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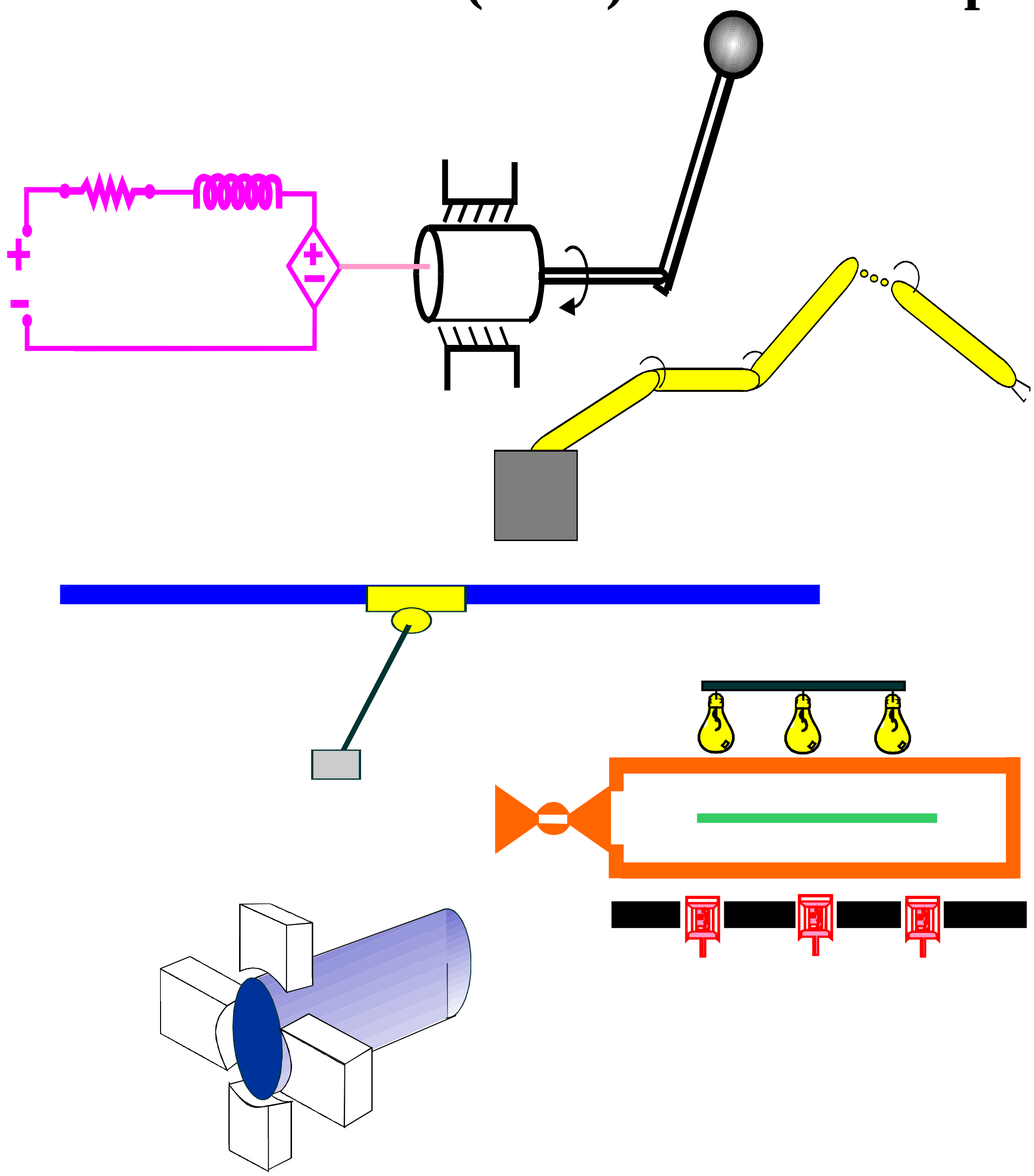

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Title: A Tuning Function-Based Robust Adaptive Controller

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A New Tuning Function-Based Robust Adaptive Controller for Parametric Strict Feedback Systems

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***Abstract*— This paper reports a new robust adaptive tracking controller for multi-input/multi-output nonlinear parametric strict-feedback systems in the presence of parametric uncertainty and *any* unknown continuous bounded additive disturbance. The proposed solution uses a new projection-like adaptation that allows the application of the standard tuning function approach, thereby avoiding overparametrization and the need for extra nonlinear damping-type terms in the control. The robust adaptive control is shown to guarantee practical tracking in the presence of the disturbance and asymptotic tracking when the disturbance disappears.**

