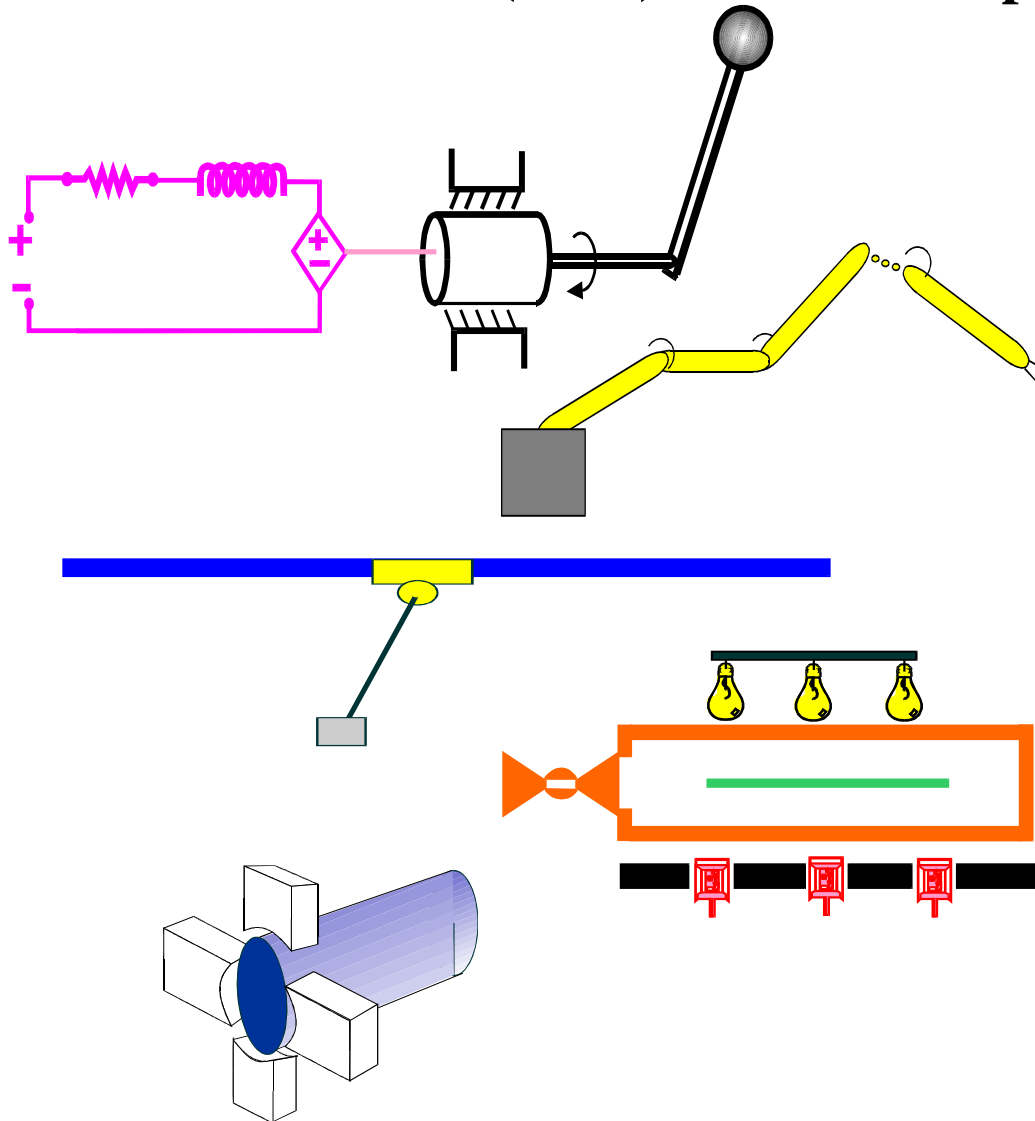


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Title: Backstepping PWM Control for Maximum Power Tracking in Photovoltaic Array Systems

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Backstepping PWM Control for Maximum Power Tracking in Photovoltaic Array Systems

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Abstract—A grid power system consisting of a photovoltaic (PV) array panel, dc-to-dc switching converter and a battery is considered in this paper. A backstepping PWM controller is developed to maximize the power of the solar generating system. The controller tracks a desired array voltage, designed online using an incremental conductance extremum-seeking algorithm, by varying the duty cycle of the switching converter. The stability of the control algorithm is demonstrated by means of Lyapunov analysis. Representative numerical results demonstrate that the grid power system can be controlled to track the maximum power point of the photovoltaic array panel.

