source,  $V_X$ , which has an internal resistance,  $R_X$ , associated with it. Also shown is the circuit symbol used to indicate the presence of such a voltmeter in a circuit.



Figure 14.4: Electrical Circuit Model for Voltmeter using a d'Arsonval Galvanometer with a Series Resistance and the Circuit Symbol Used for the Voltmeter

The maximum voltage,  $V_{FS}$ , that can be applied will depend upon the size of the series resistor,  $R_S$ . By applying Kirchhoff's voltage law to the circuit of Figure 14.4 we have,

$$V_{FS} = I_{MFS}(R_S + R_M + R_X)$$
(14.5)

**Example 14.2:** Using the meter movement previously used, suppose that it is desired to make a voltmeter with a 100V full-scale rating.

From Equation 14.5,

$$R_S = \frac{V_{FS}}{I_{MFS}} - R_M - R_X$$
$$R_S = \frac{100}{50.10^{-6}} - 5000 - R_X$$

and,

$$R_S = 1.995M\Omega - R_X$$

Often  $R_X$  is not precisely known, or is inconvenient to determine. If  $R_X$  is small enough compared to  $R_S$  to be considered negligible, it will have a small effect on the accuracy of the voltage measurement. If not, the "loading effect" of the voltmeter on the voltage source must be taken into account. A procedure for doing this will be used in Laboratory 3.

**Exercise 14.2:** Assuming that  $R_X$  is small, say 100, what value of  $R_S$  would be required to make a voltmeter with 10V full scale rating using the same meter movement as in the example above?

#### Analog Electronic Instruments

From the circuit diagram of Figure 14.4 it can be seen that if the unknown voltage source,  $V_X$ , has an internal resistance associated with it that approaches or exceeds that of the voltmeter circuit, the reading will be in error due to the voltage drop across the internal resistance of the source. The analog electronic instruments were developed to provide a high input impedance for the voltmeter in order to minimize such a "loading effect" of the measuring instrument on the circuit being measured.

The early versions of such electronic voltmeters used vacuum tube amplifiers and were consequently known as "vacuum tube voltmeters" or VTVMs. More recent instruments use the high impedance offered by field-effect transistors in operational amplifiers (FET-input op-amps). The output voltage of the op-amp can then be used to drive a dArsonval galvanometer to provide a continuous (analog) indication. However, the advent of digital circuits and indicating devices has enabled the development of digital meters, which will be briefly reviewed in the next section.

#### **Digital Instruments**

Figure .5 shows a block diagram of an electronic ammeter with a digital readout. Figure 14.5 shows a block diagram of an electronic voltmeter with a digital readout. Except for the input circuitry, both instruments are similar in design and construction.



Figure 14.5: Block Diagram of Electronic Ammeter with a Digital Display Readout



Figure 14.6: Block Diagram of Electronic Voltmeter with a Digital Display Readout

The input module of the digital electronic ammeter in Figure .5 and that of the digital electronic voltmeter in Figure 14.6 is usually built around an operational amplifier and other transistor devices. You will learn more about these in ECE 320 and ECE 321. The A/D (spoken as "A to D") converter and other digital circuit devices you will learn more about in ECE 371.

The input resistance of the digital electronic ammeter is designed to be as low as possible (ideally zero), whereas the input resistance of the digital electronic voltmeter is designed to be as high as possible (ideally infinite).

### How Good Are Your Measurements?

In engineering practice, although qualitative words like "good" and "bad" are used, more often than not quantitative descriptions are used, i.e. how good or bad, in numerical terms. When making measurements, three attributes are of interest: **accuracy**, **precision**, and **resolution**.

Accuracy is usually expressed as a percentage, and represents the difference, in percent, of the readout value of the measuring instrument compared to the actual, or true, value. Usually the **accuracy specification** of an instrument is expressed as a percentage of the **full-scale** value on a given range. For example, if you are using a voltmeter with an accuracy of 2% of full scale on the 0-10V range and you obtain a scale reading of 4.8V, you would expect the actual voltage to be within the range of  $4.8 \pm 0.2$ V.

