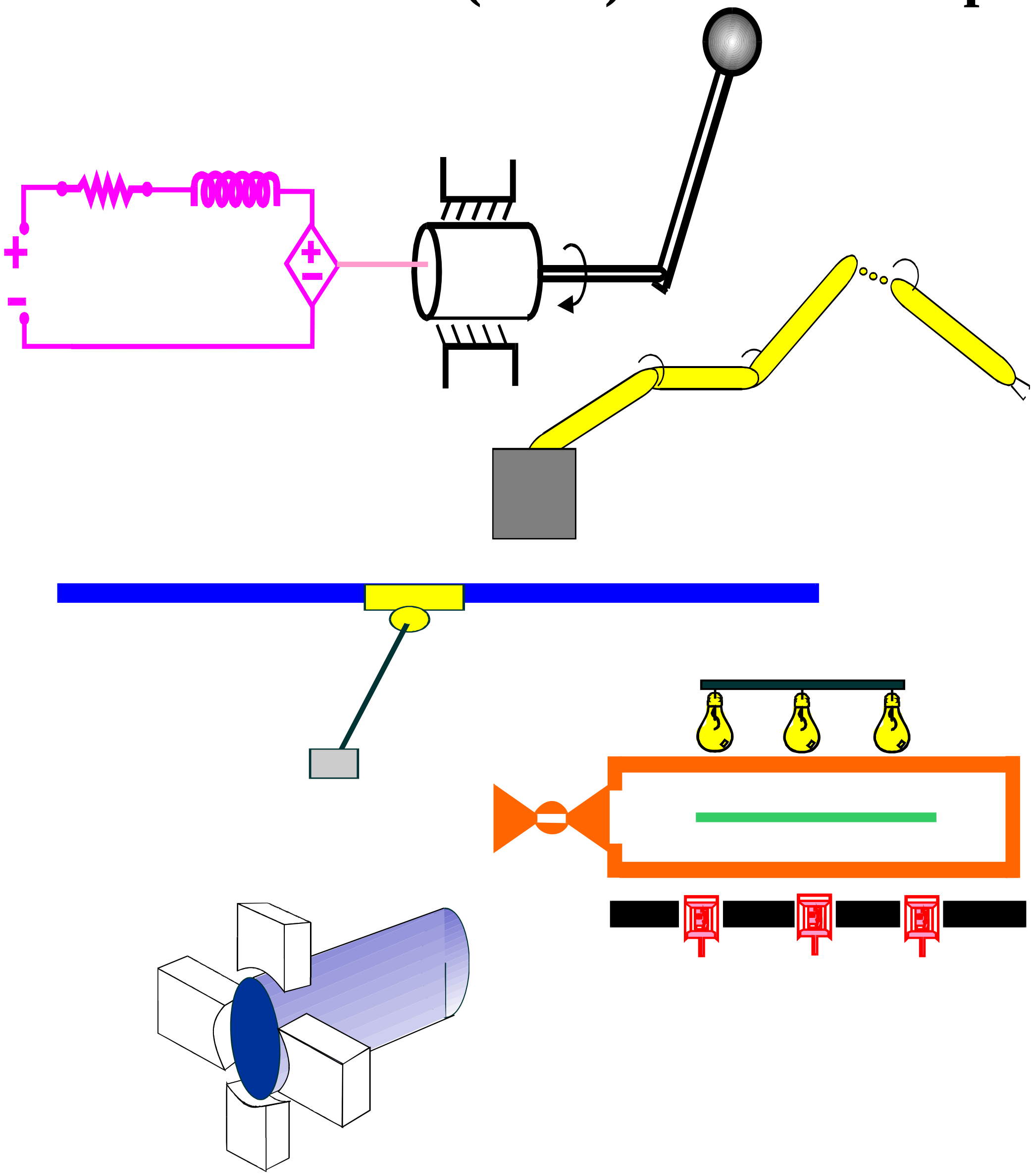
Clemson University
College of Engineering and Science
Control and Robotics (CRB) Technical Report



Number: CU/CRB/9/14/09/#1

Title: Robust Control of One Degree-Of-Freedom Exercise Machines

Authors: Apoorva Kapadia, Enver Tatlicioglu, and Darren M. Dawson

Robust Control of One Degree-Of-Freedom Exercise Machines

Apoorva Kapadia, Enver Tatlicioglu, and Darren M. Dawson

Abstract: A novel robust controller is proposed for a one degree-of-freedom exercise machine. The objective of the controller is to achieve maximal user power output while simultaneously ensuring that the system remains passive and stable in the interest of user safety. A desired velocity trajectory is designed to achieve maximum user power output. To that end, a powerful numerical extremal-seeking algorithm is employed to simultaneously optimize the user power output while also satisfying certain minimal assumptions about differentiability of the designed trajectory. A nonlinear robust controller is designed to ensure that the user input velocity tracks the desired velocity while providing passive and stable operation. Lyapunov-based stability tools are employed to prove semi-global tracking. Numerical simulation results are presented to highlight the performance and effectiveness of the proposed controller.