Keywords—Maximum power point tracking, solar power generation, robust control, Lyapunov, PWM.

I. INTRODUCTION

Solar energy is one of the more attractive sources of energy today owing to the rising costs of traditional energy sources, an increase in environmentalism and the inexhaustibility of the source of energy. The primary device for harnessing solar energy is the solar cell, which uses the photovoltaic effect to transform sunlight into electricity via a semiconductor device. Conditions such as cell parameters and atmospheric conditions (temperature and solar irradiation) affect the instantaneous energy generated by a PV array as demonstrated by the current-voltage ($i_{pv} - v_{pv}$) characteristic shown in Fig. 1 which can be mathematically described as follows [1]:

$$i_{pv} = n_p I_{ph} - n_p I_{rs} \left(e^{\frac{q v_{pv}}{n_s A K T}} - 1 \right). \quad (1)$$

where $i_{pv}(t) \in \mathbb{R}$ is the PV array output current; $v_{pv}(t) \in \mathbb{R}$ is the PV array output voltage; n_s is the number of cells connected in series; n_p represents the number of parallel modules; q is the charge of an electron; K is the Boltzmann's constant; A is the p - n junction ideality factor; and T is the cell temperature in Kelvin (K). The reverse saturation current, $I_{rs} \in \mathbb{R}$, and the photocurrent, $I_{ph} \in \mathbb{R}$, can be expressed as:

$$I_{rs} = I_{or} \left(\frac{T}{T_r} \right)^3 e^{\frac{q E_{go}}{K T} \left(\frac{1}{T_r} - \frac{1}{T} \right)}. \quad (2)$$

$$I_{ph} = (I_{sc} + K_l (T - T_r)) \frac{\lambda}{100}. \quad (3)$$

where I_{or} is the reverse saturation current at the reference temperature, T_r ; E_{go} is the band gap energy of the semiconductor; I_{sc} is the short-circuit cell current at the reference temperature and radiation; K_l is the short-circuit current temperature coefficient; and λ is the solar radiation in mW/cm^2 . Thus, the PV array output power, $P_{pv}(t)$, can be calculated as:

$$P_{pv} = i_{pv} v_{pv} = n_p I_{ph} v_{pv} - n_p I_{rs} v_{pv} \left(e^{\frac{q v_{pv}}{n_s A K T}} - 1 \right). \quad (4)$$

It can be concluded that exists a maximum power point (MPP) that varies with solar radiation and cell temperature as shown in Fig. 2. To this end, several control approaches has been developed to optimize the power output when atmospheric conditions are varying. An area of particular importance is the development of online extremum-seeking algorithms which are generally classified into incremental conductance (IncCond) [2]-[4] and ‘‘perturb and observe’’ (P&O) methods [5], [6]. Hussein et al. [2] developed a maximum power tracking (MPT) technique that is efficient in cases of rapidly changing atmospheric conditions. They showed that the maximum power operating point can be tracked accurately by measuring the solar array current and voltage, comparing the incremental and instantaneous conductances of the PV and changing the array voltage accordingly. Leyva et al. [5] demonstrated the global stability of an MPPT algorithm using Lyapunov analysis and applied it to a PV system based on the ‘‘perturb and observe’’ method. Control techniques used to directly control photovoltaic characteristics include classical control [7]-[9], fuzzy logic control [1], robust control [6], [10], variable structure [11], [12], and artificial neural networks [13]-[15]. Lian et al. [1] regulate the output power of a solar power generating system using the Takegi-Sugeno fuzzy method which includes using virtual desired variables (VDVs). Kasa et al. [6] presents a robust control method for maximum power point (MPP) tracking in a photovoltaic system where the circuit parameters are uncertain. The MPP is tracked by varying the duty ratio of the switching device in order to control the array voltage.

