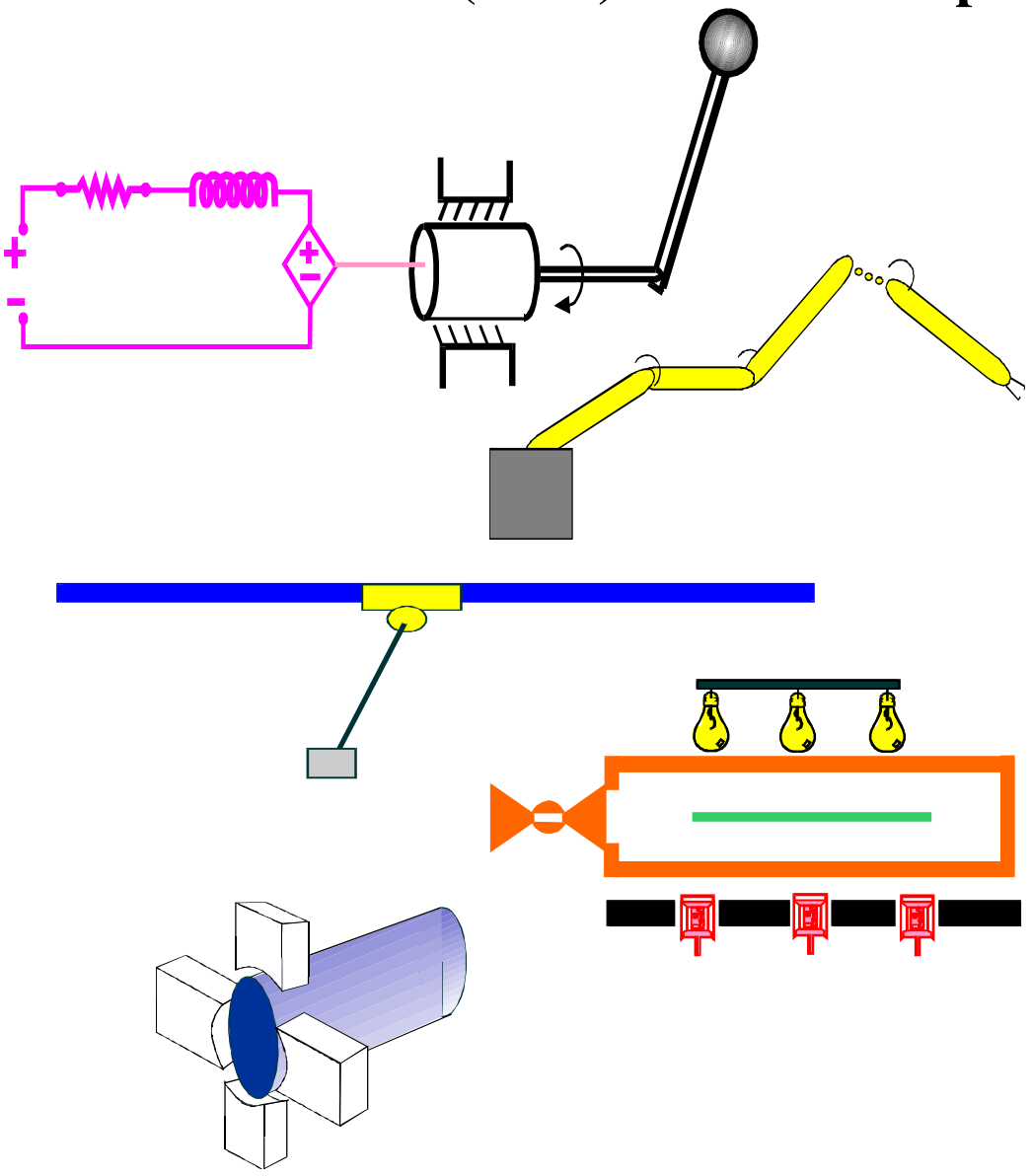
Clemson University
College of Engineering and Science
Control and Robotics (CRB) Technical Report



Number: CU/CRB/5/9/07/#1

Title: Derivation of Regression Matrices for Position based Structure from Motion for Camera-In-Hand Configuration