I. INTRODUCTION

It is safe to say that movement of our limbs is practically indispensable for daily activity. It is needed to perform one's daily routine, achieving individual athletic goals, accomplish tasks that require additional strength requirements, and even rehabilitate neuromuscular systems after trauma. To that end, resistance training has been the simplest and most common method to increase muscular strength as well as stamina. It has been noted that strength training is optimum with systems having high resistance performed over short periods of time with long rest periods [1]. Conversely, endurance training involves a high number of repetitions on lower resistance systems with shorter breaks between repetitions.

Classic resistance training methods called for the use of freeweights such as dumbells and weightlifting bars, however, over the past two decades, machines have become increasingly popular in fitness centers and home gyms. These passive open-loop devices include pulley systems, spring-loaded machines, friction-based fans or brakes, and more recently motor driven systems. Most of these machines only allow for manual resistance adjustments to increase or decrease the weight providing the resistance, and thus could be susceptible to the state-of-mind of the user resulting in diminished efficiency. Other concerns with open-loop exercise machines are listed in [2].

Using modern mechatronic apparatus that apply novel control technologies allows for newer conditioning and rehabilitation exercises, with the feedback loop providing for an enhanced qualitative effect [3]. Such devices allow for the resistance to be tailored to the users based on their needs and requirements and the muscle groups in use. One of the first attempts at an active exercise machine was presented in [4]. The development in [4] aimed to provide a virtual sensation of applying a force on a simple mass-spring or a mass-damper system with a 2 degree-of-freedom (dof) machine. As opposed to passive exercise machines, this system could be programmed to mimic almost any trajectory-based force. However, it did not provide a passive relationship with the user and no automatic optimization technique was presented.

Characteristics for a smart exercise machine were listed in [5], stating that the machine should identify the user's strength characteristics and upon which it should base its optimal exercise routine on resulting in an optimal workout. Thus the machine would be user dependent and function differently for different users, and finally, the machine would have to be safe for physical interaction with humans. The same researchers later addressed the passivity and optimization problems in [6] and [7]. The passivity of the controller was developed based on the inversely proportional relationship between the applied force and the velocity of the muscle systems known as the Hill Curve [8]. The exercise strategy was transformed into a velocity field within the exercise machine configuration space. The objective of this adaptive controller was to force the user to track an unknown desired velocity field to maximize power expenditure, while maintaining passivity. This strategy however did require a training phase to account for unknown user biomechanics. In [9], a 1 dof passive braking system was developed to provide constant rotational velocity that did not require any *a priori* user information. The passivity was incorporated into a simple PI controller that maintained constant velocity without always allowing maximum output force realization by the user. In [2], prelimiary results were presented for an active arm-curling machine that was retrofitted with a linear actuator to develop a haptic interface for controlled strength training and rehabilitation based on the Hill Curve. Similarly in [10], an arm exercise machine was supplemented with a high torque DC motor to provide active resistance. The design of an exercise machine using magneto-rheological dampers that allow varying resistance levels was highlighted in [11], however no control loop or self-optimization feature was presented. In [12], extremum velocity seeking algorithms

---

This work is supported in part by a DOE Contract and a Honda Corporation Grant.

Contact Author: Apoorva Kapadia is with the Department of Electrical & Computer Engineering, Clemson University, Clemson, SC 29634-0915 (akapadi@clemson.edu).

Enver Tatlicioglu is with the Department of Electrical & Electronics Engineering, Izmir Institute of Technology, Urla, Izmir 35430 Turkey (enver@envertatlicioglu.com).

Darren. M. Dawson is with the Department of Electrical & Computer Engineering, Clemson University, Clemson, SC 29634-0915 (darren.dawson@ces.clemson.edu).

were used to find an optimal desired velocity set-point to ensure maximum user power output while ensuring passivity of the 1 dof system. The controllers were designed under the assumption of exact model knowledge for the cases of input torque measurement as well as torque estimation.