I. INTRODUCTION

Consider the nonlinear system

$$\dot{x} = f(x, \theta, d) + g(x)u \tag{1}$$

affine in the uncertain constant vector θ, where $d(t)$ denotes unknown bounded external disturbances. The adaptive control problem consists of finding a dynamic state feedback controller

$$u = u(x, \hat{\theta}) \tag{2}$$

$$\dot{\hat{\theta}} = \tau(x, \hat{\theta}), \tag{3}$$

where $\hat{\theta}(t)$ denotes the estimate of θ, that drives the tracking error to zero or a small residual set while keeping all closed-loop signals bounded. It is well known that the performance of such adaptive controllers can significantly deteriorate and even become unstable when $d(t) \neq 0$ [12]. In this case, $\hat{\theta}(t)$ cannot be proven bounded, possibly leading to the unboundedness of other closed-loop signals. Common approaches for counteracting this problem include adding a robustifying (leakage) term to the adaptation law (3) (e.g., the σ-modification [12] and the e_1-modification [19]), or using a projection operator [12], [15], [24] to confine $\hat{\theta}(t)$ to a bounded convex set in the parameter space. Leakage modifications have the disadvantage of not recovering the disturbance-free stability performance of the unmodified adaptation law if $d(t) = 0 \ \forall t \geq T > 0$. On the other hand, projection operators preserve the ideal properties of the adaptive controller if the disturbance disappears, but require parameter bounds to be known a priori. See [4], [8], [9], [11], [13], [16], [21], [22], [23], [26] for examples of robust adaptive controllers.

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The class of systems to which adaptive control can be applied was broaden with the advent of the integrator backstepping design [15]. This systematic design procedure allows one to adaptively stabilize systems that are in the so-called parametric strict-feedback form [15]. In the standard adaptive backstepping design, the recursive procedure generates at each step a new adaptation law, thereby leading to overparametrization. Furthermore, an nth-order parametric strict-feedback system (where $n \geq 3$) requires the $(n-2)$th derivative of the first adaptation law, the $(n-3)$th derivative of the second adaptation law, and so on. In the disturbance-free case, the tuning functions method [15] avoids the differentiation of the adaptation laws as well as overparametrization. However, when $d(t) \neq 0$ and $d\hat{\theta}/dt = \text{Proj}(x, \hat{\theta})$, where $\text{Proj}(\cdot)$ denotes a standard projection operator, the tuning functions in the time derivative of the Lyapunov-like function will likely not be cancelled by the adaptation law as in the disturbance-free (no projection) case nor can be proven nonpositive. One recourse is to use the standard adaptive backstepping procedure (with overparametrization). This however would require multiple differentiations of $\text{Proj}(\cdot)$, which in most cases is Lipschitz continuous at best. As a result, the derivatives of $\text{Proj}(\cdot)$ would be discontinuous or not well defined. This differentiability problem was recently overcome in [3], where a smoothened version of the projection operator of [23] was proposed. Specifically, [3] replaces the Lipschitz continuity with the stronger property of arbitrarily many times continuous differentiability while introducing minor or no modifications to the other projection properties. The projection operator of [3], like the one in [23], is not amenable to the use of tuning functions. Another recourse is to inject extra nonlinear terms in the stabilizing functions to dominate or damp the projection-related terms left over in the time derivative of the Lyapunov-like function (see [16], [17]). This alternative avoids the overparametrization, but may lead to high control effort.

A common assumption in adaptive control designs for (1) is that the disturbance is the output of an autonomous exosystem (i.e., $\dot{d} = s(d)$) with partially known structure and having certain properties (e.g., $s(d) = Sd$ where S is an unknown stable matrix). Such an assumption naturally restricts the class of disturbances under consideration, and facilitates the control design since the internal model principle or an observer can be used to compensate or estimate the unknown disturbance. See [5], [6], [7], [10], [18], [20] for examples of such results. Among the benefits of the exosystem assumption is the lack of overparametrization in the adaptive controller.