For digital display instruments, the accuracy specification is usually given as a percentage of full scale on the range being used **plus** one digit (the lowest-order digit). Thus, for example, if a digital voltmeter with 4 digits is being used on the 020V range and the accuracy specification is (0.1% + 1 digit), a reading of 13.42V will indicate an actual input voltage in the range 13.42  $\pm$  0.03V.

**Precision** is used to indicate how closely the readout can be read. For example, an ammeter using a d'Arsonval meter movement for readout with a scale calibrated in units of 0.2 of each major subdivision can provide an estimate down to about 0.1 of a major subdivision. That is, an ammeter with a scale subdivided into 10 major subdivisions (0.1 mA per major subdivision) with these major subdivisions further partitioned into 5 minor subdivisions (of 0.02 mA each) might allow and estimate of 0.01 mA (or 10  $\mu$ ) if the indicating needle is backed with a mirror that allows the observer to carefully align the needle with the scale.

For a digital readout, the precision is determined by the number of digits available in the display. For example, a readout that has 4 digits on a voltmeter being used on the 0-20V range would have a precision of 0.01V and on the 0-200V range would have a precision of 0.1V. Note that the **precision** of

an instrument will usually be better than its **accuracy**.

**Resolution** is used to indicate the smallest change of the input variable to which the instrument (sensibly, through its readout) can respond.

Just because the original specifications for a given instrument state a given accuracy doesn't mean that the instrument is guaranteed to retain that accuracy indefinitely. Measuring instruments, like all manufactured items, are subject to "wear and tear". An instrument which is being used within its ratings i.e. hasn't been abused electrically or mechanically (like being dropped on a workbench or onto the floor) still requires periodic maintenance and recalibration. The procedure for recalibration is usually specified in the instruction manual for the instrument, provided by the manufacturer. It is the usual (good) practice for the person performing the recalibration to place a "sticker" on the outside of the instrument giving the date of servicing and the name (or initials) of the person responsible. In industrial plants, where accurate measurements are important, the calibration procedures are usually performed in an instrument shop which is equipped with "secondary standards" of voltage, current, resistance, etc. traceable to the National Institute of Standards and Technology (NIST formerly known as NBS, the National Bureau of Standards).

When using an instrument, refer to the instruction manual for correct usage and for specifications on the accuracy and resolution of the instrument. Also check to see what the last date of calibration was. The longer the interval between calibration and your date of using the instrument, the more suspect is the accuracy of your measurements. It is good practice to have a "known" voltage source (such as a "fresh" 1.5V battery) to quickly check a voltmeter or current source (such as a 1.5V battery in series with a 1.5 k resistor to provide 1 mA) to obtain a quick check of an ammeter. If several instruments are available, they can be used with the same source to obtain a comparison of readings. The best rule of measurement is "check it out before you believe the readings".

## How Good Are Their Measurements Compared To Mine?

Often the measurements made by two engineers, or two different groups of engineers, under ostensibly similar conditions, using ostensibly similar sets of instruments and equipment will result in different values, sometimes what appear to be significantly different value. The consequences may range from a "friendly dispute" to lawsuits.

Various engineering organizations, such as the American Society for Testing Materials (ASTM), go to great lengths to standardize measurement procedures and then to compare the results obtained by various people in different organizations using these procedures. One method of doing this is the so-called "round-robin" method.

In the "round-robin" method, a set of objects to be measured for example, a set of standard voltage cells will be sent from laboratory to laboratory for measurements to be made. Then each group making the measurements will report them to a central group, the referee, for comparison. Following a statistical analysis of the individual and pooled measurements, the results are reported back to all of the participants. This method helps to establish the practical limits on repeatability of the measurements made using the procedure.

