ECE 212

ELECTRICAL ENGINEERING LABORATORY II

For use in

ECE 212

Electrical Engineering Laboratory II

a companion laboratory for ECE 262, Electric Circuits II

January 2010

Dr. J. E. Harriss
Revision History

This laboratory manual is based on a compilation of laboratory experiments originally devised by Dr. A. L. Duke and Dr. L. T. Fitch. We are indebted to them for their substantial contributions to laboratory education.

Revision History:

Fall 1992 Mr. Richard Agnew and Dr. J. W. Harrison: Various additions and revisions.

Summer 1998 Dr. David L. Lubkeman: General revisions.

Summer 2008 Ms. Srimathi Govindan and Dr. James Harriss: General update for new equipment. Clarify Lab 2 and Lab 9. Add expanded Appendix, including details on oscilloscope. Unify the Appendix for ECE 212 and 309.

January 2009 Mr. Apoorva Kapadia and Dr. James Harriss: Revise Labs 1, 3, 4, 5, 6, 7, 8, 10, 11.

January 2010 Dr. James Harriss: Complete re-write to convert the labs to use the NI-ELVIS II system. Dr. Harriss gratefully acknowledges the help of Mr. Erhun Iyasere in testing the revised lab exercises.
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Introduction

This course is intended to enhance the learning experience of the student in topics encountered in ECE 262. In this lab, students are expected to gain experience in using the basic measuring devices used in electrical engineering and in interpreting the results of measurement operations in terms of the concepts introduced in the second electrical circuits course. How the student performs in the lab depends on his/her preparation, participation, and teamwork. Each team member must participate in all aspects of the lab to insure a thorough understanding of the equipment and concepts. The student, lab teaching assistant, and faculty coordinator all have certain responsibilities toward successful completion of the lab's goals and objectives.

Student Responsibilities

The student is expected to be prepared for each lab. Lab preparation includes reading the lab experiment and related textbook material. In addition to this, the lab pre-laboratory preparation may consist of performing calculations that you will need during the lab experiment. If you have questions or problems with the preparation, contact your Laboratory Teaching Assistant (LTA), but in a timely manner. Don't wait until an hour or two before and then expect to find the LTA immediately available. Active participation by each student in lab activities is expected. The student is expected to ask the teaching assistant any questions he/she may have. DO NOT MAKE COSTLY MISTAKES BECAUSE YOU DID NOT ASK A SIMPLE QUESTION. A large portion of the student's grade is determined in the comprehensive final exam, so understanding the concepts and procedure of each lab is necessary for successful completion of the lab. The student should remain alert and use common sense while performing a lab experiment. He/she is also responsible for keeping a professional and accurate record of the lab experiments in a laboratory notebook. Students should report any errors in the lab manual to the teaching assistant.

Laboratory Teaching Assistant Responsibilities

The Laboratory Teaching Assistant (LTA) shall be completely familiar with each lab prior to class. The LTA shall provide the students with a syllabus and safety review during the first class. This syllabus shall include the LTA's office hours, telephone number, and the name of the faculty coordinator. The LTA is responsible for insuring that all of the necessary equipment and/or preparations for the lab are available and in working condition. LAB EXPERIMENTS SHOULD BE CHECKED IN ADVANCE TO MAKE SURE EVERYTHING IS IN ORDER. The LTA should fully answer any questions posed by the students and supervise the students performing the lab experiments. The LTA is expected to grade the pre-labs, lab notebooks, and reports in a fair and timely manner. The reports should be returned to the students in the next lab period following submission. The LTA should report any errors in the lab manual to the faculty coordinator.
Faculty Coordinator Responsibilities

The faculty coordinator should insure that the laboratory is properly equipped, i.e., that the teaching assistants receive any equipment necessary to perform the experiments. The coordinator is responsible for supervising the teaching assistants and resolving any questions or problems that are identified by the teaching assistants or the students. The coordinator may supervise the format of the final exam for the lab. He/she is also responsible for making any necessary corrections to this manual. The faculty coordinator is responsible for insuring that the software version of the manual is continually updated and available.

Lab Policy and Grading

The student should understand the following policy.

ATTENDANCE: Attendance is mandatory and any absence must be for a valid excuse and must be documented. If the instructor is more than 15 minutes late, students may leave the lab.

LAB RECORDS: The student must:

1. Perform the Pre-Lab assignment by the beginning of each lab, and
2. Keep all work in preparation of and obtained during the lab in an approved NOTEBOOK, and
3. Prepare a lab report on selected experiments.

GRADING POLICY: The final grade of this course is based on the following:

- Laboratory notebook and in-class work
- Lab Pre-lab
- Lab reports
- Final exam

In-class work will be determined by the teaching assistant, who, at his/her discretion may use team evaluations to aid in this decision. The final exam should contain a written part and a practical (physical operations) part.