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This technical report explains the derivation of regression matrices (W(t), Π(t), Θ1, and Θ2) for position SFM, position based pixel equations, moving camera with known camera calibration matrix. The development is done using MAPLE ver. 11.0.

```
> restart;
> with(linalg) :
> xc := matrix(3, 1, [xc1, xc2, xc3]);
```

$$xc := \begin{bmatrix} xc1 \\ xc2 \\ xc3 \end{bmatrix} \quad (1)$$

xc is the known translation vector from the camera frame to the mechanical system frame, expressed in mechanical system frame.

```
> xf1 := matrix(3, 1, [xf11, xf12, xf13]);
```

$$xf1 := \begin{bmatrix} xf11 \\ xf12 \\ xf13 \end{bmatrix} \quad (2)$$

xf1 is the unknown constant Euclidean coordinates of 1st feature point relative to the base frame.

```
> xb := matrix(3, 1, [xb1, xb2, xb3]);
```

$$xb := \begin{bmatrix} xb1 \\ xb2 \\ xb3 \end{bmatrix} \quad (3)$$

xb(t) is measurable translation vector from the mechanical system frame to the base frame relative to the base frame.

```
> Rb := matrix(3, 3, [rb11, rb12, rb13, rb21, rb22, rb23, rb31, rb32, rb33]);
```

$$Rb := \begin{bmatrix} rb11 & rb12 & rb13 \\ rb21 & rb22 & rb23 \\ rb31 & rb32 & rb33 \end{bmatrix} \quad (4)$$

Rb(t) is measurable rotation matrix from the mechanical system frame to the base frame relative to the base frame.

In this section derivations of Π and Θ1 are presented: Regression matrix for 'Zi= ΠΘ'

ΠΘ = (Rc3)^T [Rb^T (Xfi - Xb) - Xc]

```
> rc3t := matrix(1, 3, [rc13, rc23, rc33]);
```

$$rc3t := \begin{bmatrix} rc13 & rc23 & rc33 \end{bmatrix} \quad (1.1)$$

rc3t is the last row of the transpose of the known rotation matrix from the camera frame to the mechanical system
frame relative to the mechanical system frame.

```
> temp1 := evalm(transpose(Rb)&·(xf1-xb)-xc) :
> z1 := simplify(evalm(rc3t&·temp1)) :
> Π := matrix(1, 4, [(rc13 rb11 + rc23 rb12 + rc33 rb13), (rc13 rb21 + rc23 rb22
      + rc33 rb23), (rc13 rb31 + rc23 rb32 + rc33 rb33), ( - rc13 rb11 xb1 - rc13 rb21 xb2
      - rc13 rb31 xb3 - rc13 xc1 - rc23 rb12 xb1 - rc23 rb22 xb2 - rc23 rb32 xb3 - rc23 xc2
      - rc33 rb13 xb1 - rc33 rb23 xb2 - rc33 rb33 xb3 - rc33 xc3)]);
```

$$\Pi := [rc13\,rb11 + rc23\,rb12 + rc33\,rb13,\ rc13\,rb21 + rc23\,rb22 + rc33\,rb23,\ rc13\,rb31 + rc23\,rb32 + rc33\,rb33,\ -rc13\,rb11\,xb1 - rc13\,rb21\,xb2 - rc13\,rb31\,xb3 - rc13\,xc1 - rc23\,rb12\,xb1 - rc23\,rb22\,xb2 - rc23\,rb32\,xb3 - rc23\,xc2 - rc33\,rb13\,xb1 - rc33\,rb23\,xb2 - rc33\,rb33\,xb3 - rc33\,xc3] \quad (1.2)$$

Π is found out to be a 1-by-4 matrix after collecting all the known parameters.

```
> θ1 := matrix(4, 1, [xf11, xf12, xf13, 1]);
```

$$\theta 1 := \begin{bmatrix} xf11 \\ xf12 \\ xf13 \\ 1 \end{bmatrix} \quad (1.3)$$

θ1 is a vector of 4 elements found out after collecting all the unknown constant parameters from (1.2).

```
> t1 := simplify(evalm(Π&·θ1)) :
> temp2 := simplify(expand(evalm(z1-t1)));
```

$$temp2 := \begin{bmatrix} 0 \end{bmatrix} \quad (1.4)$$

A simple check is performed to verify the regression matrices found in (1.3) and (1.4) are correct.

In this section derivations of W and Θ2 are presented:Regression matrix for 'WΘ'

WΘ = A1(Rc)^T [Rb^T (Xfi - Xb) - Xc]

```
> A1 := matrix(2, 3, [a1, a2, a3, 0, a4, a5]);
```

$$A1 := \begin{bmatrix} a1 & a2 & a3 \\ 0 & a4 & a5 \end{bmatrix} \tag{2.1}$$

A1 is a known constant camera's intrinsic calibration matrix.

```
> Rc := matrix(3, 3, [rc11, rc12, rc13, rc21, rc22, rc23, rc31, rc32, rc33]);
```

$$Rc := \begin{bmatrix} rc11 & rc12 & rc13 \\ rc21 & rc22 & rc23 \\ rc31 & rc32 & rc33 \end{bmatrix} \tag{2.2}$$