In general, there are many types of errors that can interfere with an accurate measurement of a

variable such as a voltage value. These can be partitioned [1,2] into three categories: human errors, systematic errors, and random errors.

Human errors are caused by a loss of concentration, ignorance, or laziness on the part of the human being making and/or recording the measurement(s). Such errors include: choosing the wrong scale or even instrument, misreading the scale, transposing numbers when recording, etc. For this reason, it is helpful to have one or more lab partners observing or repeating the measurements in order to catch such errors before they become a problem.

Systematic errors may be due to equipment (instrument) errors or to environmental conditions. An example of the instrument error is the use of an instrument that is out of calibration, or one for which the zero has not been properly adjusted. Many instruments are temperature sensitive, and will give erroneous readings when used out of their specified ambient temperature range. They may also be sensitive to the presence of large magnetic fields or sensitive to high humidity. In some cases, the effects of environmental conditions may be accounted for if known, but quite often either an accurate measurement of actual instrument temperature or its local humidity are not known, so correction is not possible.

**Random errors** are those errors which are due to unknown causes and which are operative when the human errors and the systematic errors have been brought under control or otherwise accounted for. These random errors can be minimized or accounted for by the use of multiple measurements and statistical analysis. Statistical analysis terminology and methodology are discussed in Appendix C of this manual.

### Appendix B References

[1] F. Spitzer and B. Howard, *Principles of Modern Instrumentation*. New York: Holt, Rinehart and Winston, 1972.

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[3] S. Wolf. *Guide to Electronic Instrument and Laboratory Practice*. Englewood Cliffs, NJ: Prentice-Hall. 1973.

[4] H.V. Malmstadt, C.G. Enke, S.R. Crouch, and G. Horlick. *Electronic Measurement for Scientists*. Menlo Park, CA: W.A. Benjamin, Inc. 1974.

[5] H.H. Chiang. *Electrical and Electronic Instrumentation*. New York, NY: John Wiley and Sons. 1984.

[6] F.F. Mazda. *Electronic Instruments and Measurement Techniques*. Cambridge (U.K.): Cambridge University Press, 1987.

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# Appendix C - Fundamentals of Statistical Analysis

When the human errors and systematic errors associated with a measurement procedure have been eliminated or brought under control, there will still be some variability in the measurements when they are repeated, or when some other person, presumably equally skilled, makes the measurement. These are called random errors. They are characterized by irregularity, which may appear to be a source of potential disagreement or confusion. However, the use of statistical analysis methods makes possible the extraction of essential information. Two characteristics of the measurements are of special interest: the average (arithmetic mean) value of a series of measurements and some measure of the spread, or dispersion, i.e., the degree of variability about the mean.

Consider a series of measurements:  $x_1, x_2, x_3, ..., x_n$  of some quantity. For example, 6 measurements of a voltage source over a period of an hour, repeated for 5 consecutive hours. Table 15.1 provides values for each of these measurements.

Measurement .	Hour 1	Hour 2	Hour 3	Hour 4	Hour 5
1	5.043	5.048	5.040	5.044	5.047
2	5.038	5.043	5.044	5.044	5.041
3	5.036	5.046	5.042	5.039	5.040
4	5.035	5.042	5.039	5.036	5.042
5	5.040	5.040	5.043	5.045	5.037
6	5.043	5.038	5.041	5.400	5.039

Table 15.1: Voltage Measurements of a Source Over a 5-Hour Interval

For each sample of n measurements one can calculate an average or arithmetic mean value from,

$$x_{avg} = \frac{\sum_{j=1}^{n} x_j}{n}$$
(15.1)

and a variance,

$$S^{2} = \frac{\sum_{j=1}^{n} (x_{j} - x_{avg})^{2}}{n-1}$$
(15.2)

More often, the square root of the variance, called the **standard deviation**, s, is used rather than the variance. As can be seen from Equation 15.2, the variance is the **mean-square deviation** of each measurement from the average value. Therefore, the standard deviation is the **root-mean- square deviation** of the set of measurements.