In this paper, we consider the tracking problem for parametric strict-feedback systems with parametric uncertainty and additive disturbances that are *not* exosystem-generated. Our goal is to design a robust adaptive control that is not overparametrized. We address this problem by incorporating a projection-like operator inspired by [1], [2] in the adaptive backstepping control that enables the use of the standard tuning function approach. Thereby, we avoid the need for injecting extra nonlinear damping-type terms in the control. A Lyapunov analysis is used to prove practical tracking (i.e., the steady-state tracking error can be made as small as desired) in the presence of the disturbance and asymptotic tracking when the disturbance disappears. The main contributions of this work are that the proposed solution: i) can handle *any* unknown continuous bounded additive disturbance, and ii) leads to a control law simpler than the ones in [16], [17]. The authors of [25] studied a more general class of multi-input/multi-output (MIMO) strict-feedback systems with additive disturbance than the one in this paper; however, the proposed robust adaptive controller is overparametrized. Finally, we note that adaptive control presented in this paper can be easily integrated with the dynamic robust control of [4] to guarantee asymptotic tracking if the disturbance never disappears. This extension requires some additional assumptions on the disturbance, which are outlined later in the paper.

The paper is organized as follows. Section II introduces the class of MIMO nonlinear systems and states the control objective. Some preliminaries concerning the parameter estimation are given in Section III. The design and stability analysis of the robust adaptive controller are presented in Section IV. An extension to the proposed control is briefly discussed in Section V. A numerical example is reported in Section VI while concluding remarks are given in Section VII.

II. Problem Statement

We consider MIMO parametric strict-feedback systems of the form

$$\dot{x}_1 = \varphi_1^\intercal(x_1)\,\theta + x_2 + d_1 \tag{4a}$$

$$\vdots$$

$$\dot{x}_i = \varphi_i^\intercal(x_1,...,x_i)\,\theta + x_{i+1} + d_i \tag{4b}$$

$$\vdots$$

$$\dot{x}_n = \varphi_n^\intercal(x_1,...,x_n)\,\theta + d_n + u \tag{4c}$$

where $x_i(t) \in \mathbb{R}^{m_i}$, $i = 1,...,n$ are the system states, $\varphi_i \in \mathbb{R}^{p\times m_i}$, $i = 1,...,n$ are known nonlinearities, $\theta \in \mathbb{R}^p$ is an uncertain constant parameter vector, $d_i(t) \in \mathbb{R}^{m_i}$, $i = 1,...,n$ are unknown additive disturbances, $u \in \mathbb{R}^{m_n}$ is the control input, and $y = x_1$ is the system output. We make the following assumptions regarding the system:

A1. $\varphi_i \in \mathcal{C}^{n-i}$, $i = 1,...,n$.

A2. $d_i \in \mathcal{C}^0$, $i = 1,...,n$ and $\|d_i(t)\|_{\mathcal{L}_\infty} \le \overline{d}_i$ where $\overline{d}_i$ are unknown positive constants.

A3. The elements of θ satisfy $\underline{\theta}_i < \theta_i < \overline{\theta}_i$, $i = 1,...,p$ where $\underline{\theta}_i, \overline{\theta}_i$ are known bounds.

Let the output tracking error be defined as

$$e = y - y_r \tag{5}$$

where the $\mathcal{C}^n$ reference trajectory $y_r(t)$ is such that

$$y_r^{(i)}(t) \in \mathcal{L}_\infty,\ i = 0,...,n+1, \tag{6}$$

and $(\cdot)^{(i)}(t)$ denotes the ith derivative with respect to time. Our goal is to construct a $\mathcal{C}^0$ state feedback control $u(x_1,...,x_n)$ that ensures the boundedness of all closed-loop signals and $e(t) \to 0$ as $t \to \infty$ (or $\|e(t)\| \to \varepsilon$ as $t \to \infty$ where ε can be made arbitrarily small).

For the sake of reducing the notational complexity, the control construction that follows is presented for the case where $m_i = 1$, $i = 1,...,n$ and $d_i = 0$, $i = 1,...,n-1$. Note however that the main result is readily applicable to the MIMO case and when the disturbance is present in each equation of (4).

III. Preliminaries

To facilitate the design of the robust adaptive controller, we introduce the following parameter transformation. Let

$$\begin{aligned} &\overline{\theta} = \left(\overline{\theta}_1,\ldots,\overline{\theta}_p\right)^\top \qquad \underline{\theta} = \left(\underline{\theta}_1,\ldots,\underline{\theta}_p\right)^\top \\ &\Delta = \left\|\overline{\theta} - \underline{\theta}\right\| \qquad \tau = \frac{\overline{\theta} - \theta}{\Delta}, \end{aligned} \tag{7}$$

then $\|\underline{\theta}\| < \|\theta\| < \|\overline{\theta}\|$ and $\|\tau\| < 1$, where $\|\cdot\|$ denotes the 2-norm.