Rc is known constant rotation matrix from camera frame to the mechanical system frame relative to the mechanical system frame.

```
> Wtheta2 := simplify(combine(expand(evalm(A1&·transpose(Rc)&·evalm(transpose(Rb)
      &·(xf1-xb)-xc)))));
> w1 := collect(Wtheta2[1, 1], [xf11, xf12, xf13]):
> w2 := collect(Wtheta2[2, 1], [xf11, xf12, xf13]):
> W := matrix(2, 4, [(a3 rc13 rb11 + a3 rc23 rb12 + a2 rc32 rb13 + a2 rc12 rb11
      + a3 rc33 rb13 + a1 rc31 rb13 + a1 rc21 rb12 + a1 rc11 rb11 + a2 rc22 rb12),
      (a1 rc31 rb23 + a1 rc21 rb22 + a2 rc22 rb22 + a3 rc23 rb22 + a2 rc12 rb21
      + a3 rc13 rb21 + a2 rc32 rb23 + a3 rc33 rb23 + a1 rc11 rb21), (a3 rc23 rb32
      + a3 rc13 rb31 + a3 rc33 rb33 + a1 rc31 rb33 + a1 rc21 rb32 + a1 rc11 rb31
      + a2 rc32 rb33 + a2 rc22 rb32 + a2 rc12 rb31), -a1 rc11 rb21 xb2 - a1 rc11 rb31 xb3
      - a2 rc12 rb11 xb1 - a2 rc12 rb21 xb2 - a2 rc12 rb31 xb3 - a3 rc13 rb11 xb1
      - a3 rc13 rb21 xb2 - a3 rc13 rb31 xb3 - a1 rc21 rb12 xb1 - a1 rc21 rb22 xb2
      - a1 rc21 rb32 xb3 - a2 rc22 rb12 xb1 - a2 rc22 rb22 xb2 - a2 rc22 rb32 xb3
      - a3 rc23 rb12 xb1 - a3 rc23 rb22 xb2 - a3 rc23 rb32 xb3 - a1 rc31 rb13 xb1
      - a1 rc31 rb23 xb2 - a1 rc31 rb33 xb3 - a2 rc32 rb13 xb1 - a2 rc32 rb23 xb2
      - a2 rc32 rb33 xb3 - a3 rc33 rb13 xb1 - a3 rc33 rb23 xb2 - a3 rc33 rb33 xb3
      - a1 rc21 xc2 - a1 rc11 xc1 - a2 rc12 xc1 - a3 rc13 xc1 - a2 rc22 xc2 - a3 rc23 xc2
      - a1 rc31 xc3 - a2 rc32 xc3 - a3 rc33 xc3 - a1 rc11 rb11 xb1, (a5 rc33 rb13
      + a5 rc13 rb11 + a4 rc22 rb12 + a5 rc23 rb12 + a4 rc32 rb13 + a4 rc12 rb11),
      (a4 rc12 rb21 + a4 rc32 rb23 + a5 rc33 rb23 + a5 rc13 rb21 + a5 rc23 rb22
      + a4 rc22 rb22), (a5 rc23 rb32 + a4 rc22 rb32 + a5 rc33 rb33 + a4 rc12 rb31
      + a5 rc13 rb31 + a4 rc32 rb33), - a4 rc12 xc1 - a5 rc13 xc1 - a5 rc23 xc2
      - a4 rc32 xc3 - a5 rc33 xc3 - a4 rc22 rb22 xb2 - a4 rc22 rb32 xb3
      - a5 rc23 rb12 xb1 - a5 rc23 rb22 xb2 - a5 rc23 rb32 xb3 - a4 rc32 rb13 xb1
      - a4 rc32 rb23 xb2 - a4 rc32 rb33 xb3 - a5 rc33 rb13 xb1 - a5 rc33 rb23 xb2
      - a5 rc33 rb33 xb3 - a4 rc22 xc2 - a5 rc13 rb11 xb1 - a4 rc12 rb31 xb3
      - a4 rc12 rb21 xb2 - a5 rc13 rb21 xb2 - a5 rc13 rb31 xb3 - a4 rc22 rb12 xb1
      - a4 rc12 rb11 xb1]);
```

$$W := [[a1\,rc21\,rb12 + a1\,rc31\,rb13 + a1\,rc11\,rb11 + a2\,rc32\,rb13 + a2\,rc22\,rb12 \tag{2.3}$$