Equations 15.1 and 15.2 have been used to calculate the average voltage, variance, and standard deviation for each of the sets of voltage measurements in Table 15.1. The results are shown in Table 15.2.

Parameter	Hour 1	Hour 2	Hour 3	Hour 4	Hour 5
$V_{av}$	5.039	5.043	5.042	5.041	5.041
s <sup>2</sup>	1.18•10-5	1.38•10 <sup>-5</sup>	0.35•10 <sup>-5</sup>	1.27•10 <sup>-5</sup>	1.16•10 <sup>-5</sup>
S	0.0034	0.0037	0.0019	0.0036	0.0034
	All Hours				
$(V_{av})_{av}$	5.041				
(S)av	0.0015				

Table 15.2: Statistics for Voltage Measurements of a Source Over a 5-hour Interval

Also calculated and shown in Table 15.2 is the average of the sample averages and the standard deviation of these averages. It can be seen that the sample averages are more tightly grouped about the average of the averages (or the "grand mean") than are the individual measurements grouped about their respective sample mean values.

Why would one go through such calculations? What is the interpretation of the results? From the original data tabulations in Table 15.1 it can be seen that the maximum voltage measured was 5.047V and the minimum was 5.035V. This corresponds to a difference of 12mV, which compared to the grand mean value of 5.041V is only 0.24% for the range (Max-Min) and only  $\pm 0.12\%$  about the grand mean. On a percentage basis, these results look very good. However, there are situations when the ability to resolve a few millivolts might be important, so it is important to characterize the ability of the voltmeter to make precise measurements. Another item of interest is to estimate how much of a deviation might be expected over a longer period of time

Again looking at Table 15.1, it can be seen that over any of the one hour periods the readings fluctuated up and down around the grand mean value. Only twice in a 5-hour period was the grand mean value obtained (2 out of 30 readings). This wandering back and forth is characteristic of the variation expected from the random interplay of various, uncontrolled, physical factors. The average value of a measurement might be thought about as "information" and the random fluctuations as "noise" that interferes (in this case rather slightly) with the process of obtaining the information. We only took 5 samples (one per hour) of 6 readings each. What would we expect from, say, 20 readings per hour taken over the period of a month? Or perhaps, through automation, 1200 readings per hour, taken over a month. What information would we gain by greatly increasing the number of observations, and of course, greatly increasing the work of analyzing the data? In other words, how good are our inferences made from a few samples about the "true" values of mean and variance (or standard deviation)?

The theory of measurement errors and their determination is based on the following observations [1]:

- 1. The most probable value that can be assigned to a measured magnitude, on the basis of equally trustworthy direct measurements, is the arithmetic mean.
- 2. In any large number of measurements positive and negative errors of the same magnitude are equally likely to occur.
- 3. Small errors are much more likely to occur than large ones.
- 4. All of the errors of measurement in a given series lie between equal positive and negative limits.

These observations have led to the use of the **normal distribution** function to model the expected error distribution of a large (approaching infinity) series of measurements of the same kind. This large (infinite) reference group is called the **population**. The average value of the population (of measurements) is given the symbol  $\mu$ . Its variance is symbolized as  $\sigma^2$  and its standard deviation is  $\sigma$ . In terms of these parameters, the normal distribution function is,

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(x-\mu)^2}{2\sigma^2}}$$
(15.3)

Figure 15.1 shows a plot of this distribution function in terms of a **normalized variable**.

$$z = \frac{x - \mu}{\sigma} \tag{15.4}$$

or,

$$f(z) = \frac{1}{\sqrt{2\pi}} e^{\frac{-z^2}{2}}$$
(15.5)

to give a "universal curve". A related curve is that of the integral of Equation 15.5, or,

$$f(z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{z} e^{\frac{-u^2}{2}} du$$
 (15.6)



