**CLEMSON UNIVERSITY**

**Department of Mechanical Engineering**

**PhD Qualifiers exam**

**Mathematics**

ID # _________________________

Solve six problems out of the following seven problems.
Detail your approach to finding the solutions. We want to see your thought process even though you may not reach the final solution. Giving results without details will not be given credit. All problems carry equal weight.

This is a closed-book and closed calculator, two-hour examination.
No calculator of any kind allowed.

Only six out of the following seven problems will be graded.
Choose the problem you do not want graded by completing the following.

Do not grade problem # ____________

**Problem 1**: Use the matrix

$$A = \begin{bmatrix} 7 & 2 \\ -4 & 1 \end{bmatrix}$$

(a) Find the eigenvalues and corresponding eigenvectors of *A*.

(b) Find the nonsingular matrix P and diagonal matrix D such that $A = PDP^{-1}$.

(c) Use the diagonalization in (b) to compute $A^5$.

**Problem 2**: Find all values of *h* and *k* such that the system has (a) no solution, (b) a unique solution, and (c) many solutions. Give separate answers for each part.

$$\begin{aligned} x_1 + h x_2 &= 2 \\ 4x_1 + 8x_2 &= k \end{aligned}$$

**Problem 3**: Using separation of variables, find solution *u(x,y)* of the equation

$$y\frac{\partial u}{\partial x} - x\frac{\partial u}{\partial y} = 0$$

**Problem 4**: Express the Taylor series of the function $e^x$ about *x=0*. Write out at least four nonzero terms in this expansion.

Then use this result and a power series for *y(x)*, to solve the following second-order differential equation with initial conditions. Write out at least four nonzero terms in the series solution for *y*.

$$y'' - e^x y = 0$$
$$y(0) = 1$$
$$y'(0) = 0$$

**Problem 5**: Find the directional derivative of $\vec{\nabla} \bullet \vec{u}$ where $\vec{u} = x^4\vec{i} + y^4\vec{j} + z^4\vec{k}$ at the point P:(4,4,2) in the direction of the outward normal from the surface of the sphere given by $S = x^2 + y^2 + z^2 = 36$.

**Problem 6**: Find the Fourier series approximation for the function

$f(t) = |t|, \quad -2 < t < 2$. Here T=4 is the period of the function.

**Problem 7**: For the system shown,

$$\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 2y = F(t)$$

Find the response to a unit impulse $F(t) = \delta(t)$ and sketch the response.