The goal of the exercise machine controller presented in this paper is to assist the user accomplish an optimal level of exercise in a relatively short amount of time. To that end, a robust nonlinear controller for a 1 dof exercise machine is detailed to achieve two goals, that of seeking out an extremal velocity set-point to be tracked as the desired trajectory and maximizing user power output. The extremal velocity is propagated using a numerical method, namely the Golden Section Search of Brent's Method outlined in [13]. To ensure user safety, the controller is shown to be passive with respect to the user input. No knowledge of the user input biomechanics is required, and the restrictive assumption of exact machine dynamics information is relaxed and compensated for in the proposed controller. Lyapunov-based analysis techniques are presented to highlight the stability of the controller along with a dynamic simulation.

The paper is organized as follows, Section II details the system model along with the model assumptions. Section III outlines the control development while the stability analysis is presented in Section IV. Section V highlights the numerical extremum seeking algorithm and we end with simulation results in section VI along with concluding remarks in section VII.

II. DYNAMIC MODEL

The model for a 1-DOF exercise machine, a simplified version of a manipulator dynamic model, is assumed to be of the following form:

$$J\ddot{q} = \tau\left(\dot{q}\right) + u, \tag{1}$$

where $J \in \mathbb{R}^{+}$ denotes the uncertain constant inertia of the machine, $\dot{q}(t)$, $\ddot{q}(t) \in \mathbb{R}$ denote the angular velocity and acceleration of the machine, respectively, $\tau\left(\dot{q}\right) \in \mathbb{R}$ represents the measurable user torque input, and $u(t) \in \mathbb{R}$ represents the motor control input. In the subsequent development, it is assumed that the user input torque is second-order differentiable, $\left(i.e.,\ \tau\left(\dot{q}\right) \in C^{2}\right)$ and without loss of generality, to be unidirectional satisfying the following inequalities,

$$0 \leq \tau\left(\dot{q}\right) \leq \tau_{max}, \tag{2}$$

where $\tau_{max} \in \mathbb{R}^{+}$ denotes the maximum possible user-applied torque input into the system.

III. CONTROLLER DESIGN

A. Control Objectives

Our controller is designed so as to maximize the user's power output, denoted by, $p\left(\dot{q}\right) \in \mathbb{R}$ while ensuring that the exercise machine tracks a desired velocity $\dot{q}_d(t) \in \mathbb{R}$. As defined in [6], the modified user power output is of the following form

$$p\left(\dot{q}\right) = \tau\left(\dot{q}\right)\dot{q}^{\rho}(t), \tag{3}$$

where $\rho \in \mathbb{R}^{+}$ is a constant. It should be noted that to achieve maximum user power output, the desired trajectory $\dot{q}_d(t) \in \mathbb{R}$ must eventually achieve an optimal unknown user-dependent velocity setpoint, denoted by $\dot{q}_d^{*} \in \mathbb{R}^{+}$. An additional control objective is the passivity, which is achieved by ensureing the machine will remain passive with respect to the user's power input. The passivity objective is achieved if the following inequality is satisfied [6]

$$\int_{t_0}^{t} \tau(\sigma)\dot{q}(\sigma)d\sigma \geq -c^{2}, \tag{4}$$

where $c \in \mathbb{R}^{+}$ is a bounding constant. Satisfaction of the inequality in (4) ensures that the flow of energy in the system occurs only from the user into the machine thus resulting in safe operation by the user.

B. Control Development

Let $e(t) \in \mathbb{R}$ be the velocity tracking error signal defined as

$$e \triangleq \dot{q} - \dot{q}_d, \tag{5}$$

where $\ddot{q}_d(t) \in \mathbb{R}$ is the desired velocity. In the subsequent development the standard assumption that $\dot{q}_d(t)$, $\ddot{q}_d(t)$ and $\dddot{q}_d(t) \in \mathcal{L}_{\infty}$ will be made. To facilitate the controller development, a filtered tracking error, denoted by $r(t) \in \mathbb{R}$, is defined as follows

$$r \triangleq \dot{e} + e. \tag{6}$$

Taking the time derivative of (6) and then multiplying with J results in the following expression

$$J\dot{r} = J\dddot{q} - J\dddot{q}_d + J\dot{e}, \tag{7}$$

in which the second-order time derivative of (5) was also utilized.