Figure 15.1: Normal Distribution Function Versus z



Figure 15.2: Cumulative Normal Distribution Function Versus z

The integral in Equation 15.6 is known as the cumulative normal distribution function. Values of this integral for positive values of z (i.e. for  $x > \mu$ ) are given in Table 15.3. Values of the integral for z < 0 (i.e. for  $x < \mu$ ) are obtained from the relation,

$$P(-z) = 1 - P(z) \tag{15.7}$$

z (col + row)	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.50000	0.50399	0.50798	0.51197	0.51595	0.51994	0.52392	0.52790	0.53188	0.53586
0.1	0.53983	0.54380	0.54776	0.55172	0.55567	0.55962	0.56356	0.56749	0.57142	0.57535
0.2	0.57926	0.58317	0.58706	0.59095	0.59483	0.59871	0.60257	0.60642	0.61026	0.61409
0.3	0.61791	0.62172	0.62552	0.62930	0.63307	0.63683	0.64058	0.64431	0.64803	0.65173
0.4	0.65542	0.65910	0.66276	0.66640	0.67003	0.67364	0.67724	0.68082	0.68439	0.68793
0.5	0.69146	0.69497	0.69847	0.70194	0.70540	0.70884	0.71226	0.71566	0.71904	0.72240
0.6	0.72575	0.72907	0.73237	0.73565	0.73891	0.74215	0.74537	0.74857	0.75175	0.75490
0.7	0.75804	0.76115	0.76424	0.76730	0.77035	0.77337	0.77637	0.77935	0.78230	0.78524
0.8	0.78814	0.79103	0.79389	0.79673	0.79955	0.80234	0.80511	0.80785	0.81057	0.81327
0.9	0.81594	0.81859	0.82121	0.82381	0.82639	0.82894	0.83147	0.83398	0.83646	0.83891
1.0	0.84134	0.84375	0.84614	0.84849	0.85083	0.85314	0.85543	0.85769	0.85993	0.86214
1.1	0.86433	0.86650	0.86864	0.87076	0.87286	0.87493	0.87698	0.87900	0.88100	0.88298
1.2	0.88493	0.88686	0.88877	0.89065	0.89251	0.89435	0.89617	0.89796	0.89973	0.90147
1.3	0.90320	0.90490	0.90658	0.90824	0.90988	0.91149	0.91309	0.91466	0.91621	0.91774
1.4	0.91924	0.92073	0.92220	0.92364	0.92507	0.92647	0.92785	0.92922	0.93056	0.93189
1.5	0.93319	0.93448	0.93574	0.93699	0.93822	0.93943	0.94062	0.94179	0.94295	0.94408
1.6	0.94520	0.94630	0.94738	0.94845	0.94950	0.95053	0.95154	0.95254	0.95352	0.95449
1.7	0.95543	0.95637	0.95728	0.95818	0.95907	0.95994	0.96080	0.96164	0.96246	0.96327
1.8	0.96407	0.96485	0.96562	0.96638	0.96712	0.96784	0.96856	0.96926	0.96995	0.97062
1.9	0.97128	0.97193	0.97257	0.97320	0.97381	0.97441	0.97500	0.97558	0.97615	0.97670
2.0	0.97725	0.97778	0.97831	0.97882	0.97932	0.97982	0.98030	0.98077	0.98124	0.98169
2.1	0.98214	0.98257	0.98300	0.98341	0.98382	0.98422	0.98461	0.98500	0.98537	0.98574
2.2	0.98610	0.98645	0.98679	0.98713	0.98745	0.98778	0.98809	0.98840	0.98870	0.98899
2.3	0.98928	0.98956	0.98983	0.99010	0.99036	0.99061	0.99086	0.99111	0.99134	0.99158
2.4	0.99180	0.99202	0.99224	0.99245	0.99266	0.99286	0.99305	0.99324	0.99343	0.99361
2.5	0.99379	0.99396	0.99413	0.99430	0.99446	0.99461	0.99477	0.99492	0.99506	0.99520
2.6	0.99534	0.99547	0.99560	0.99573	0.99585	0.99598	0.99609	0.99621	0.99632	0.99643
2.7	0.99653	0.99664	0.99674	0.99683	0.99693	0.99702	0.99711	0.99720	0.99728	0.99736
2.8	0.99744	0.99752	0.99760	0.99767	0.99774	0.99781	0.99788	0.99795	0.99801	0.99807
2.9	0.99813	0.99819	0.99825	0.99831	0.99836	0.99841	0.99846	0.99851	0.99856	0.99861
3.0	0.99865	0.99869	0.99874	0.99878	0.99882	0.99886	0.99889	0.99893	0.99896	0.99900

