Decomposition-based Optimization for Complex Systems

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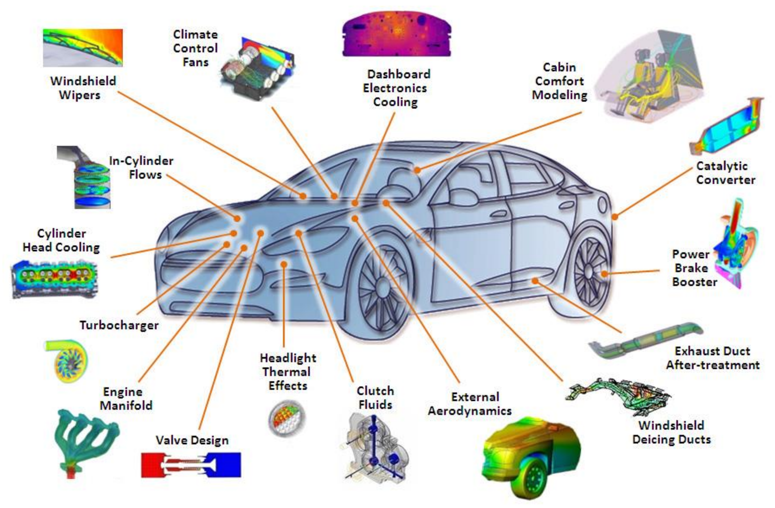
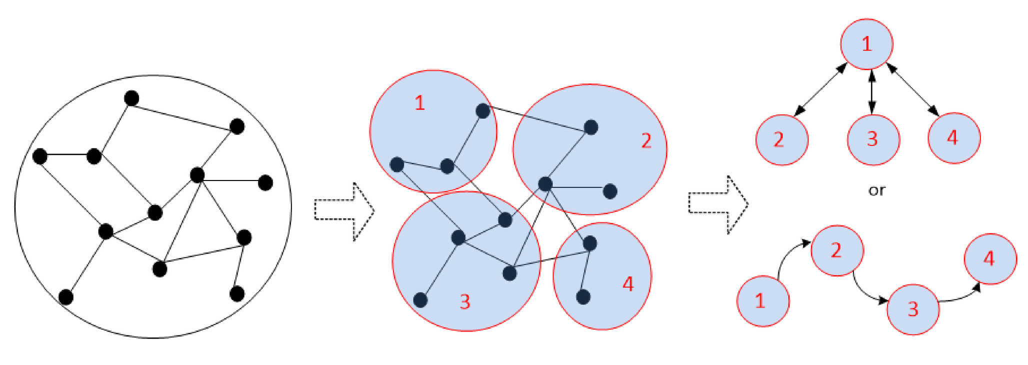
Introduction



- Many products involve various knowledge in **different disciplines**. They are too **complex** to be addressed by a single designer or even a single design group.
- One approach for solving the complex problem is to **divide the product design task into smaller and more manageable design problems, which is called decomposition**.
- Decomposition process consists of two steps:
  (1) **Partitioning** a system into smaller elements that can be designed autonomously
  (2) **Coordination** of individual elements towards an optimal and consistency system



Process of decomposition

This research focuses on making decomposition-based optimization of complex system **more accurate and efficient.**