$$+ a2\,rc12\,rb11 + a3\,rc23\,rb12 + a3\,rc13\,rb11 + a3\,rc33\,rb13,\ a3\,rc23\,rb22$$
$$+ a3\,rc13\,rb21 + a2\,rc32\,rb23 + a3\,rc33\,rb23 + a1\,rc11\,rb21 + a2\,rc12\,rb21$$
$$+ a1\,rc21\,rb22 + a1\,rc31\,rb23 + a2\,rc22\,rb22,\ a2\,rc32\,rb33 + a3\,rc13\,rb31$$
$$+ a1\,rc11\,rb31 + a2\,rc22\,rb32 + a3\,rc23\,rb32 + a1\,rc31\,rb33 + a2\,rc12\,rb31$$
$$+ a3\,rc33\,rb33 + a1\,rc21\,rb32,\ -a1\,rc31\,rb23\,xb2 - a1\,rc31\,rb33\,xb3$$
$$- a2\,rc32\,rb13\,xb1 - a2\,rc32\,rb23\,xb2 - a3\,rc33\,xc3 - a3\,rc23\,xc2 - a2\,rc22\,xc2$$
$$- a1\,rc21\,xc2 - a3\,rc13\,xc1 - a2\,rc12\,xc1 - a1\,rc11\,xc1 - a3\,rc33\,rb23\,xb2$$
$$- a3\,rc33\,rb33\,xb3 - a1\,rc31\,xc3 - a2\,rc32\,rb33\,xb3 - a3\,rc33\,rb13\,xb1$$
$$- a2\,rc32\,xc3 - a1\,rc11\,rb11\,xb1 - a1\,rc21\,rb12\,xb1 - a1\,rc21\,rb22\,xb2$$
$$- a3\,rc13\,rb21\,xb2 - a3\,rc13\,rb31\,xb3 - a3\,rc23\,rb12\,xb1 - a3\,rc23\,rb22\,xb2$$
$$- a3\,rc23\,rb32\,xb3 - a1\,rc31\,rb13\,xb1 - a1\,rc21\,rb32\,xb3 - a2\,rc22\,rb12\,xb1$$
$$- a2\,rc22\,rb22\,xb2 - a2\,rc22\,rb32\,xb3 - a1\,rc11\,rb21\,xb2 - a1\,rc11\,rb31\,xb3$$
$$- a2\,rc12\,rb11\,xb1 - a2\,rc12\,rb21\,xb2 - a2\,rc12\,rb31\,xb3 - a3\,rc13\,rb11\,xb1\,],$$
$$[a4\,rc22\,rb12 + a5\,rc23\,rb12 + a4\,rc12\,rb11 + a5\,rc13\,rb11 + a5\,rc33\,rb13$$
$$+ a4\,rc32\,rb13,\ a5\,rc13\,rb21 + a4\,rc32\,rb23 + a4\,rc22\,rb22 + a5\,rc23\,rb22$$
$$+ a5\,rc33\,rb23 + a4\,rc12\,rb21,\ a5\,rc33\,rb33 + a5\,rc13\,rb31 + a4\,rc32\,rb33$$
$$+ a4\,rc22\,rb32 + a5\,rc23\,rb32 + a4\,rc12\,rb31,\ -a5\,rc13\,xc1 - a4\,rc12\,xc1$$
$$- a5\,rc33\,xc3 - a4\,rc32\,xc3 - a5\,rc23\,xc2 - a4\,rc22\,xc2 - a5\,rc33\,rb23\,xb2$$
$$- a5\,rc13\,rb31\,xb3 - a5\,rc33\,rb33\,xb3 - a5\,rc33\,rb13\,xb1 - a4\,rc32\,rb23\,xb2$$
$$- a4\,rc22\,rb32\,xb3 - a4\,rc32\,rb33\,xb3 - a4\,rc32\,rb13\,xb1 - a4\,rc12\,rb11\,xb1$$
$$- a4\,rc12\,rb21\,xb2 - a5\,rc23\,rb32\,xb3 - a4\,rc12\,rb31\,xb3 - a5\,rc13\,rb11\,xb1$$
$$- a5\,rc13\,rb21\,xb2 - a4\,rc22\,rb12\,xb1 - a4\,rc22\,rb22\,xb2 - a5\,rc23\,rb22\,xb2$$
$$- a5\,rc23\,rb12\,xb1\,]\,]$$

W is found out to be a 2-by-4 matrix after collecting all the known parameters.

```
> θ2 := matrix(4, 1, [xf11, xf12, xf13, 1]);
```

$$\theta 2 := \begin{bmatrix} xf11 \\ xf12 \\ xf13 \\ 1 \end{bmatrix} \qquad (2.4)$$

θ2 is a vector of 4 elements found out after collecting all the unknown parameters.

```
> t2 := simplify(evalm(W&·θ2)):
> temp3 := simplify(expand(evalm(Wtheta2 - t2)));
```

$$temp3 := \begin{bmatrix} 0 \\ 0 \end{bmatrix} \qquad (2.5)$$

A simple check is performed to verify the regression matrices found in (2.3) and (2.4) are correct.

It should be noted from (1.4) and (2.4) that θ1 and θ2 are same.