PRE- and CO-REQUISITES: The lab course is to be taken during the same semester as ECE 262, but receives a separate grade. If ECE 262 is dropped, the ECE 212 must be dropped also. Students are required to have completed ECE 202, MTHSC 206 and PHYS 221 with a C or better grade in each. Students are also assumed to have completed a programming class and be familiar with the use of a computer-based word processor application program.

THE INSTRUCTOR RESERVES THE RIGHT TO ALTER ANY PART OF THIS INFORMATION AT HIS HER DISCRETION IF CIRCUMSTANCES SHOULD DICTATE. Any changes should be announced in class and distributed in writing to the students prior to their effect.
Course Goals and Objectives

The Electrical Circuits Laboratory II is designed to provide the student with the knowledge to use basic measuring instruments and techniques with proficiency. These techniques are designed to complement the concepts introduced in ECE 262. In addition, the student should learn how to effectively record experimental results and present these results in a written report. More explicitly, the class objectives are:

1. To gain proficiency in the use of common measuring instruments;

2. To enhance understanding of advanced electric circuit analysis concepts.
   - Inductance, Capacitance, and reactance
   - AC voltage and current addition. Phasors
   - AC power (real and reactive, instantaneous and average)
   - Series and parallel resonant circuit behavior
   - Passive Filters
   - Transfer functions
   - Transformers
   - Two-port network analysis;

3. To develop communication skills through
   a) maintenance of succinct but complete laboratory notebooks as permanent, written descriptions of procedures, results, and analyses,
   b) verbal interchanges with the Laboratory Instructor and other students, and
   c) preparation of succinct but complete laboratory reports;

4. To compare theoretical predictions with experimental results and to resolve any apparent differences.
Use of Laboratory Instruments

One of the major goals of this lab is to familiarize the student with the proper equipment and techniques for making electrical measurements. Some understanding of the lab instruments is necessary to avoid personal or equipment damage. By understanding the device's purpose and following a few simple rules, costly mistakes can be avoided. You have already, in ECE 211, learned these rules, but they are repeated for convenience and emphasis below. Most of the instrumentation used in this laboratory is implemented through National Instruments NI-ELVIS II breadboard and circuit analysis system.

For details about the NI-ELVIS instruments, refer to the ELVIS Operation Manual at http://www.clemson.edu/ces/departments/ece/resources/lab_manuals.html.

In general, all devices have physical limits. These limits are specified by the device manufacturer and are referred to as the device RATING. The ratings are usually expressed in terms of voltage limits, current limits, or power limits. It is up to the engineer to make sure that these ratings (limit valves) are not exceeded in device operation. The following rules provide a guideline for instrument protection.

**Instrument Protection Rules**

1. Set instrument scales to the highest range before applying power.

2. When using an oscilloscope, especially one with a cathode ray tube, do not leave a bright dot or trace on the screen for long periods of time. To avoid burning the image into the screen, reduce the intensity until the dot or trace is barely visible.

3. Be sure instrument grounds are connected properly. Avoid accidental grounding of "hot" leads, i.e., those that are above ground potential. (See especially “Avoiding Grounding Errors with Oscilloscope” in Appendix C.)

4. Check polarity markings and connections of instruments and components carefully before connecting or turning on power.

5. **Never connect an ammeter across a voltage source.** Only connect ammeters in series with loads. An ammeter is a low-resistance device that, if connected in parallel, will short out most components and usually destroy the ammeter or its protecting fuse.

6. Do not exceed the voltage and current ratings of instruments or other circuit elements. This particularly applies to wattmeters since the current or voltage rating may be exceeded with the needle still on the scale.

7. Be sure any fuse or circuit breaker is of suitable value. When connecting electrical elements to make up a circuit, it is easy to lose track of various points in the network and accidentally connect a wire to the wrong place. A procedure to follow that helps to avoid this is to connect the main series portion of the network first, then go back and add the elements in parallel. As an element is added, place a small check (✓) by it on your circuit diagram. This will help you keep track of your progress in assembling the whole circuit. Then go back and verify all connections before turning on the power. [One day someone's life may depend upon your making sure that all has been done correctly.]
Laboratory Notebooks and Reports

The Laboratory Notebook

The student records and interprets his/her experiments via the laboratory notebook and the laboratory report. The laboratory notebook is essential in recording the methodology and results of an experiment. In engineering practice, the laboratory notebook serves as an invaluable reference to the technique used in the lab and is essential when trying to duplicate a result or write a report. Therefore, it is important to learn to keep an accurate notebook.