As the primary step in control design, to partially feedback linearize the system, the control input is designed as follows

$$u \triangleq T - \tau, \tag{8}$$

where $T(t) \in \mathbb{R}$ is an auxiliary control input that is yet to be designed. Substituting the control input in (8) into the system dynamics in (1) results in the following simplified system model

$$J\ddot{q} = T. \tag{9}$$

Substituting the time derivative of (9) into (7) results in the following expression

$$J\dot{r} = \dot{T} - \dddot{q}_d + J\dot{e}. \tag{10}$$

To facilitate the control design, the expression in (10) can be rewritten as

$$J\dot{r} = \dot{T} + \tilde{N} + N_d - e, \tag{11}$$

where $\tilde{N}(t) \in \mathbb{R}$ is an auxiliary function defined as

$$\tilde{N} \triangleq N - N_d, \tag{12}$$

in which $N(t) \in \mathbb{R}$ is defined as

$$N \triangleq -J\dddot{q}_d + J\dot{e} + e, \tag{13}$$

with $N_d(t) \in \mathbb{R}$ defined as

$$N_d \triangleq N|_{\dot{q}=\dot{q}_d,\ddot{q}=\ddot{q}_d} = -J\dddot{q}_d. \tag{14}$$

It should be noted that $\tilde{N}(t)$, defined in (12), can be bounded as

$$|\tilde{N}(t)| \leq \rho\left(|z(t)|\right)\|z(t)\|, \tag{15}$$

where $z(t) \in \mathbb{R}^2$ is defined as

$$z \triangleq \left[e \quad r\right]^T, \tag{16}$$

while $\rho\left(\|z(t)\|\right) \in \mathbb{R}^+$ is a non-decreasing bounding function in $\|z(t)\|$. Further, it can be seen that $N_d(t)$ and $\dot{N}_d(t)$ are bounded given that the higher order derivatives of $\dot{q_d}(t)$ are bounded

$$\|N_d(t)\| \leq \zeta_1 \quad \|\dot{N}_d(t)\| \leq \zeta_2, \tag{17}$$

where ζ_1 and $\zeta_2 \in \mathbb{R}^+$ are known bounding constants.

Thus, based on the structure of (11) and the subsequent stability analysis, the controller is designed to be of the following form [14]

$$\begin{aligned} T = \left(k_s+1\right)&\left[e\left(t_0\right) - e(t) - \textstyle\int_{t_0}^{t} e(\sigma)d\sigma\right] \\ &- \left(\beta_1+\beta_2\right)\textstyle\int_{t_0}^{t} sgn(e(\sigma))d\sigma \end{aligned} \tag{18}$$

where $k_s, \beta_1, \beta_2 \in \mathbb{R}^+$ represent control gains and $sgn(\cdot)$ is the standard signum function. In (18) the term $e\left(t_0\right)$ was used to ensure that $u\left(t_0\right) = 0$. Thus, the time derivative of (18) is given as

$$\dot{T} = -\left(k_s+1\right)r - \left(\beta_1+\beta_2\right)sgn(e). \tag{19}$$

Substituting (19) in (11) results in the following closed-loop error system

$$J\dot{r} = -\left(k_s+1\right)r - \left(\beta_1+\beta_2\right)sgn(e) + \tilde{N} + N_d - e. \tag{20}$$

IV. STABILITY ANALYSIS

Theorem 1: The controllers in (8) and (18) guarantee that all system signals are bounded under closed-loop operation and the velocity tracking control objective is met in the sense that $\dot{q}(t) \to \dot{q}_d(t)$ as $t \to \infty$ provided that

$$\beta_1 > \zeta_1 + \zeta_2. \tag{21}$$

Proof: See Appendix I. ■

Theorem 2: The controllers in (8) and (18) guarantee that the closed-loop system is passive with respect to the user power.

Proof: See Appendix II. ■

V. DESIRED TRAJECTORY GENERATOR

When $\dot{q}(t)$ tracks $\dot{q}_d(t)$ and assuming $\dot{q}_d(t) \to \dot{q}_d^*$ as $t \to \infty$, the modified user power output from (3) can be rewritten using the following approximation

$$p \cong \tau\left(\dot{q}_d\right)\dot{q}_d, \tag{22}$$

where $\rho = 1$, thus ensuring that at q_d^*, the user power output will be maximized. Hitherto, $\dot{q}_d^*$ is unknown, thus a powerful numerical extremum-seeking algorithm is employed to find the desired extremal velocity. While there are several methods that can be employed, Brent's Method was found to be the simplest and most powerful. Brent's Method can be defined as a complex root-finding algorithm that combine the bisection and the inverse quadratic interpolation methods and is considered an upgrade over the secant method. Its use here is attributed to the fact that only two initial guesses are required by the algorithm with the only caveat that the extremum $\dot{q}_d^*$ be within the bounds of the two guesses. Additionally, the only input required by the algorithm is the function using the unknown extremal constant. In this case, the function is the user input power $p(t)$, being measured in (3). The resulting output of the extremum-seeking algorithm is then filtered to produce a smooth curve. Details of the implementation of this algorithm can be found in [12].