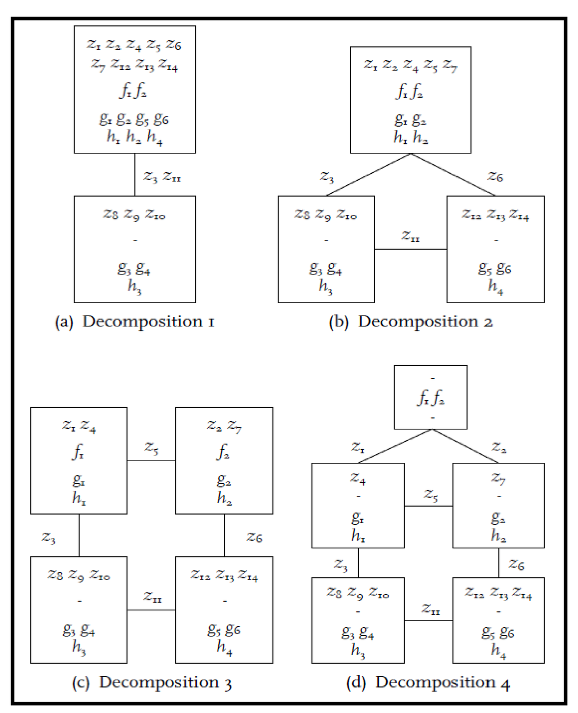
- **An illustrative example:**

$$\min_{z_1,\dots,z_{14}} \quad f = z_1^2 + z_2^2$$

subject to

$$\begin{aligned}
g_1 &= (z_3^{-2} + z_4^2)z_5^{-2} - 1 \le 0\\
g_2 &= (z_5^2 + z_6^{-2})z_7^{-2} - 1 \le 0\\
g_3 &= (z_8^2 + z_9^2)z_{11}^{-2} - 1 \le 0\\
g_4 &= (z_8^{-2} + z_{10}^2)z_{11}^{-2} - 1 \le 0\\
g_5 &= (z_{11}^2 + z_{12}^{-2})z_{13}^{-2} - 1 \le 0\\
g_6 &= (z_{11}^2 + z_{12}^2)z_{14}^{-2} - 1 \le 0\\
h_1 &= (z_3^2 + z_4^{-2} + z_5^2)z_1^{-2} - 1 = 0\\
h_2 &= (z_5^2 + z_6^2 + z_7^2)z_2^{-2} - 1 = 0\\
h_3 &= (z_8^2 + z_9^{-2} + z_{10}^{-2} + z_{11}^2)z_3^{-2} - 1 = 0\\
h_4 &= (z_{11}^2 + z_{12}^2 + z_{13}^2 + z_{14}^2)z_6^{-2} - 1 = 0\\
&z_1, \dots, z_{14} > 0
\end{aligned}$$

For this example, we tried:
4 partition methods: decom1, 2, 3, 4
3 coordination methods:
Augmented Lagrangian Coordination:
- ENMOM, INMOM, ADMOM



Four partitions

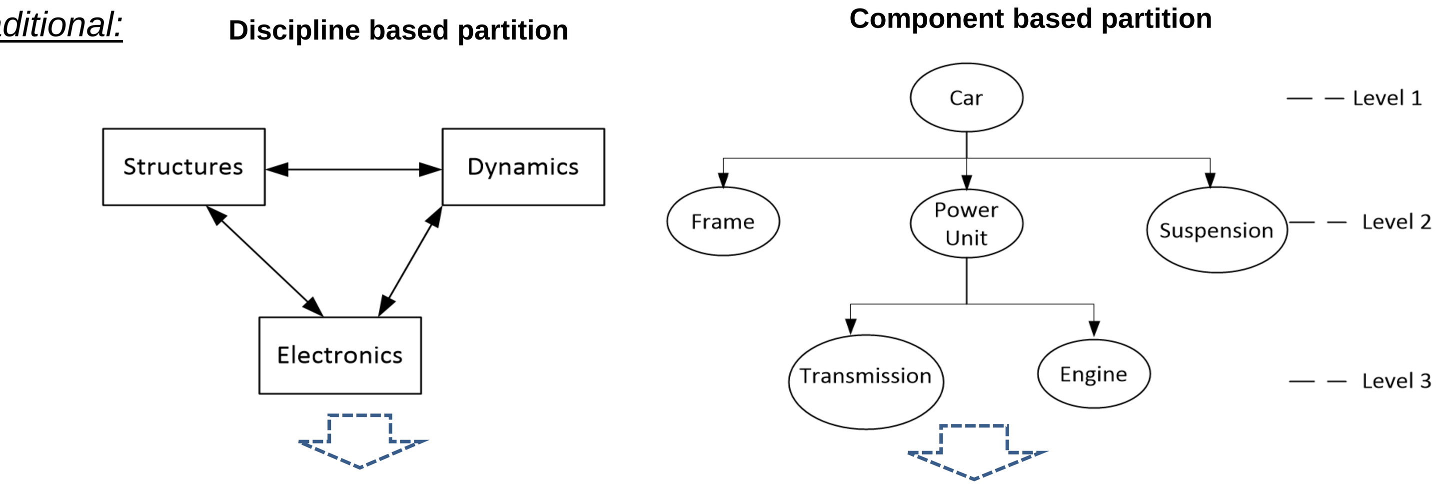
Time of computation/sec

Partition /coordination method	ENMOM	INMOM	ADMOM
Partition 1	3.01	1.55	1.82
Partition 2	26.99	5.93	2.66
Partition 3	14.47	8.04	3.01
Partition 4	380.75	11.02	3.45