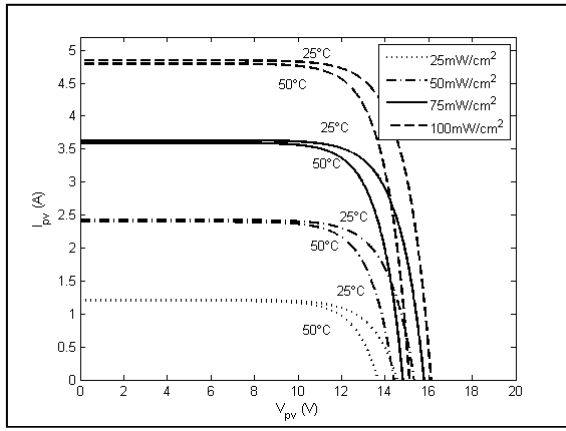


Figure 1. Current-voltage characteristics of a PV array

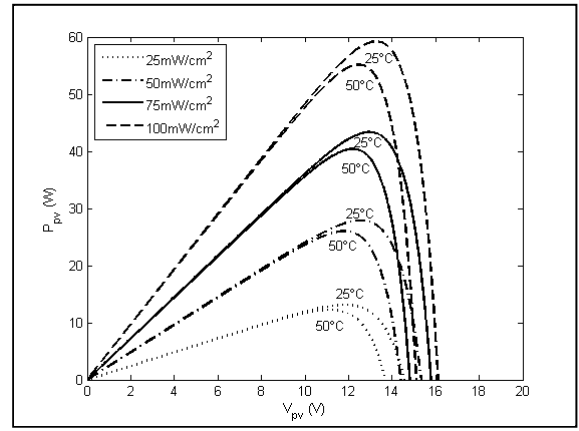


Figure 2. Power-voltage characteristics of a PV array

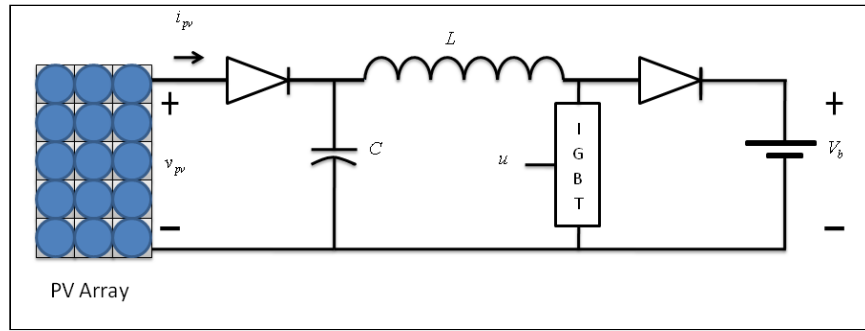


Figure 3. The system structure of the photovoltaic array system

Valenciaga et al. [11] designed a variable structure controller to regulate the output power of a standalone hybrid generation system consisting of a PV array, wind turbine, a storage battery bank and a variable monophasic load.

Aside from maximizing the output power, another common application for photovoltaic arrays is load matching [6], [16], [17]. Saied et al. [16] maximized the output mechanical energy of a DC motor, driving a mechanical load, connected to a PV array via a dc-dc converter with varying atmospheric conditions. Yadaiah et al. [6] developed a controller algorithm to match a solar cell array to a mechanical load using artificial neural networks.

In this paper, a control strategy is developed to maximize the power of a solar generating system while including the dynamics of the DC-DC converter that assumed absent in some papers. The control objective is to determine the maximum power operating point (MPOP) by tracking a desired array voltage which can be achieved by modulating the pulse width of the switch control signal (increasing or decreasing the duty ratio of the switching converter). The desired array voltage is designed online using a filtered incremental conductance MPP tracking algorithm. The proposed strategy ensures that the MPOP is determined and the tracking errors are globally asymptotically regulated. The stability of the control algorithm is verified by Lyapunov analysis.