VI. SIMULATION RESULTS

A numerical simulation was conducted to illustrate the performance of the controllers presented in (8) and (18). Based on the assumptions made in the system model, the user input torque was modeled to have the following form,

$$\tau = b.exp\left(-a\dot{q}\right), \tag{23}$$

where a and $b \in \mathbb{R}^+$ are amplification constants. Substituting (23) in (3) yields

$$p = \left(b.exp\left(-a\dot{q}\right)\right)\dot{q}^{\rho}. \tag{24}$$

The power expression in (24) is maximized for the following value of q_d^*.

$$q_d^* = \frac{\rho}{a} \tag{25}$$

The inertia of the system and the control gains were given the following values

$$\begin{array}{ccc} a = 0.1 & b = 16 & k_s = 5 \\ \beta_1+\beta_2 = 2 & J = 1\left[kg.m^2\right] & \rho = 1 \end{array} \tag{26}$$

and all initial conditions were taken to be zero. Figure 1 shows the user input torque $\tau(t)$, while the user angular velocity $\dot{q}(t)$ is presented in Figure 5 . The desired angular velocity $\dot{q}_d(t)$, computed using Brents method is depicted in Figure 2. The error between user velocity and desired velocity $e(t)$, is shown in Figure 3, while the system control input $u(t)$, is presented in Figure 4. From Figure 3, it is clear that the velocity error signal is driven to zero and the simulation is validated by the fact that based on the choice of the control gains $a = 0.1$ and $\rho = 1$, from (26), $\dot{q}_d^* = 10$ as seen in Figure 2.

VII. CONCLUSION

In this paper, a robust controller for a one degree-of-freedom exercise machine was developed. The design ensured maximum user power output while keeping the system passive and stable in the interest of user safety. A powerful numerical extremum seeking algorithm was used to find the unknown user-dependent velocity set-point for the generated desired velocity trajectory. The controller was designed to tracked the desired velocity trajectory while simultaneously keeping the system passive. The stability analysis provided

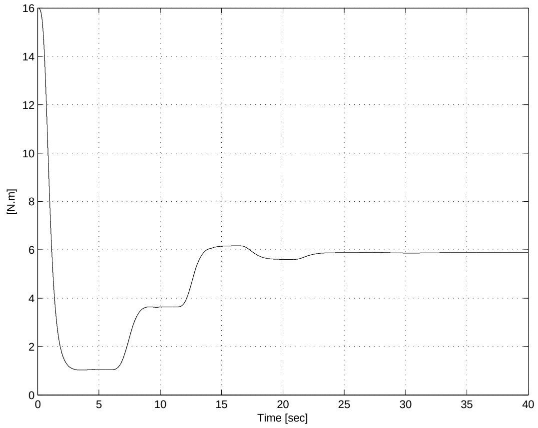


Fig. 1. User Input Torque.

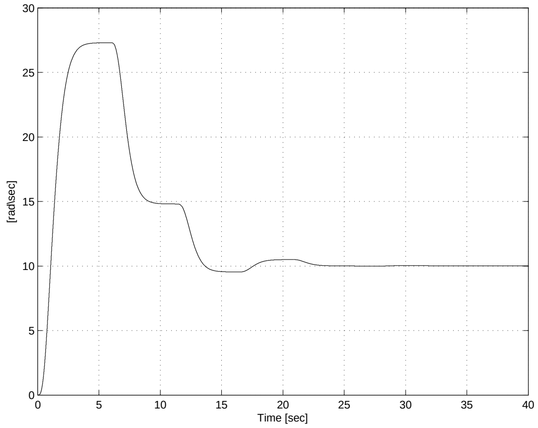


Fig. 2. Desired Angular Velocity.