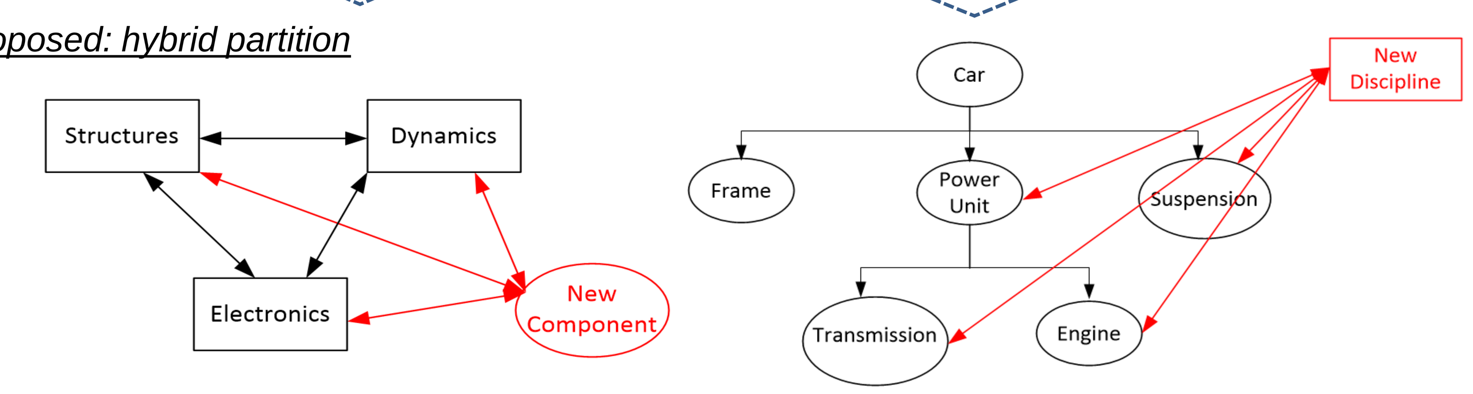
Choosing appropriate partition and coordination methods can greatly improve the optimization efficiency .

Partition

Traditional:



Proposed: hybrid partition



Coordination

w – penalty weights used to drive the inconsistency between different sub-problems to zero

Traditional:

$$\mathbf{w}^{k+1} = \begin{cases} \mathbf{w}^k & if \ \left|\mathbf{c}^k\right| \le \gamma \left|\mathbf{c}^{k-1}\right| \\ \beta \mathbf{w}^k & if \ \left|\mathbf{c}^k\right| > \gamma \left|\mathbf{c}^{k-1}\right| \end{cases}$$
$$where \ \ \beta > 1, 0 < \gamma < 1$$

w is always increasing

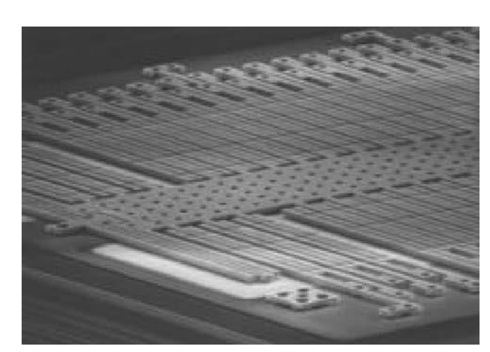
Proposed:

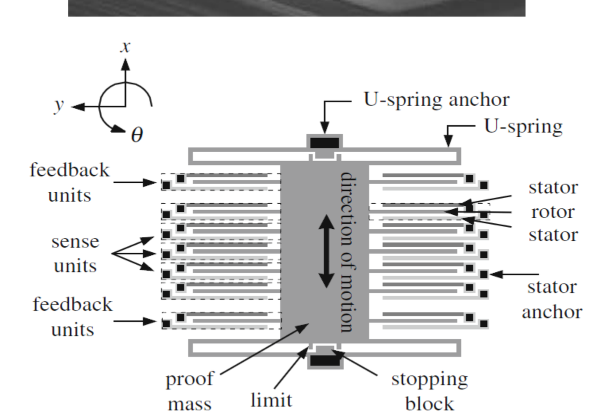
$$\mathbf{w}_i^{k+1} := \begin{cases} \tau^{incr}\mathbf{w}_i^k & if \ \left\|\mathbf{r}_i^k\right\|_2 > \mu \left\|\mathbf{s}_i^k\right\|_2 \\ \mathbf{w}_i^k / \tau^{decr} & if \ \left\|\mathbf{s}_i^k\right\|_2 > \mu \left\|\mathbf{r}_i^k\right\|_2 \\ \mathbf{w}_i^k & if \ otherwise \end{cases}$$
$$where \ \mu > 1, \tau^{incr} > 1, \tau^{decr} > 1$$

w can either increase or decrease

Numerical tests

Test problem:





