Spring 2010 Qualifying tests

Heat transfer

Closed book exam.
The three problems must be worked out.
Box all answers.

Alloted time: 2 hours

Problem 1: Transient conduction

A thin metallic foil is sandwiched between two electrically insulating slabs of constant thermal conductivity k (figure 1). Each slab has a thickness L and is infinite in the two other directions. It is assumed that the contact thermal resistance between the metallic foil and the slabs are negligible.

Should the foil be traversed by an electric current, it generates a heat rate which is evacuated through the two slabs.

The two slabs are initially at a uniform temperature T_i. At time $t = 0$ and for any time $t > 0$:

- A constant electric power is applied to the metallic foil which produces a constant heat flux $q_0''/2$ in each slab,
- The external faces of the slabs at $x = -L$ and $x = L$ are maintained at a constant temperature T_0 greater than the initial temperature T_i.

1. Consider the transient temperature profile in the slabs. Solving the heat diffusion equation requires a set of boundary conditions. Write these boundary conditions:
   - In the left slab:
     - At $x = -L$,
     - At $x = 0_-$,
   - In the right slab:
     - At $x = 0_+$,
     - At $x = L$,

2. Without solving any equation, draw the temperature profiles $T(x)$ in the two slabs on the graph of figure 2 at the following times:
   - Time $t < 0$,
   - Time $t \to \infty$,
   - Two intermediate times t_1 and t_2 such that $0 < t_1 < t_2 < \infty$

Clearly justify the shape of each profile.

3. Without solving any equation, draw the curve representing the heat flux $q_x''(L,t)$ on the external face of the right slab as a function of time t on the graph of figure 3. Clearly justify the shape of the curve.

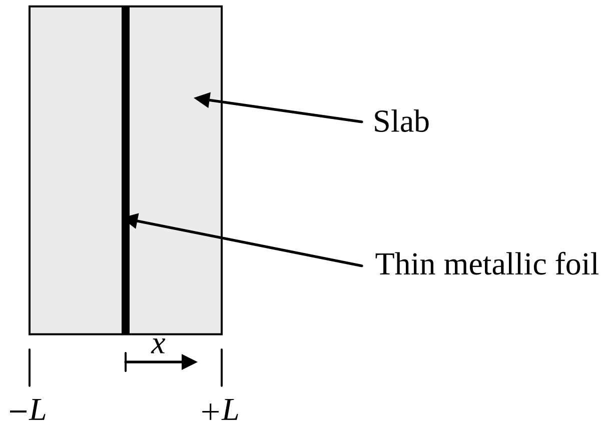


Figure 1

Answers:

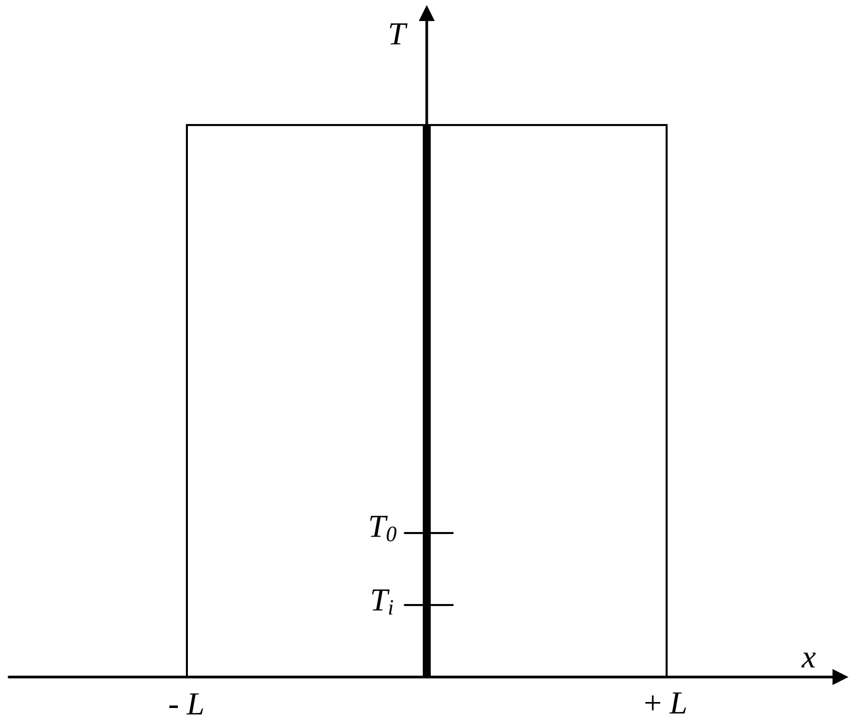


Figure 2

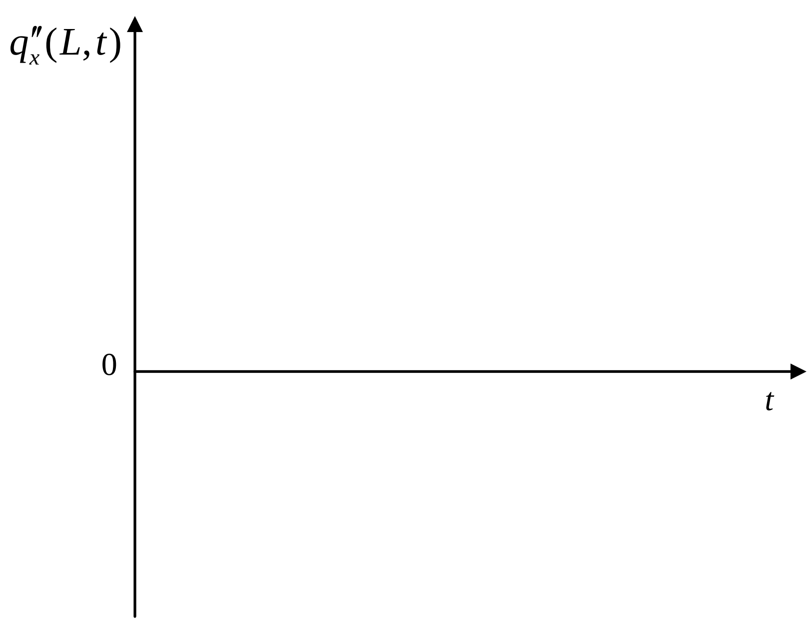
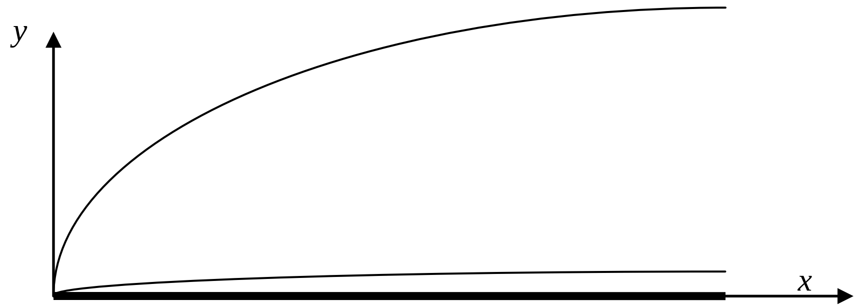


Figure 3

Problem 2: Liquid metal boundary layer

Consider a boundary layer flow of a liquid metal over a horizontal, isothermal flat plate at temperature T_w.



The following conditions or assumptions are satisfied:

C1 Steady state conditions prevail
C2 The problem is two-dimensional in x and y
C3 The plate is isothermal
C4 The boundary layer approximations are valid
C5 The velocity in the thermal boundary layer is uniform and equal to the uniform velocity in the free stream u_∞. The temperature T_∞ in the free stream region is uniform.
C6 The physical properties are constant and evaluated at the film temperature.

1. Indicate in the figure the hydrodynamic and thermal boundary layers.

2. What is the definition of the film temperature?

3. If conditions C1, C2 and C4 are satisfied the internal energy equation reduces to:

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}+\frac{\upsilon}{c_p}\left(\frac{\partial u}{\partial y}\right)^2 \quad (1)$$

   where υ is the kinematic viscosity.
   What is the physical significance of the last term of equation (1)?

4. Using the continuity equation and conditions C5 and C6 show that this equation reduces to:

$$u_\infty\frac{\partial T}{\partial x}=\alpha\frac{\partial^2 T}{\partial y^2} \quad (2)$$

5. Let δ_t be the thickness of the thermal boundary layer. Integrating equation (2) from $y=0$ to $y=\delta_t$ find the expression of δ_t in terms of x, u_∞ and α.

6. Give the expression of δ_t / x in terms of the Péclet number. Give the physical significance of the Péclet number.

Problem 3: Radiation

Two flat plates are located in a large room as shown. The following data are given:

$F_{1-2} = 0.4$

$T_1 = 1000\ \mathrm{K},\ A_1 = 0.5\ \mathrm{m}^2,\ \varepsilon_1 = 0.8$

$T_2 = 500\ \mathrm{K},\ A_2 = 0.8\ \mathrm{m}^2,\ \varepsilon_2 = 0.2$

$T_3 = 300\ \mathrm{K},\ \varepsilon_3 = 0.5$

1. Draw the radiation network and label all the junctions and resistances.

2. Find values of all resistances and the values of E_{b1}, E_{b2} and E_{b3}.

3. Explain how you would determine the heat transfer rate from plate 1 to 2 from the data given. List the appropriate equations but do **NOT** solve these equations.