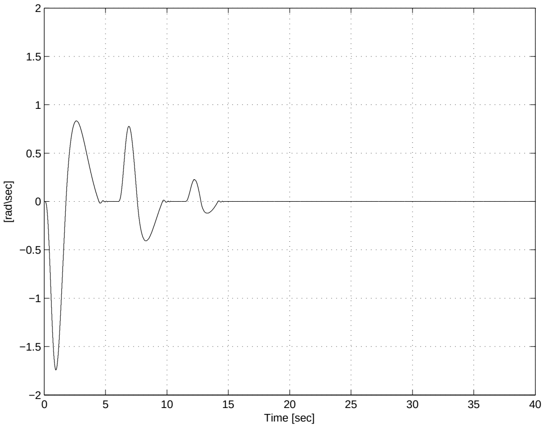


Fig. 3. Angular Velocity Error.

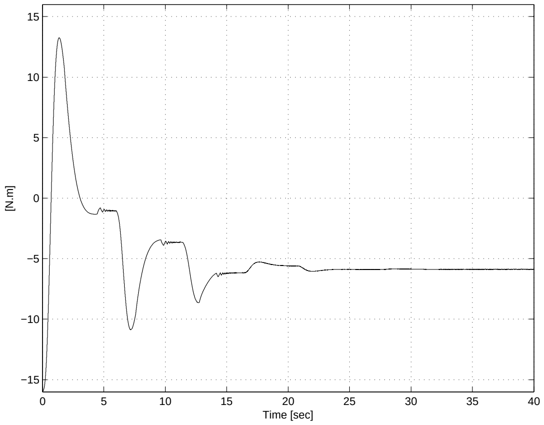


Fig. 4. Machine Control Input.

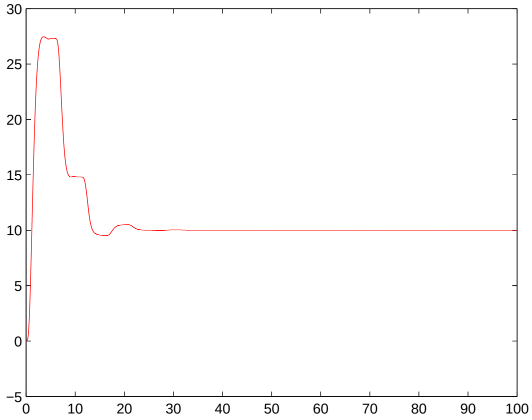


Fig. 5. User Input Velocity.

semi-global tracking proved through a Lyapunov-type analysis. Numerical simulation results were provided to highlight the performance the proposed controller.

APPENDIX I
PROOF OF THEOREM 1

Lemma 1: Let $L_1(t)$ and $L_2(t) \in \mathbb{R}$ be defined such that

$$L_1 \triangleq r\left(N_d + \beta_1 sgn(e)\right), \tag{27}$$

$$L_2 \triangleq \beta_2 \dot{e}\ sgn(e), \tag{28}$$

and if β_1 and β_2 are selected so as to satisfy (21)

$$\int_{t_0}^{t} L_1(\sigma)d\sigma \leq \zeta_1$$

$$\int_{t_0}^{t} L_2(\sigma)d\sigma \leq \zeta_2, \tag{29}$$

where ζ_1 and $\zeta_2 \in \mathbb{R}^+$ are constants such that

$$\zeta_1 \triangleq \beta_1 |e\left(t_0\right)| - e\left(t_0\right) N_d\left(t_0\right),$$

$$\zeta_2 \triangleq \beta_2 |e\left(t_0\right)|. \tag{30}$$

Proof: See [12]. ■

Let $P_1(t)$ and $P_2(t) \in \mathbb{R}$ be defined such that

$$P_1 \triangleq \zeta_1 - \int_{t_0}^{t} L_1(\tau)d\tau \geq 0, \tag{31}$$

$$P_2 = \zeta_2 - \int_{t_0}^{t} L_2(\tau)d\tau \geq 0. \tag{32}$$

The results from Lemma 1 can be applied to show $P_1(t)$ and $P_2(t)$ are non-negative. Let $V(y,t) \in \mathbb{R}$ represent a non-negative function defined as

$$V \triangleq \frac{1}{2}e^2 + \frac{1}{2}Jr^2 + P_1 + P_2, \tag{33}$$

where $y(t) \in \mathbb{R}^4$ is defined as

$$y(t) \triangleq \left[\begin{array}{ccc} z^T & \sqrt{P_1} & \sqrt{P_2} \end{array} \right]^T. \tag{34}$$