Now, consider the function

$$\begin{aligned} W \;=\; & -\tau^\top\Gamma^{-1}\ln(\hat{\tau}) - (1_p - \tau)^\top\Gamma^{-1}\ln(1_p - \hat{\tau}) \\ & +\tau^\top\Gamma^{-1}\ln(\tau) \\ & +(1_p - \tau)^\top\Gamma^{-1}\ln(1_p - \tau) \end{aligned} \tag{8}$$

where $\hat{\tau} \in \mathbb{R}^p$ denotes the estimate of the transformed parameter τ, $\ln(\xi) := \left(\ln\xi_i,\ldots,\ln\xi_p\right)^\top$, $1_p := (1,\ldots,1)_{p\times 1}$, and Γ is a positive-definite diagonal matrix. The above function has the following special properties.

Lemma 1: The function (8) is nonnegative and radially unbounded in $\mathcal{S} := \{\hat{\tau} \in \mathbb{R}^p \mid 0 < \hat{\tau}_i < 1\ \forall i\}$.

Proof: We write $W(\hat{\tau}) = \sum_{i=1}^p W_i(\hat{\tau}_i)$ where

$$\begin{aligned} W_i \;=\; & -\tau_i\Gamma_{ii}^{-1}\ln(\hat{\tau}_i) - (1-\tau_i)\Gamma_{ii}^{-1}\ln(1-\hat{\tau}_i) \\ & +\tau_i\Gamma_{ii}^{-1}\ln(\tau_i) \\ & +(1-\tau_i)^\top\Gamma_{ii}^{-1}\ln(1-\tau_i). \end{aligned} \tag{9}$$

It follows from (9) that

$$\frac{dW_i}{d\hat{\tau}_i} = \frac{-(\tau_i - \hat{\tau}_i)}{\Gamma_{ii}\hat{\tau}_i(1-\hat{\tau}_i)}. \tag{10}$$

It is easy to check that for $\hat{\tau}_i \in (0,1)$,

$$\frac{dW_i}{d\hat{\tau}_i} = \begin{cases} <0, & \hat{\tau}_i < \tau_i \\ =0, & \hat{\tau}_i = \tau_i \\ >0, & \hat{\tau}_i > \tau_i; \end{cases} \tag{11}$$

therefore, $W_i(\hat{\tau}_i)$ has a global minimum at $\hat{\tau}_i = \tau_i$. In fact, $W_i(\hat{\tau}_i = \tau_i) = 0$ from (9). Finally, it is not difficult to see that

$$\lim_{\hat{\tau}_i \to 0^+} W_i(\hat{\tau}_i) = +\infty \text{ and } \lim_{\hat{\tau}_i \to 1^-} W_i(\hat{\tau}_i) = +\infty. \quad (12)$$

■

Remark 1: The function in (8) was inspired by work of [1], [2], where a similar function was used in the stability analysis of a saturated dynamic controller.

IV. Robust Adaptive Control Law

Step 1

Let $z_1 = e$, and use (4a) and (7) to obtain

$$\begin{aligned} \dot{z}_1 &= \varphi_1^\intercal \theta + x_2 - \dot{y}_r \\ &= \varphi_1^\intercal \left(\overline{\theta} - \tau\Delta\right) - \dot{y}_r + \alpha_1 + z_2 \end{aligned} \quad (13)$$

where $z_2 = x_2 - \alpha_1$. We design the stabilizing function α_1 as

$$\alpha_1 = -c_1 z_1 - \varphi_1^\intercal \left(\overline{\theta} - \hat{\tau}\Delta\right) + \dot{y}_r \quad (14)$$

where $c_1 > 0$.

We now define

$$V_1 = \frac{1}{2} z_1^2 + W\left(\hat{\tau}\right) \quad (15)$$

where W was defined in (8).

Taking the derivative of (15) along (13) and (14) yields

$$\dot{V}_1 = -c_1 z_1^2 - \tilde{\tau}^\top \left(\varsigma_1 + \Gamma^{-1} N^{-1} \dot{\hat{\tau}}\right) + z_1 z_2 \quad (16)$$

where

$$\tilde{\tau} = \tau - \hat{\tau}, \quad \varsigma_1 = \varphi_1 z_1 \Delta, \quad N = \operatorname{diag}\left[\hat{\tau}_i \left(1 - \hat{\tau}_i\right)\right]_{p \times p}. \quad (17)$$