The laboratory notebook should:

- Be kept in a sewn and bound or spiral bound notebook.
- Contain the experiment's title, the date, the equipment and instruments used, any pertinent circuit diagrams, the procedure used, the data (often in tables when several measurements have been made), and the analysis of the results.
- Contain plots of data and sketches when these are appropriate in the recording and analysis of observations.
- Be an accurate and permanent record of the data obtained during the experiment and the analysis of the results. You will need this record when you are ready to prepare a lab report.

The Laboratory Report

The laboratory report is the primary means of communicating your experience and conclusions to other professionals. In this course you will use the lab report to inform your LTA what you did and what you have learned from the experience. Engineering results are meaningless unless they can be communicated to others.

Your laboratory report should be clear and concise. The lab report shall be typed on a word processor. As a guide, use the format on the next page. Use tables, diagrams, sketches, and plots, as necessary to show what you did, what was observed, and what conclusions you draw from this. Even though you will work with one or more lab partners, your report must (shall) be the result of your individual effort in order to provide you with practice in technical communication.

You will be directed by your LTA to prepare a lab report on a few selected lab experiments during the semester. Your assignment might be different from your lab partner's assignment.
Format of Lab Report

LABORATORY XX

TITLE

- Indicate the lab title and number.

NAME – Give your name.

LAB PARTNER(S) - Specify your lab partner's name.

DATE - Indicate the date the lab was performed.

OBJECTIVE - Clearly state the objective of performing the lab.

EQUIPMENT USED - Indicate which equipment was used in performing the experiment. The manufacturer and model number should be specified.

PROCEDURE - Provide a concise summary of the procedure used in the lab. Include any modifications to the experiment.

DATA - Provide a record of the data obtained during the experiment. Data should be retrieved from the lab notebook and presented in a clear manner using tables.

OBSERVATIONS AND DISCUSSIONS - The student should state what conclusions can be drawn from the experiment. Plots, charts, other graphical medium, and equations should be employed to illustrate the student's viewpoint. Sources of error and percent error should be noted here.

QUESTIONS - Questions pertaining to the lab may be answered here. These questions may be answered after the lab is over.

CONCLUSIONS - The student should present conclusions, which may be logically deduced, from his/her data and observations.

SIGNATURE - Include the statement "This report is accurate to the best of my knowledge and is a true representation of my laboratory results."

SIGNED _______________________________________
Laboratory 1
Orientation

INTRODUCTION: In the first lab period, the students should become familiar with the location of equipment and components in the lab, the course requirements, and the teaching instructor. Students should also make sure that they have all of the co- and pre-requisites for the course at this time.

OBJECTIVE: To familiarize the students with the lab facilities, equipment, standard operating procedures, lab safety, and the course requirements.

PRE-LAB: Read the introduction and Appendix A (Safety) of this manual.

EQUIPMENT NEEDED: Lab Manual.

PROCEDURE: During the first laboratory period, the instructor will provide the students with a general idea of what is expected from them in this course. Each student will receive a copy of the syllabus, stating the instructor's office hours and contact information. In addition, the instructor will review the safety concepts of the course. The instructor will indicate which word processor should be used for the lab reports. The students should familiarize themselves with the preferred word processor software.

During this period the Instructor will briefly review the measuring instruments and other equipment that will be used. The location of instruments, equipment, and components (e.g. resistors, capacitors, connecting wiring) will be indicated. The guidelines for instrument use will be reviewed. Most of the instrumentation used in this laboratory is implemented through National Instruments NI-ELVIS II breadboard and circuit analysis system.

You should record in your notebook the information from your LTA.

REPORT: No report is due next time.
Laboratory 2

Average and RMS Values

INTRODUCTION: Waveforms of voltage and current that vary periodically with time may be characterized by their average value or their root mean square (r.m.s) value. The latter is used to determine the power supplied, dissipated, or stored by a circuit element. Some of the measuring instruments you will use respond to average values of voltage or current, while others respond to r.m.s values.

EDUCATIONAL OBJECTIVE:
(1) Learn how to determine the values of r.m.s voltage for three types of waveforms: a sinusoid, a square wave, and a triangular wave.
(2) Understand the difference between a true-r.m.s and a conventional multimeter.

EXPERIMENTAL OBJECTIVE:
Determine whether the voltage metering function of the NI-ELVIS’s Digital Multimeter (DMM) measures true RMS voltage for three types of waveforms: a sinusoid, a square wave, and a triangular wave.

PRE-LAB:
Reading:
Read Appendix B and Appendix C of this manual, paying particular attention to the methods of using measurement instruments.

Written:
(a) Using the definition of average value (equation 2.1) and r.m.s value (equation 2.2), calculate the average voltage, the average absolute voltage, and the r.m.s voltage values for the symmetrical sine, square, and triangular waveforms, assuming that the peak value of each waveform is 2 V.
(b) Compute the voltage values that would be reported by non-true r.m.s voltmeters. You may use the equations derived in the Background section for these calculations. Show all of your calculations in your lab notebook and summarize the results in a table.
(c) After you have done these calculations, review the laboratory exercise procedures and plan how you will use the experience gained in these calculations to find the values sought.