It can be seen that the expression in (33) can be bounded such that

$$W_1(y) \leq V(y,t) \leq W_2(y), \tag{35}$$

where $W_1(y) = \lambda_1 \|y(t)\|^2$ and $W_2(y) = \lambda_2 \|y(t)\|^2$, with

$$\begin{array}{c} \lambda_1 \triangleq \frac{1}{2} min\left\{1, J\right\} \\ \lambda_2 \triangleq max\left\{1, \frac{1}{2}J\right\}. \end{array} \tag{36}$$

The time derivative of (33) is given as follows

$$\dot{V} \triangleq e\dot{e} + Jr\dot{r} + \dot{P}_1 + \dot{P}_2. \tag{37}$$

After substituting (6) and (27) along with the time derivatives of (31) and (32) into (37) and then simplifying yields the following expression

$$\dot{V} = -e^2 - r^2 - k_s r^2 + r\tilde{N} - \beta_2 |e|. \tag{38}$$

This can further be rewritten as

$$\dot{V} \leq -\|z\|^2 - k_s r^2 + r\tilde{N} - \beta_2 |e|. \tag{39}$$

The expression in (39) can be rewritten by using the expression in (15) such that

$$\dot{V} \leq -\|z\|^2 - \left(k_s abs(r)^2 + r\rho(\cdot)\|z\|\right) - \beta_2 |e|. \tag{40}$$

After completing the squares in the bracketed term in (39), it can be seen that

$$\dot{V} \leq -\left(1 - \frac{\rho^2(\cdot)}{4k_s}\right)\|z\|^2 - \beta_2 |e|. \tag{41}$$

The expression in (41) can be further simplified as

$$\dot{V} \leq W(y) - \beta_2 |e|, \tag{42}$$

where

$$k_s > \frac{\rho^2(\cdot)}{4}$$

or

$$\|z\| < \rho^{-1}\left(2\sqrt{k_s}\right),$$

and $W(y) = -\gamma \|z\|^2$ where $\gamma \in \mathbb{R}^+$ is a constant. From (42) and the definition of $W(y)$, the regions $\mathcal{D}$ and $\mathcal{S}$ can thus be specifically defined as

$$\mathcal{D} \triangleq \left\{ y : \|y\| \leq \rho^{-1}\left(2\sqrt{k_s}\right)\right\} \tag{43}$$

$$\mathcal{S} \triangleq \left\{ y \in \mathcal{D} : W_2(y) < \lambda_1 \left(\rho^{-1}\left(2\sqrt{k_s}\right)\right)^2 \right\} \tag{44}$$

It should be noted that the region of attraction of (44) can be made arbitrarily large to encompass any initial condition simply by increasing the control gain k_s, thus yielding a semi-global stability result. Thus, the region of attraction can be calculated as

$$W_2\left(y\left(t_0\right)\right) < \lambda_1 \left(\rho^{-1}\left(2\sqrt{k_s}\right)\right)^2,$$

implying

$$\|y\left(t_0\right)\| < \sqrt{\frac{\lambda_1}{\lambda_2}}\rho^{-1}\left(2\sqrt{k_s}\right)^2, \tag{45}$$

and rearranging results in

$$k_s > \frac{1}{4}\rho^2\left(\sqrt{\frac{\lambda_1}{\lambda_2}}\|y\left(t_0\right)\|\right). \tag{46}$$

Thus by using the filtered tracking error (6) and (5) in (34, $y\left(t_0\right)$ takes the form

$$\|y\left(t_0\right)\| = \sqrt{e^2\ y\left(t_0\right) + \left(\dot{e}\left(t_0\right) + e\left(t_0\right)\right)^2 + \zeta_1 + \zeta_2}. \tag{47}$$

Thus from (33) and (38), it is clear that $V(t) \in \mathcal{L}_\infty$; hence $e(t), r(t) \in \mathcal{L}_2 \cup \mathcal{L}_\infty$. Also, from (40) it can be concluded that $e(t) \in \mathcal{L}_1$. Thus from (6), it is clear that $\dot{e}(t) in \mathcal{L}_\infty$. Using the fact that $\dot{q}_d(t) \in \mathcal{L}_\infty$, from (5). From these statements of signal boundedness, it can be easily inferred that $u(t) \in \mathcal{L}_\infty$. From (20), it is clear that $\dot{r}(t) \in \mathcal{L}_\infty$. Since $e(t)$, $r(t) \in \mathcal{L}_2 \cup \mathcal{L}_\infty$ and $\dot{e}(t)$, $\dot{r}(t) \in \mathcal{L}_\infty$, it can be concluded that $|r(t)|$, $|e(t)| \to 0$ as $t \to \infty$. It is thus clear $\dot{q}(t) \to \dot{q}_d(t)$ and $\dot{q}_d(t) \to \dot{q}_d^*$.