Step 2

After taking the derivative of z_2, we obtain

$$\begin{aligned} \dot{z}_2 &= \varphi_2^\intercal \left(\overline{\theta} - \tau\Delta\right) - \frac{\partial \alpha_1}{\partial x_1} \left[\varphi_1^\intercal \left(\overline{\theta} - \tau\Delta\right) + x_2\right] - \frac{\partial \alpha_1}{\partial \hat{\tau}} \dot{\hat{\tau}} \\ &\quad - \frac{\partial \alpha_1}{\partial y_r} \dot{y}_r - \frac{\partial \alpha_1}{\partial \dot{y}_r} \ddot{y}_r + \alpha_2 + z_3 \end{aligned} \quad (18)$$

where $z_3 = x_3 - \alpha_2$. We design the stabilizing function α_2 as

$$\begin{aligned} \alpha_2 &= -c_2 z_2 - \varphi_2^\intercal \left(\overline{\theta} - \hat{\tau}\Delta\right) + \frac{\partial \alpha_1}{\partial x_1} \left[\varphi_1^\intercal \left(\overline{\theta} - \hat{\tau}\Delta\right) + x_2\right] \\ &\quad + \frac{\partial \alpha_1}{\partial y_r} \dot{y}_r + \frac{\partial \alpha_1}{\partial \dot{y}_r} \ddot{y}_r - z_1 + \phi_2 \end{aligned} \quad (19)$$

where $c_2 > 0$ and ϕ_2 is a tuning function [15] to be selected.

If

$$V_2 = V_1 + \frac{1}{2} z_2^2, \quad (20)$$

its derivative along (18) and (19) is

$$\begin{aligned} \dot{V}_2 &= -c_1 z_1^2 - c_2 z_2^2 - \tilde{\tau}^\top \left(\varsigma_2 + \Gamma^{-1} N^{-1} \dot{\hat{\tau}}\right) \\ &\quad + z_2 \left(\phi_2 - \frac{\partial \alpha_1}{\partial \hat{\tau}} \dot{\hat{\tau}}\right) + z_2 z_3 \end{aligned} \quad (21)$$

where

$$\varsigma_2 = \varsigma_1 + \left(\varphi_2 - \frac{\partial \alpha_1}{\partial x_1} \varphi_1\right) z_2 \Delta. \quad (22)$$

If we set the tuning function ϕ_2 to

$$\phi_2 = -\frac{\partial \alpha_1}{\partial \hat{\tau}} N \Gamma \varsigma_2, \quad (23)$$

we can rewrite (21) as

$$\begin{aligned} \dot{V}_2 &= -c_1 z_1^2 - c_2 z_2^2 - \tilde{\tau}^\top \left(\varsigma_2 + \Gamma^{-1} N^{-1} \dot{\hat{\tau}}\right) \\ &\quad - z_2 \frac{\partial \alpha_1}{\partial \hat{\tau}} N \Gamma \left(\varsigma_2 + \Gamma^{-1} N^{-1} \dot{\hat{\tau}}\right) + z_2 z_3. \end{aligned} \quad (24)$$

Step i ($3 \leq i \leq n-1$)

Let $z_i = x_i - \alpha_{i-1}$, and take its derivative to obtain

$$\begin{aligned} \dot{z}_i &= \varphi_i^\intercal \left(\overline{\theta} - \tau\Delta\right) - \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_j} \left[\varphi_j^\intercal \left(\overline{\theta} - \tau\Delta\right) + x_{j+1}\right] \\ &\quad - \frac{\partial \alpha_{i-1}}{\partial \hat{\tau}} \dot{\hat{\tau}} - \sum_{j=1}^{i} \frac{\partial \alpha_{i-1}}{\partial y_r^{(j-1)}} y_r^{(j)} + \alpha_i + z_{i+1}. \end{aligned} \quad (25)$$

Now, design the stabilizing function α_i as

$$\begin{aligned} \alpha_i &= -c_i z_i - \varphi_i^\intercal \left(\overline{\theta} - \hat{\tau}\Delta\right) \\ &\quad + \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_j} \left[\varphi_j^\intercal \left(\overline{\theta} - \hat{\tau}\Delta\right) + x_{j+1}\right] \\ &\quad + \sum_{j=1}^{i} \frac{\partial \alpha_{i-1}}{\partial y_r^{(j-1)}} y_r^{(j)} - z_{i-1} + \phi_i \end{aligned} \quad (26)$$

where $c_i > 0$ and ϕ_i is the tuning function.