The rest of the paper is organized as follows. The dynamic model of the solar generating system is described in

Section II. In Section III, a backstepping array voltage tracking controller is designed along with the corresponding closed-loop error system. The stability analysis of the closed-loop error system is discussed in Section IV. In Section V, the desired array voltage trajectory is generated followed by numerical simulation results in Section VI. Concluding remarks are presented in Section VII.

II. PHOTOVOLTAIC ARRAY SYSTEM DYNAMICS

The solar generation model consists of a PV array module, dc-to-dc boost converter and a battery as shown in Fig. 3. The converter transfers power from the PV array terminals to the battery bank, indirectly controlling the voltage of the PV array panel, $v_{pv}(t)$ and thus the array power generation.

The dynamic model of the solar generation system can be expressed by an instantaneous switched model as follows:

$$C\dot{v}_{pv} = i_{pv} - i_L \quad (5)$$

$$L\dot{i}_L = v_{pv} - (1-u)V_b \quad (6)$$

where L and $i_L(t)$ represents the dc-to-dc converter storage inductance and the current across it; $V_b \in \mathbb{R}$ is the voltage of the storage battery and $u(t) \in \mathbb{R}$ is the switched control signal that can only take the discrete values 0 (switch open)

and 1 (switch closed). Using the state averaging method [18], the switched model can be redefined by the average PWM model as follows:

$$C\dot{V}_{pv} = I_{pv} - I_L. \quad (7)$$

$$L\dot{I}_L = V_{pv} - D'V_b. \quad (8)$$

where $V_{pv}(t) \in \mathbb{R}$ and $I_{pv}(t) \in \mathbb{R}$ are the average states of the output voltage and current of the solar cell; $I_L(t) \in \mathbb{R}$ is the average state of the inductor current; $D'(t) \in \mathbb{R}$ is the limited duty ratio function of the off-state of the switched control signal, $u(t)$.

To facilitate control development, the following model characteristics are assumed:

Assumption 1: $V_{pv}(t)$, $I_{pv}(t)$, $I_L(t)$ and $V_b(t)$ are measurable.

Assumption 2: C and L are known constants.

Assumption 3: V_b is modeled as a constant value due to its slow charge dynamics [5].

Assumption 4: $I_{pv}(t)$ is bounded provided that $V_{pv}(t)$ is bounded.

Assumption 5: $\dot{I}_{pv}(t)$ can be upper bounded by a positive constant such that $|\dot{I}_{pv}| < \mu$ where $\mu \in \mathbb{R}^+$.

III. CONTROLLER DESIGN

The control objective is to maximize the power extracted from a solar generating system, $P_{pv}(t)$ by tracking a developed desired array voltage, $V_d(t) \in \mathbb{R}$, such that $V_{pv}(t) \rightarrow V_d(t)$ as $t \rightarrow \infty$. This is achieved by varying $D'(t)$, the duty ratio of the off-state of the switched control signal.

Remark 3.1: The desired array voltage, $V_d(t)$, is designed online using a numerical-based extremum-seeking algorithm, as shown in Section IV, to maximize the extracted power $P_{pv}(t)$ such that $V_d(t) \rightarrow V^*$, where V^* is the unknown optimal array voltage, implies that $P_{pv}(t)$ tends to P_{\max} , the maximum power point (MPP). Additionally, $V_d(t)$ is designed to be sufficiently differentiable, that is $V_d(t), \dot{V}_d(t), \ddot{V}_d(t) \in \mathcal{L}_\infty$.

A. Error System Development

To quantify the state control objective, tracking errors denoted by denoted by $e(t) \in \mathbb{R}$ and $z(t) \in \mathbb{R}$ are defined as follows

$$e = V_d - V_{pv}. \quad (9)$$

$$z = I_L - I_D. \quad (10)$$

where $I_D(t) \in \mathbb{R}$ denotes the subsequently designed desired storage inductor current.

From the definition of the tracking errors in (9) and (10), and the system dynamics in (7) and (8), an open loop system is developed as follows:

$$C\dot{e} = C\dot{V}_d - I_{pv} + z + I_D. \quad (11)$$

$$L\dot{z} = V_{pv} - D'V_b - L\dot{I}_D. \quad (12)$$

B. Control Input Design

The control inputs will be designed based on the subsequent stability analysis as well as the structure of the open loop error systems in (11) and (12).