APPENDIX II
PROOF OF THEOREM 2

The substitution of the modified form of (5) into the left hand side of (4) yields

$$\int_{t_0}^{t} \tau(\sigma)\dot{q}(\sigma)d\sigma = \int_{t_0}^{t} \tau(\sigma)\left(e(\sigma) + \dot{q}_d(\sigma)\right)d\sigma. \tag{48}$$

From the assumptions made in the system model that desired trajectory is in the same direction as the user input (thus positive) and the user input torque is unidirectional, the right most term from (48) will always be positive. From the results of Theorem 1, we know that $e(t) \in \mathcal{L}_1$, thus using (2), it can be seen that

$$\int_{t_0}^{t} \tau(\sigma)e(\sigma)d\sigma \geq -\tau_{max}\int_{t_0}^{t} |e(\sigma)|d\sigma \geq -c^2. \tag{49}$$

Thus the passivity condition is satisfied. More information on passivity and its applications can be found in [15].

REFERENCES

[1] L. Bowling, *Resistance Training*. Durham, NC: Carolina Academic Press, 2007.

[2] C. R. Carignan and J. Tang, "A haptic control interface for a motorized exercise machine," in *Proc. IEEE Int. Conf. Robot. Autom.*, Pasadena, CA, 2008, pp. 2055–2060.

[3] R. F. Erlandson, "Applications of robotic/mechatronic systems in special education, rehabilitation therapy, and vocational training: A paradigm shift," *IEEE Trans. Rehab. Eng.*, vol. 3, no. 1, pp. 22–34, Mar. 1995.

[4] H. Kazerooni and M. G. Ker, "A virtual exercise machine," in *Proc. IEEE Int. Conf. Robot. Autom.*, Atlanta, GA, 1993, pp. 232–238.

[5] P. Li and R. Horowitz, "Intelligent control of an exercise machine," in *Proc. Int. Work. Adv. Motion Control*, Mie, 1996, pp. 271–276.

[6] ——, "Control of smart exercise machines - part 1: Problem formulation and nonadaptive control," *IEEE Trans. Mechatronics*, vol. 2, no. 4, pp. 237–247, Dec. 1997.

[7] P. Y. Li and R. Horowitz, "Control of smart exercise machines - part 2: Self-optimizing control," *IEEE Trans. Mechatronics*, vol. 2, no. 4, pp. 248–258, Dec. 1997.

[8] A. V. Hill, "The heat of shortening and the dynamic constants of muscle," *Proc. Royal Soc. B*, vol. 126, no. 843, 1938.

[9] T. Kikuchi, J. Furusho, and K. Oda, "A virtual exercise machine," in *Proc. IEEE Int. Conf. Robot. Autom.*, Atlanta, GA, 1993, pp. 232–238.

[10] A. Van-Reet and M. G. Feemster, "Development of an electrically actuated exercise system," in *Proc. Int. Symposium Sys. Theory*, 2004, pp. 246–250.

[11] B. Levins and I. Gravagne, "A magnetically controllable valve to vary the resistance of hydraulic dampers for exercise machines," in *Proc. Int. Conf. Adv. Intell. Mechatronics*, Monterey, CA, 2005, pp. 492–497.

[12] X. T. Zhang, D. M. Dawson, W. E. Dixon, and B. Xian, "Extremum seeking nonlinear controllers for a human exercise machine," *IEEE Trans. Mechatronics*, vol. 11, no. 2, pp. 233–240, Apr. 2006.

[13] W. H. Press, S. A. Teukolsky, W. T. Vetterling, and B. P. Flannery, *Numerical Recipes in C: The Art of Scientific Computing, Second Edition*. Cambridge, UK: Cambridge Univ. Press, 1988.

[14] B. Xian, M. de Queiroz, and D. M. Dawson, *A Continuous Control Mechanism for Uncertain Nonsmooth Nonlinear Systems in Optimal Control, Stabilization and Nonsmooth Analysis*. Heidelberg, Germany: Springer-Verlag, 2004.

[15] C. Desoer and M. Vidyasagar, *Feedback Systems: Input-Output Properties*. New York, NY: Academic Press, 1975.