Set $V_i = V_{i-1} + \frac{1}{2} z_i^2$ and take its derivative along (25) and (26) to obtain

$$\begin{aligned} \dot{V}_i &= -\sum_{j=1}^{i} c_j z_j^2 - \tilde{\tau}^\top \left(\varsigma_i + \Gamma^{-1} N^{-1} \dot{\hat{\tau}}\right) \\ &\quad - \sum_{j=1}^{i-2} \left(z_{j+1} \frac{\partial \alpha_j}{\partial \hat{\tau}}\right) N \Gamma \left(\varsigma_{i-1} + \Gamma^{-1} N^{-1} \dot{\hat{\tau}}\right) \\ &\quad + z_i \left(\phi_i - \frac{\partial \alpha_{i-1}}{\partial \hat{\tau}} \dot{\hat{\tau}}\right) + z_i z_{i+1} \end{aligned} \quad (27)$$

where

$$\varsigma_i = \varsigma_{i-1} + \left(\varphi_i - \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_j} \varphi_j\right) z_i \Delta. \quad (28)$$

We first set the tuning function ϕ_i to

$$\phi_i = -\frac{\partial \alpha_{i-1}}{\partial \hat{\tau}} N \Gamma \varsigma_i + \bar{\phi}_i, \quad (29)$$

where $\bar{\phi}_i$ is an auxiliary tuning term, so we can rewrite (27) as

$$\begin{aligned}\dot{V}_i &= -\sum_{j=1}^{i} c_j z_j^2 - \tilde{\tau}^\top \left(\varsigma_i + \Gamma^{-1} N^{-1} \dot{\hat{\tau}} \right) \\ &\quad -\sum_{j=1}^{i-2} \left(z_{j+1} \frac{\partial \alpha_j}{\partial \hat{\tau}} \right) N\Gamma \left(\varsigma_{i-1} + \Gamma^{-1} N^{-1} \dot{\hat{\tau}} \right) \\ &\quad -z_i \frac{\partial \alpha_{i-1}}{\partial \hat{\tau}} N\Gamma \left(\varsigma_i + \Gamma^{-1} N^{-1} \dot{\hat{\tau}} \right) + z_i \bar{\phi}_i \\ &\quad + z_i z_{i+1}. \end{aligned} \tag{30}$$

From (30) it is clear that $\bar{\phi}_i$ needs to be designed as

$$\bar{\phi}_i = -\sum_{j=1}^{i-2} \left(z_{j+1} \frac{\partial \alpha_j}{\partial \hat{\tau}} \right) N\Gamma \left(\varphi_i - \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_j} \varphi_j \right) \Delta \tag{31}$$

so that we have

$$\begin{aligned}\dot{V}_i &= -\sum_{j=1}^{i} c_j z_j^2 - \tilde{\tau}^\top \left(\varsigma_i + \Gamma^{-1} N^{-1} \dot{\hat{\tau}} \right) \\ &\quad -\sum_{j=1}^{i-1} \left(z_{j+1} \frac{\partial \alpha_j}{\partial \hat{\tau}} \right) N\Gamma \left(\varsigma_i + \Gamma^{-1} N^{-1} \dot{\hat{\tau}} \right) \\ &\quad + z_i z_{i+1}. \end{aligned} \tag{32}$$

Step n

Taking the derivative of $z_n = x_n - \alpha_{n-1}$ yields

$$\begin{aligned}\dot{z}_n &= \varphi_n^\intercal \left(\overline{\theta} - \tau\Delta \right) - \sum_{j=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial x_j} \left[\varphi_j^\intercal \left(\overline{\theta} - \tau\Delta \right) + x_{j+1} \right] \\ &\quad - \frac{\partial \alpha_{n-1}}{\partial \hat{\tau}} \dot{\hat{\tau}} - \sum_{j=1}^{n} \frac{\partial \alpha_{n-1}}{\partial y_r^{(j-1)}} y_r^{(j)} + d_n + u. \end{aligned} \tag{33}$$

The control input is designed as

$$\begin{aligned} u &= -(c_n + k_n) z_n - \varphi_n^\intercal \left(\overline{\theta} - \hat{\tau}\Delta \right) \\ &\quad + \sum_{j=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial x_j} \left[\varphi_j^\intercal \left(\overline{\theta} - \hat{\tau}\Delta \right) + x_{j+1} \right] \\ &\quad + \sum_{j=1}^{n} \frac{\partial \alpha_{n-1}}{\partial y_r^{(j-1)}} y_r^{(j)} - z_{n-1} + \phi_n \end{aligned} \tag{34}$$

where $c_n, k_n > 0$ and ϕ_n is the final tuning function.