The designed desired storage inductor current, $I_D(t)$ is designed as

$$I_D = -C\dot{V}_d + I_{pv} - k_e e. \quad (13)$$

The duty ratio, $D'(t)$ is designed as follows

$$D' = \frac{1}{V_b} \left[V_{pv} + LC\ddot{V}_d + Lk_e \left(\dot{V}_d - \frac{I_{pv}}{C} + \frac{I_L}{C} \right) + e + k_z z + k_1 \text{sgn}(z) \right]. \quad (14)$$

where $k_e, k_1, k_z \in \mathbb{R}^+$ are control gains, and $\text{sgn}(\cdot)$ is the standard signum function.

Substituting (13) and (14) into the open loop error dynamics of (11) and (12), results in the following closed loop error system

$$C\dot{e} = -k_e e + z. \quad (15)$$

$$L\dot{z} = -k_z z - e - k_1 \text{sgn}(z) - L\dot{I}_{pv}. \quad (16)$$

IV. STABILITY ANALYSIS

Theorem 1: Given the closed loop error system in (15) and (16), the tracking error signals defined in (9) and (10) are globally asymptotically regulated in the sense that

$$e(t), z(t) \rightarrow 0 \text{ as } t \rightarrow \infty. \quad (17)$$

Proof: A non-negative scalar function, denoted by $V(t) \in \mathbb{R}$, is defined as

$$V = \frac{1}{2}Ce^2 + \frac{1}{2}Lz^2. \quad (18)$$

After taking the time derivative of (18) and making the appropriate substitutions from (15) and (16), the following expression is obtained

$$\dot{V} = e[-k_e e + z] + z[-k_z z - e - k_1 \operatorname{sgn}(z) - L\dot{I}_{pv}]. \quad (19)$$

$$\dot{V} = -k_e e^2 - k_z z^2 - k_1 |z| - L\dot{I}_{pv} z. \quad (20)$$

From (20), $\dot{V}(t)$ can be upper bound as follows

$$\dot{V} \leq -k_e e^2 - k_z z^2 - k_1 |z| + L|\dot{I}_{pv}| |z|. \quad (21)$$

If the control gain k_1 is designed such that $k_1 > L\mu$ then from Assumption 5, $\dot{V}(t)$ can be upper bound as follows

$$\dot{V} \leq -k_e e^2 - k_z z^2. \quad (22)$$

From (18) and (22), it is straightforward to see that $e(t), z(t) \in \mathcal{L}_\infty$. Since $e(t) \in \mathcal{L}_\infty$, (9) can be used along with Remark 3.1 to show that $V_{pv}(t) \in \mathcal{L}_\infty$. Based on the above boundedness statements, (13) can be used along with Remark 3.1 and Assumption 4 to show that $I_d(t) \in \mathcal{L}_\infty$. After utilizing the fact that $I_D(t), z(t) \in \mathcal{L}_\infty$, from (10), it is clear that $I_L(t) \in \mathcal{L}_\infty$. The expression in (14), Remark 3.1 and Assumptions 3 and 4, can be used along with the above boundedness statements to show that $D'(t) \in \mathcal{L}_\infty$. The above boundedness statements can be utilized along with (7), (8) and Assumption 3 to show that $\dot{V}_{pv}(t), \dot{I}_L(t) \in \mathcal{L}_\infty$. Above boundedness statements can be used along with Remark 3.1, and the time derivative of (9) to show that $\dot{e}(t) \in \mathcal{L}_\infty$. The time derivative of (13) can be used along with the above boundedness statements, Remark 3.1 and Assumption 5 to show that $\dot{I}_D(t) \in \mathcal{L}_\infty$. After taking the time derivative of (10), it can be concluded that $\dot{z}(t) \in \mathcal{L}_\infty$. After employing a corollary to Barbalat's lemma [19], it is easy to show that $e(t), z(t) \rightarrow 0$ as $t \rightarrow \infty$.