Table 15.3: Values of the cumulative distribution function, P(z), for positive values of z. The value of z is determined by z = row heading + column heading

A plot of the cumulative normal distribution function versus z is given in Figure 15.2. Properties of the normal distribution function and the cumulative normal distribution functions are discussed in [2], where there are numerous examples of the applications of these functions in statistical analysis.

From Equations 15.6 and 15.7, as well as the values of P(z) given in Table 15.3, it can be seen that the limit of P(z) as  $z \to \infty$  is 1.0000... Thus, P(z) is a function that varies smoothly from 0 (at  $\infty$ ) to 1.0000... (at  $+\infty$ ). It is interpreted as a measure of the **probability** that x, the measured variable, lies within a given range about the "true" value,  $\mu$ .

Let's explore more of this interpretation. Suppose we want to find the probability that a value x is within one standard deviation of the mean; that is, the probability that  $|x-\mu| < 1\sigma$ , or P(|z| < 1).

From Equation 15.6, we can write,

$$P(|z| < 1.00) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{1.00} e^{\frac{-u^2}{2}} du - \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{-1.00} e^{\frac{-u^2}{2}} du$$

which is simply,

$$P(|Z| < 1.00) = P(1.00) - P(-1.00)$$

From Equation 15.7,

$$P(|Z| < 1.00) = P(1.00) - (1 - P(1.00)) = 2P(1.00) - 1$$

Table 15.3 shows values of P(z) for  $z \ge 0$ . Looking at the values in Table 15.3, we find that at z = 1.00, P(z) = 0.84134. Therefore, the probability the |z| < 1, or equivalently, that  $|x - \mu| < 1\sigma$ , is:

$$P(|Z| < 1.00) = 2P(0.84134) - 1 = 0.68268$$

Thus, about 68% of the time, we expect the measured value x to lie within 1 of . By the same reasoning, we expect the measurement to lie within  $\pm 2\sigma$  about 95.5% of the time and within  $\pm 3\sigma$  about 99.7% of the time. Since we assume the difference between x and  $\mu$  is due to random error, the same probabilities also apply to the errors  $|x - \mu|$ .

This is all well and good, but when we really don't know  $\mu$  and  $\sigma$ , how can we make any predictions about the expected error of our readings? It happens that both of these statistical parameters that are sufficient to describe the population (from Equations 15.5 and 15.6) can be estimated from our samples. Of course, you realize that as we went from 6 samples/hour to 20 samples/hour to 1200 samples/hour to "infinity", we came into the all-encompassing knowledge of how to calculate, very precisely, the expected error. Thus, it would seem, the smaller our sample is, the less precise will be our estimate of the population statistical parameters  $\mu$  and  $\sigma$ .