Given that $V_n = V_{n-1} + \frac{1}{2} z_n^2$, it follows from (33) and (34) that

$$\begin{aligned}\dot{V}_n &= -\sum_{j=1}^{n} c_j z_j^2 - \tilde{\tau}^\top \left(\varsigma_n + \Gamma^{-1} N^{-1} \dot{\hat{\tau}} \right) \\ &\quad -\sum_{j=1}^{n-2} \left(z_{j+1} \frac{\partial \alpha_j}{\partial \hat{\tau}} \right) N\Gamma \left(\varsigma_{n-1} + \Gamma^{-1} N^{-1} \dot{\hat{\tau}} \right) \\ &\quad + z_n \left(\phi_n - \frac{\partial \alpha_{i-1}}{\partial \hat{\tau}} \dot{\hat{\tau}} \right) + z_n d_n - k_n z_n^2 \end{aligned} \tag{35}$$

where

$$\varsigma_n = \varsigma_{n-1} + \left(\varphi_n - \sum_{j=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial x_j} \varphi_j \right) z_n \Delta. \tag{36}$$

Setting the tuning function to

$$\begin{aligned}\phi_n &= -\frac{\partial \alpha_{n-1}}{\partial \hat{\tau}} N\Gamma \varsigma_n \\ &\quad -\sum_{j=1}^{n-2} \left(z_{j+1} \frac{\partial \alpha_j}{\partial \hat{\tau}} \right) N\Gamma \left(\varphi_n - \sum_{j=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial x_j} \varphi_j \right) \Delta \end{aligned} \tag{37}$$

gives

$$\begin{aligned}\dot{V}_n &= -\sum_{j=1}^{n} c_j z_j^2 - \tilde{\tau}^\top \left(\varsigma_n + \Gamma^{-1} N^{-1} \dot{\hat{\tau}} \right) \\ &\quad -\sum_{j=1}^{n-1} \left(z_{j+1} \frac{\partial \alpha_j}{\partial \hat{\tau}} \right) N\Gamma \left(\varsigma_n + \Gamma^{-1} N^{-1} \dot{\hat{\tau}} \right) \\ &\quad + z_n d_n - k_n z_n^2. \end{aligned} \tag{38}$$

We now design the adaptation law as

$$\dot{\hat{\tau}} = -N\Gamma\varsigma_n, \quad \hat{\tau}(0) \in \mathcal{S}, \tag{39}$$

which ensures $\hat{\tau}(t) \in \mathcal{S}$ for all time due to the form of N (see (17)). Finally, we get after completing the squares

$$\dot{V}_n \leq -\sum_{j=1}^{n} c_j z_j^2 + \frac{d_n^2}{4k_n} \leq -\min_j (c_j) \|z\|^2 + \frac{\overline{d}_n^2}{4k_n} \tag{40}$$

where assumption A2 was used and $z := (z_1, \ldots, z_n)^\top$. Since V_n is nonnegative in $\mathbb{R}^n \times \mathcal{S}$ and $\dot{V}_n$ is negative in the set

$$\left\{ z \in \mathbb{R}^n \mid \|z\| > \frac{\overline{d}_n}{2\sqrt{\left(k_n \min_j (c_j) \right)}} \right\}, \tag{41}$$

we know that $z(t)$ is ultimately bounded and the bound can be made arbitrarily small. The boundedness of all other closed-loop signals can be easily checked by signal tracing arguments. It also follows from (40) and Barbalat's lemma that if $d_n(t) \to 0$ for $t \geq T > 0$, then $z(t) \to 0$ as $t \to \infty$.

V. Extension

Due to the nature of the above formulation, the proposed adaptive control can be fused with the dynamic robust mechanism reported in [4] to ensure asymptotic tracking even if the disturbance never disappears. However, assumptions A1 and A2 given in Section II would have to be replaced, respectively, by

A1. $\varphi_i \in \mathcal{C}^{n+1-i}$, $i = 1, ..., n$.

A2. $d_n \in \mathcal{C}^2$ and $\|d_n(t)\|_{\mathcal{L}_\infty} \leq \overline{d}_0$, $\left\|\dot{d}_n(t)\right\|_{\mathcal{L}_\infty} \leq \overline{d}_1$, and $\left\|\ddot{d}_n(t)\right\|_{\mathcal{L}_\infty} \leq \overline{d}_2$ where $\overline{d}_0, \overline{d}_1, \overline{d}_2$ are known positive constants.

Furthermore, it would require that the disturbance be matched in the sense that $d_i = 0, i = 1, \ldots, n-1$ in (4). The derivation of this new control law is straightforward given [4] and the formulation in Section IV of the present paper, so the details are omitted here.