V. GENERATING $V_d(t)$ ONLINE

In Remark 3.1, it is assumed the desired array voltage, $V_d(t)$, can be designed such that $V_d(t), \dot{V}_d(t)$ and $\ddot{V}_d(t)$ are bounded and $V_d(t) \rightarrow V^*(t)$, where $V^*(t)$ is the unknown optimal array voltage that maximizes the solar power extracted, $P_{pv}(t)$. The extremum-seeking algorithm used in this paper is the incremental conductance MPP tracking algorithm [2]. Unlike many other MPT algorithms,

there is no significant loss of efficiency in cases with rapidly changing atmospheric conditions. This algorithm utilizes zero slope regulation to track the maximum power point by comparing the incremental and instantaneous conductances of the PV array and varying the desired voltage, $V_d(t)$ accordingly. Additionally, the algorithm accounts for changes in the atmospheric conditions when the array is operating at maximum power by checking if incremental current is nonzero.

To ensure that $V_d(t), \dot{V}_d(t)$ and $\ddot{V}_d(t)$ are bounded, a filter-based form of the incremental conductance algorithm is used, wherein at each iteration, the discrete guess, $V_d[n]$, is passed through a set of third order stable and proper low pass filters to generate continuous bounded signals for $V_d(t), \dot{V}_d(t)$ and $\ddot{V}_d(t)$. The following filters were used in this study

$$V_d(t) = \frac{\zeta_3}{s^3 + \zeta_1 s^2 + \zeta_2 s + \zeta_3} V_d[n] \quad (23)$$

$$\dot{V}_d(t) = \frac{\zeta_3 s}{s^3 + \zeta_1 s^2 + \zeta_2 s + \zeta_3} V_d[n] \quad (24)$$

$$\ddot{V}_d(t) = \frac{\zeta_3 s^2}{s^3 + \zeta_1 s^2 + \zeta_2 s + \zeta_3} V_d[n] \quad (25)$$

where $s \in \mathbb{C}$ is the Laplace variable, $\zeta_1, \zeta_2, \zeta_3 \in \mathbb{R}^+$ are filter constants and $n \in \mathbb{Z}^+$. The algorithm waits until certain error thresholds are met before making the next guess (i.e., if $|V_d(t) - V_d[n]| \leq e_1$ and $|V_{pv}(t) - V_d(t)| \leq e_2$ then $n = n + 1$; where $e_1, e_2 \in \mathbb{R}^+$ are threshold constants).

VI. SIMULATION RESULTS

A numerical case study is presented in this section to demonstrate the performance of the control strategy proposed in this paper using MATLAB/Simulink. In this simulation, the obtained average closed loop duty ratio $D(t)$, where $D(t) \triangleq 1 - D'(t)$, is calculated using the actual sampled state variables instead of their averaged values and then used to construct $u(t)$ in (5) by modulating the width of a 2 kHz pulse train. The state variables are sampled at 100 kHz.

The simulation plant and control parameters used in the simulation are listed below in Table 1.

TABLE I. SIMULATION PARAMETERS

Parameters	Value	Unit
A	1.6	
C	470	μF
E_{go}	1.1	V
I_{or}	2.0793×10^{-6}	A

I_{sc}	4.8	A
K	1.3805×10^{-23}	Nm/K
k_1	0.01	
k_e	8	
K_I	2.06×10^{-3}	A/°C
k_z	2	
L	4×10^{-3}	H
n_p	1	
n_s	25	
q	1.6×10^{-19}	C
T_r	301.18	K
V_b	24	V

The atmospheric conditions are assumed to be varying as shown in Fig. 4. The tracking error $e(t)$ is shown in Fig. 5. From these figures, it can be seen that the tracking error is asymptotically regulated. The synthesized duty ratio function, $D(t)$, is shown in Fig. 6 to be bounded. The results of the filter-based incremental conductance algorithm are shown in Fig. 7 and 8. In Fig. 7, the desired array voltage $V_d(t)$ is illustrated. It is clear that $V_d(t)$ is sufficiently differentiable. In Fig. 8, the time evolution of the actual power obtained and the maximum power obtainable from the PV array are presented. It should be noted that the actual power converges to the neighborhood of the maximum power. Fig. 9 shows the time evolution of the power obtained using a commonly used strategy wherein the incremental conductance algorithm was applied directly to the duty ratio, $D(t)$, in essence ignoring the dc-dc converter dynamics [3]. This approach is compared to the strategy proposed in this paper using the integral square error performance measure as shown in Fig. 10. It is clear that the proposed strategy has a better performance. Overall, the simulation results demonstrated that the proposed control strategy effectively tracks the maximum power point of the photovoltaic array in the midst of varying weather conditions.