It happens that the equations used to calculate the arithmetic mean, Equation 15.1, and the variance, Equation 15.2 give a "maximum likelihood estimator" of  $\mu$  and a "maximum likelihood estimator" of  $\sigma^2$ . How good are these estimates? It depends upon the number of measurements, n, used to obtain the estimates. The "goodness" of the estimate is usually expressed in terms of "confidence intervals". The details of doing this are discussed in [2] in Chapter 11, "Small Sampling Theory". Figure 15.3 shows the effect of sample size on the percentage error in the estimation of  $\mu$ . In this figure the difference between the estimate obtained by using Equation 15.1 and  $\mu$  is compared to the estimate of  $\sigma$  obtained by using Equation 15-2 to obtain a "universal curve":

$$y(n) = \frac{\delta}{s} = \frac{x_{av} - \mu}{s} \tag{15.8}$$

Thus, for a given value of s and sample size n, the value of the true mean  $\mu$  is expected to lie within the range  $x_{av} \pm y(n) * s$ . This expectation is expressed as probability. For the example shown in Figure 15.3, a probability of 90% has been selected. The resulting limiting values represented by the curves are called the "90% confidence limits" and may be interpreted as meaning that 9 times out of 10 the estimate of  $\mu$  lies within the limits shown.



Figure 15.3: 90% Confidence Limit, y(n), on the Error in Estimating  $\mu$  using  $x_a$  versus Sample Size n.

Figure 15.4 shows the effect of sample size on the error in the estimation of  $\sigma$  using the value of s calculated from Equation 15.2. Thus,  $\sigma$  lies between the limits,

$$s.r_l(n) < \sigma < s.r_u(n) \tag{15.9}$$

where  $r_l(n)$  is obtained from the lower curve of Figure 15.4 and  $r_u(n)$  is obtained from the upper curve of Figure 15.4. The curves shown in Figure 15.4 are calculated using the 90-percent confidence limits.



Figure 15.4: 90% Confidence Limits  $r_l(n)$  and  $r_u(n)$  on the Estimate of  $\sigma$  using s, Versus Sample Size, n

Let us now reexamine the data of Table 15.1. Since the hour-to-hour averages appear to behave randomly, we will combine all of the measurements to get a "pooled" sample size of 30 measurements.

For the pooled sample,  $V_{av} = 5.041$  and  $s^2 = (0.00326)^2$ . Using the curve in Figure 15.3, for n = 30, y(n) = 0.316 and the "population mean value" of the measurements,  $V_{\mu}$ , is expected, 9 times out of 10, to lie within the limits,

$$V_{av} - y(n).s < V_{\mu} < V_{av} + y(n).s$$

or,

$$5.040V < V_{\mu} < 5.042V$$

The standard deviation of measurements in the sample population is expected to lie in the range defined by Equation 15.9. Thus,

 $0.00274V < \sigma < 0.00424V$ 

It can also be seen that if we had used only 6 measurements (one hour's worth) for our calculations, the uncertainty in both the mean value and the standard deviation would have been much larger.

## Appendix C References

[1] A. de F. Palmer. *The Theory of Measurements*. New York, NY: McGraw-Hill Publishing Co. 1912. See Chapter IV.

[2] M. R. Spiegel. *Statistics, 2nd Edition.* New York, NY: Schaum's Outline Series (Mc- Grave-Hill), 1988.

# **Appendix D - Resistor Identification**

Color	Value	Multiplier	Tolerance
Black	0	1	-
Brown	1	10	-
Red	2	100	-
Orange	3	1,000	-
Yellow	4	10,000	-
Green	5	$10^{5}$	-
Blue	6	$10^{6}$	-
Violet	7	$10^{7}$	-
Gray	8	$10^{8}$	-
White	9	$10^{9}$	-
Gold	-	$10^{-1}$	5%
Silver	-	$10^{-2}$	10%
None	-	-	20%

#### Table 16.1: Resistor Color Code

Table 16.2: Standard Resistor Values

10	18	33	56
$11^{\dagger}$	$20^{\dagger}$	$36^{\dagger}$	62
12	22	39	68
$13^{\dagger \ddagger}$	$24^{\dagger}$	43	75
15	27	47	82
$16^{\dagger}$	$30^{\dagger}$	51	91

† Value unavailable for  $10^{-1}$  multiplier. ‡ Value unavailable for  $10^{0}$  multiplier.