VI. Simulation

For simulation purposes, we considered the parametric strict-feedback system of (4) with the following model

$$\varphi_1 = \begin{bmatrix} -x_1^2 \\ \frac{1}{4}x_1^3 \end{bmatrix} \quad \varphi_2 = \begin{bmatrix} -x_1^2 \\ \frac{1}{4}x_2^3 \end{bmatrix} \quad \varphi_3 = \begin{bmatrix} -x_2 \\ \frac{1}{4}x_1^2 x_3 \end{bmatrix}, \tag{42}$$

$$\theta = \begin{bmatrix} 2 \\ 1 \end{bmatrix} \quad d_3(t) = \begin{cases} \sin 3t & 0 \le t \le 15 \\ 0 & \text{otherwise.} \end{cases}$$

The reference trajectory was selected as

$$y_r = \sin(t)\left(1 - \exp\left(-\frac{1}{3}t^3\right)\right). \tag{43}$$

The initial conditions of the system were set to $x_1(0) = x_2(0) = x_3(0) = 0$, and $\hat{\tau}(0) = (0.6, 0.4)^\top$. The bounds on the uncertain parameter vector were set to $\underline{\theta} = (1.5, 0.677)^\top$ and $\overline{\theta} = (3, 2)^\top$. The control gains were selected as

$$c_1 = 1, \quad c_2 = 10, \quad c_3 = 10, \quad k_3 = 10, \quad \Gamma = I_2. \tag{44}$$

The simulation results are given in Figures 1 and 2. Figure 1 shows the output tracking error $e(t)$ and control input $u(t)$. Figure 2 shows the transformed and actual parameter estimates $\hat{\tau}(t)$ and $\hat{\theta}(t)$, respectively. The simulations confirm the results predicted by the theoretical analysis.

VII. Conclusions

We introduced a new robust adaptive tracking controller with no overparametrization for nonlinear parametric strict-feedback systems subjected to unknown additive disturbance. The class of systems considered here did not require the disturbance to be generated by an exosystem; rather, we only assumed continuity and boundedness of the disturbance signal. A new projection-like operator was applied to the adaptive backstepping design that enabled the use of the standard tuning function approach and avoided the need for additional nonlinear damping terms in the control. The proposed control guarantees practical tracking in the presence of the disturbance and asymptotic tracking when the disturbance disappears. Further, the proposed adaptive design can be readily augmented with the dynamic robust control of [4] to guarantee asymptotic tracking if the disturbance never disappears.

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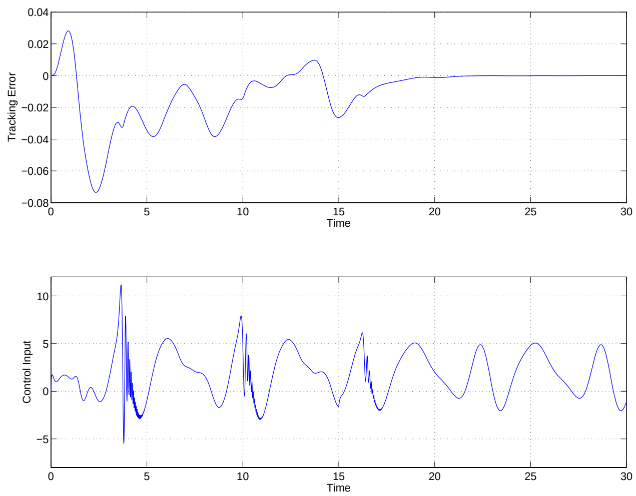

Fig. 1. Top plot: tracking error $e(t)$. Bottom plot: control input $u(t)$.

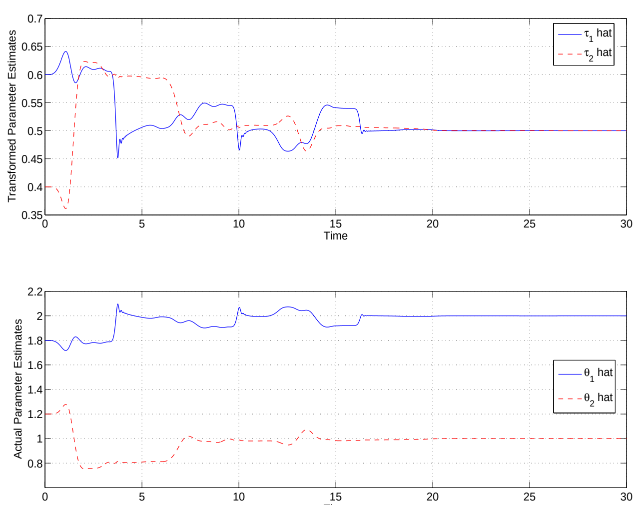

Fig. 2. Top plot: transformed parameter estimate $\hat{\tau}(t)$. Bottom plot: actual parameter estimate $\hat{\theta}(t)$.