Optimization objective:
the footprint area A

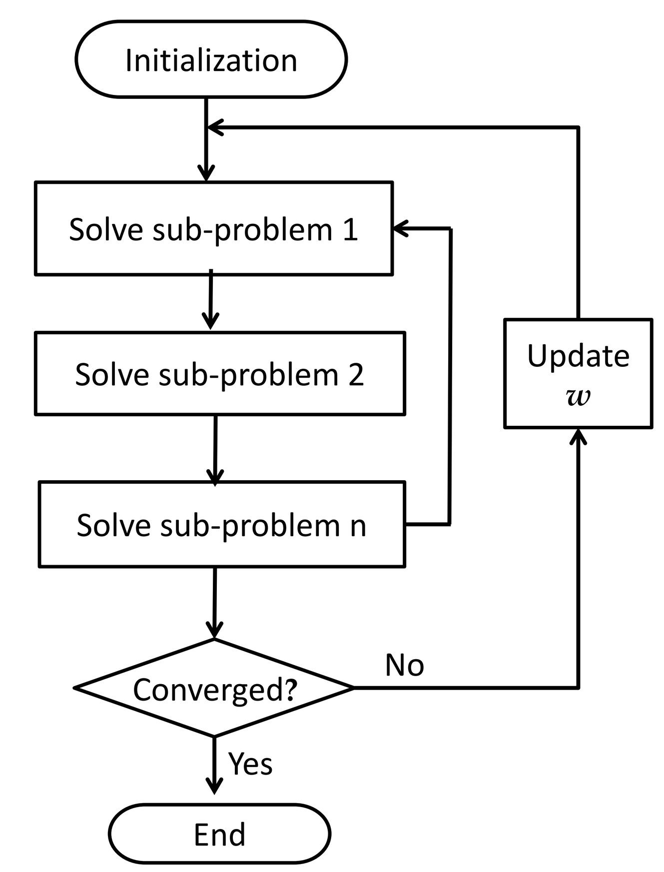
Constraints:
design requirements in the four disciplines

Design variables:
Geometry & circuit variables

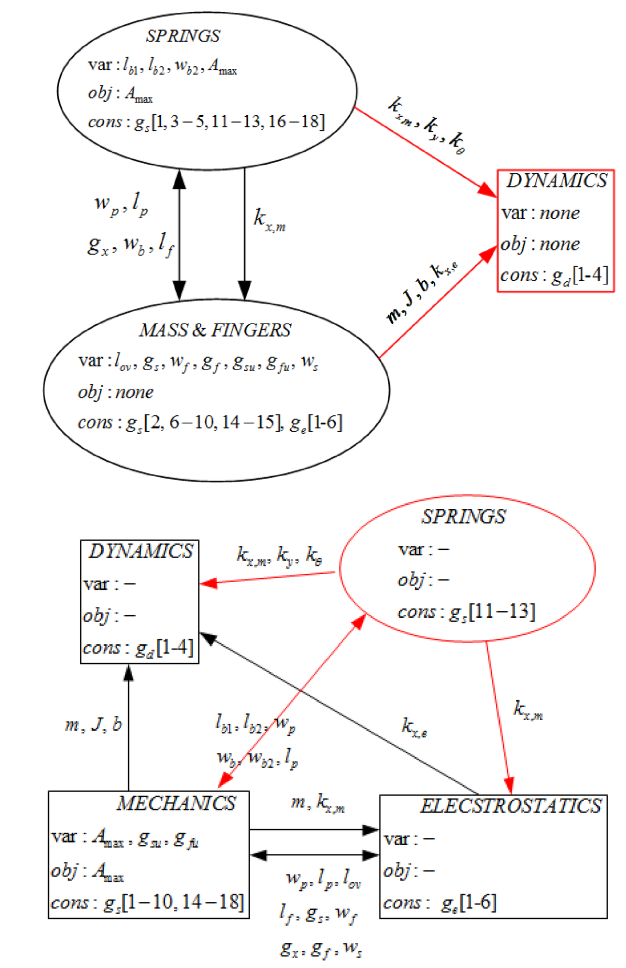
Reference solution:
Through solving the problem as a whole, without decomposition

More experiments have been conducted on other engineering problems, such as portal frame design and Golinski's speed reducer design problems

Flow chart:



Hybrid partitions:



Results:

Proposed partitions:

Ref. = 0.0453		Objective(mm²)	# of Iter	Max_vari _error
Hybrid 1 90%	Min	0.0458	459	0.7549
	Mean	0.0467	538	1.1135
	Max	0.0533	769	2.4103
Hybrid 2 90%	Min	0.0457	367	0.9253
	Mean	0.0460	434	0.9589
	Max	0.0475	589	1.1281

Proposed coordination:

	traditional		proposed update strategy	
	Objective(mm²)/ error	# of iters	Objective(mm²) /error	# of iters
Max	0.0893/10.7%	40	0.0813/0.7%	172
Mean	**0.0901/11.6%**	**49**	**0.0816/1.1%**	**247.5**
Min	0.0910/12.8%	76	0.0820/1.6%	321

Conclusions

(1) Explored the effects of different partitions and coordination on optimization.
(2) Proposed two kinds of hybrid partition.
(3) Solved two hybrid partitions with good solution accuracy.
(4) Proposed a new weight update strategy for sub-problem coordination.
(5) Verified the efficacy of the proposed update strategy.

Future work

(1) Study the convergence property of the proposed update from the mathematical perspective.
(2) Extend the proposed weight update to other coordination methods.

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