VII. CONCLUSIONS

A backstepping PWM control strategy has been developed for a solar generating system to maximize the power extracted from a photovoltaic array in varying weather conditions. A desired array voltage is designed online using an extremum-seeking algorithm to seek the unknown optimal array voltage while remaining bounded and sufficiently differentiable. To track the designed trajectory, a tracking controller is developed to modulate the duty cycle of the boost converter. The proposed controller is proven to yield global asymptotic stability with respect to the tracking errors via Lyapunov analysis. Simulation results are provided to verify the effectiveness of this approach.

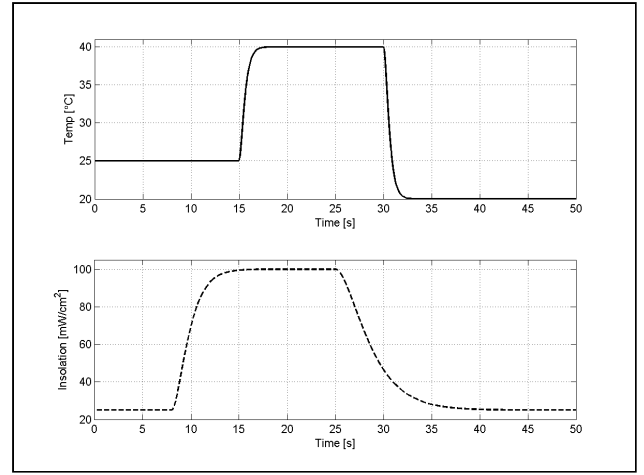


Figure 4. Temperature and solar radiation conditions present in the simulation

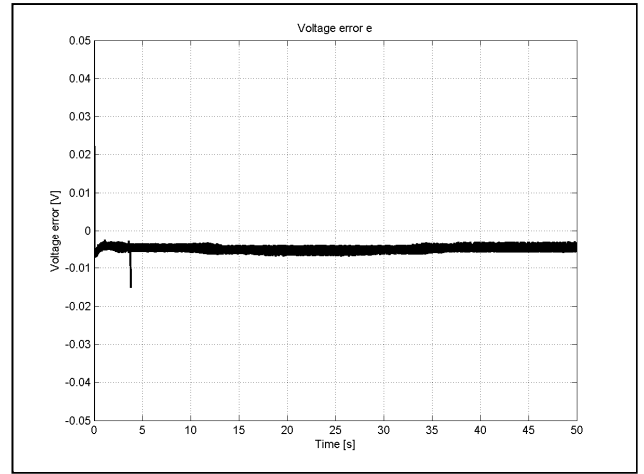


Figure 5. Array voltage Tracking error $e(t)$

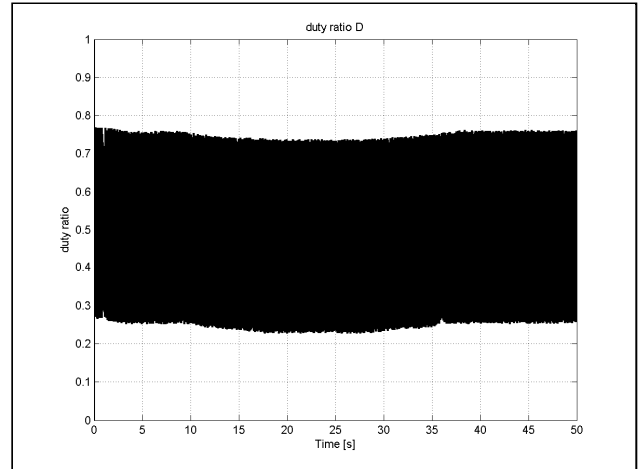


Figure 6. Duty ratio $D(t)$

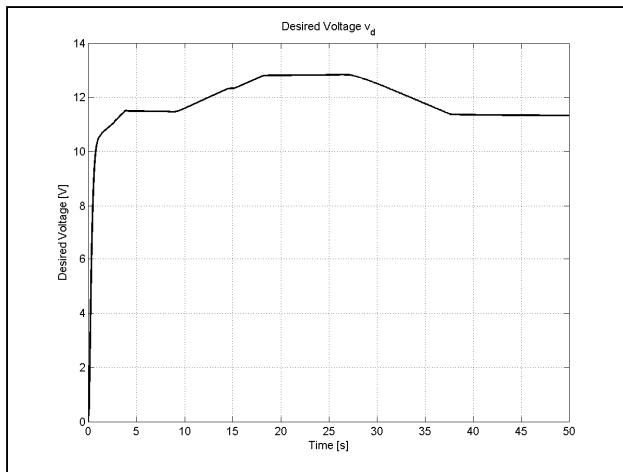


Figure 7. Desired array voltage $V_d(t)$

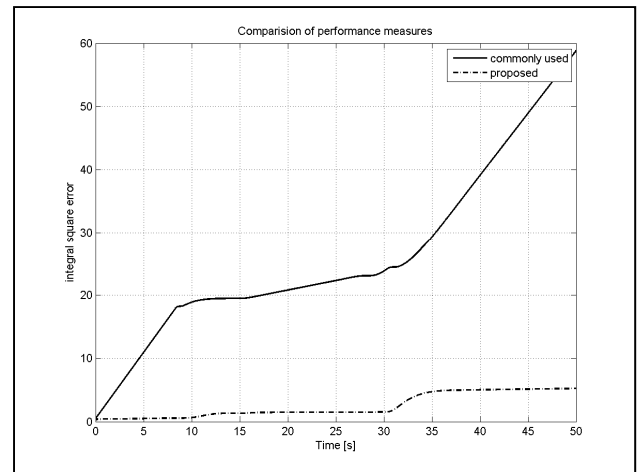


Figure 10. Integral square error performance measure

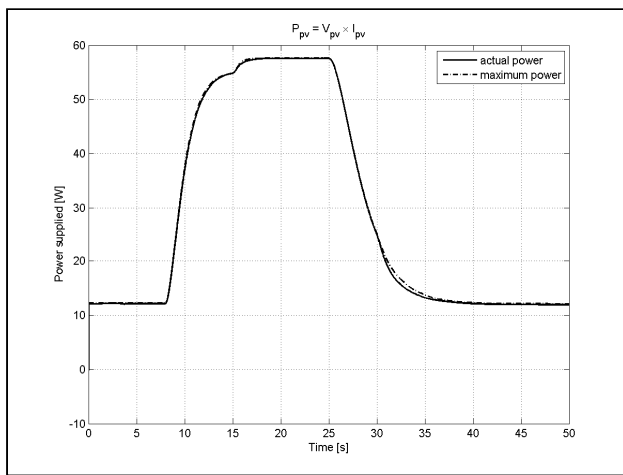


Figure 8. Actual and maximum PV array power

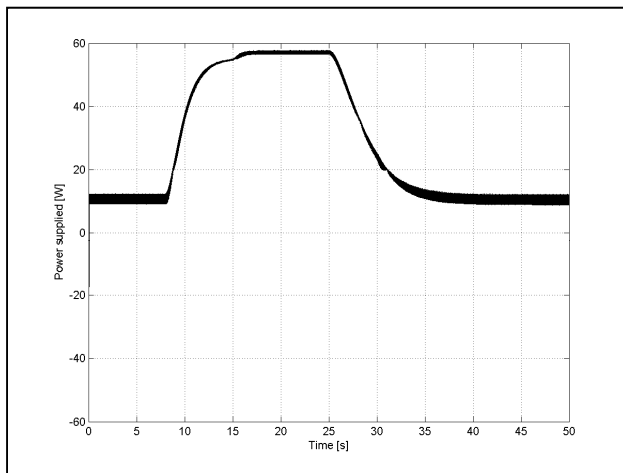


Figure 9. Power obtained when converter dynamics are ignored

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