PhD Qualifying Examination
Department of Mechanical Engineering
Clemson University

Vibrations & Dynamics

This is a two-hour closed books and notes exam. Only nonprogrammable calculators are allowed. Please start each problem on a new piece of paper and show all work in a clear manner.

HONORS PLEDGE: "I have neither given nor received aid on this examination"

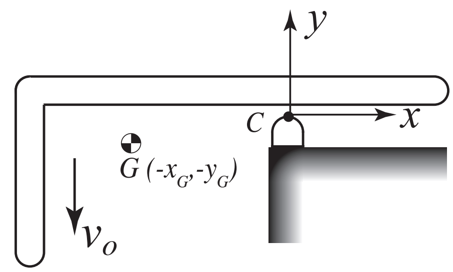
ID number______________________________

Problem	**Points**	
1	**25**	
2	**25**	
3	**25**	
4	**25**	
Total	**100**	

Please indicate only your Student ID number on these and any extra pages. DO NOT use your name.

Student ID# ________________________________

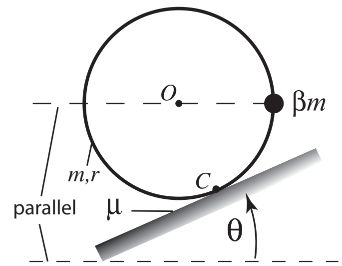
1. (25 pts.) An L-shaped bar with TOTAL mass m is in pure translation with velocity v_0 when it strikes a grounded surface at point C. The bar's center of gravity is located at point G with coordinates $(-x_G, -y_G)$ in the coordinate system shown with origin at C. <u>What is the angular velocity of the bar immediately after impact assuming a coefficient of restitution e?</u>



Your final answer should be an expression containing the parameters given above and on the figure. and a mass moment of inertia. You may also use the mass moment of inertia about any point you wish (e.g., $I_?$) as long as you define the point clearly. Your score will depend mostly on how well you demonstrate your understanding of the fundamental principles needed to solve this problem.

Student ID# ______________________________

2. (25 pts.) An advertisement for the Honda Accord has tires initially at rest on an incline, as shown in the upper frame here. The balance is accomplished by adding a weight to each tire. Use the simplified model (shown in the lower frame) of a particle of mass βm embedded in solid disk of mass m level with the disk center O.

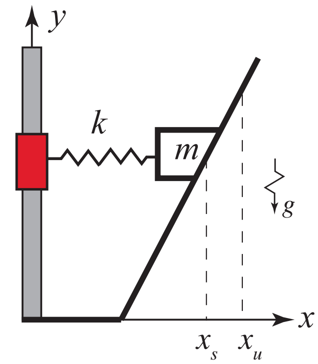


a) Assuming sufficient friction, what relationship between β and θ must exist for the simplified model to be in equilibrium?

b) What is the minimum coefficient of static friction μ for a static equilibrium position to be possible?

c) Now assume that the coefficient of static friction is too small to prevent slipping, so the wheel is held at rest by an external force in the position shown. Use Newton's equations to derive expressions for the angular acceleration as well as the acceleration of the center of mass immediately after the wheel is released.

3. (25 pts.) A mass m attached to a linear spring of constant k slides along the frictionless inclined surface that follows the equation $y = bx - c$ as shown in the figure. The spring is attached to a massless collar that can slide vertically on the pole without friction. Assume that the spring is unstretched at x_u.



a) Determine the position x_s where the system is in static equilibrium.

b) Derive the equation of motion in terms of coordinate x.

c) Detemine the equation of motion for small motions $\hat{x}$ about the equilibrium position x_s.

You may assume that the spring is always horizontal and that the mass never strikes the floor.

Student ID# ______________________________

4. (25 pts.) The figure depicts a lumped-parameter model for a general vibration system with two lumped masses in the absence of damping. The two coordinates x_1 and x_2 describe the translational motion of the two masses in horizontal direction. Please do the following for $k_1 = K$, $k_2 = 2K$, $k_3 = 2K$, $k_4 = 4K$, $k_5 = 3K$, $m_1 = M$, and $m_2 = 2M$.

   a) Write the equations of motion of this system.
   b) Represent the complete equations of motion in Part a) in matrix form and identify the mass and stiffness matrices.
   c) Find the natural frequencies and mode shapes for this system. Show clearly the relative motion of the masses in a physical sketch of each mode shape.