EQUIPMENT NEEDED:
• NI-ELVIS II, including
  • Function Generator
  • Digital Multimeter (DMM)
  • Oscilloscope
• Resistance Decade Box
BACKGROUND

Measurements of AC signal:

- **Amplitude** is a nonnegative scalar measure of a wave's maximum magnitude of oscillation. In electrical engineering it may be thought of as the maximum absolute value reached by a voltage or current waveform as measured from the center of the oscillation. An amplitude measurement may be reported as peak, peak-to-peak, average, or RMS.

- **Peak** amplitude is the height of an AC waveform as measured from the center of the oscillation to the highest positive or lowest negative point on a graph. Also known as the crest amplitude of a wave.

![Peak Amplitude Diagram](image-url)

- **Peak-to-peak** amplitude is the total height of an AC waveform as measured from maximum positive to minimum negative peaks (the highest peak to the lowest valley) on a graph of the waveform. Often abbreviated as “P-P”, e.g., $V_{p-p}$ or $V_{pp}$.

![Peak-to-Peak Amplitude Diagram](image-url)

- **Average** value is the arithmetic “mean” of a waveform’s values over one cycle. The average value of any waveform with equal-area portions above and below the “zero” line on a graph is zero. However, often as a practical measure of amplitude, a waveform may be characterized by its average absolute value, calculated as the arithmetic mean of the absolute values of the waveform.

- “RMS” stands for *Root Mean Square*, and is a way of expressing an AC quantity of voltage or current in terms functionally equivalent to DC. For example, 10 volts AC RMS is the amount of AC voltage that would produce the same amount of heat dissipation across a resistor of given value as a 10 volt DC power supply. Also known as the “equivalent” or “DC equivalent” value of an AC voltage or current.
Analog, electromechanical meter movements respond proportionally to the average absolute value of an AC voltage or current. When RMS indication is desired, the meter's calibration must be adjusted accordingly, usually by applying a constant multiplicative factor assumed for a sinusoidal wave. This means that the accuracy of an electromechanical meter's RMS indication is dependent on the purity of the waveform and whether it is the same wave shape as the waveform used in calibrating.

In this experiment, we will work with periodic waveforms having period $T$ and amplitude $V_m$. Specifically, we will work with the following three types of waveforms:

1. A sinusoidal wave voltage: $v(t) = V_m \cos(2\pi t/T)$.

\[ v(t) = V_m \text{ for } 0 \leq t < \frac{T}{2}; T \leq t < \frac{3T}{2}; \text{ etc} \]

2. A square-wave voltage:

\[ v(t) = -V_m \text{ for } \frac{T}{2} \leq t < T; \frac{3T}{2} \leq t < 2T; \text{ etc} \]

3. A triangular-wave voltage:

\[ v(t) = 4V_m \left(\frac{(t-nT)}{T}\right) \text{ for } (n-\frac{1}{4})T \leq t < (n+\frac{1}{4})T \]
\[ v(t) = V_m \left(\frac{(t-(n+\frac{1}{4})T)}{T}\right) \text{ for } (n+\frac{1}{4})T \leq t < (n+\frac{3}{4})T \]

where $n = 0, 1, 2, 3, \ldots$

AC voltage and current waveforms are further depicted in a number of ways. Two of the most common and useful are the average and the root-mean-square (rms) values.

The average value of a time-varying waveform $x(t)$ with period $T$ is given by:

\[ X_{\text{ave}} = \frac{1}{T} \int_0^T x(t)dt \quad (2.1) \]

The root-mean-square value, useful for power calculations, is defined by:

\[ X_{\text{rms}} = \sqrt{\frac{1}{T} \int_0^T x^2(t)dt} \quad (2.2) \]

Since the sine, square, and triangular waveforms are symmetrical about the time axis, they all have mathematical average voltages of zero. However, each waveform will have an rms value, and a summary of the calculation steps for relating the voltage magnitude to the rms value for each waveform is shown below:
Sinusoidal Voltage:

\[ V_{rms} = \sqrt{\frac{1}{T} \int_{0}^{T} v^2(t) dt} = \sqrt{\frac{1}{T} \int_{0}^{T} \left( V_m \sin \left( \frac{2\pi}{T} t \right) \right)^2 dt} = V_m \sqrt{\frac{1}{T} \int_{0}^{T} \left( \frac{1}{2} - \frac{\cos \left( \frac{2\pi}{T} t \right)}{2} \right) dt} \]

\[ = V_m \sqrt{\frac{1}{T} \left[ \frac{1}{2} t \right]_{0}^{T} + \frac{\sin \left( \frac{2\pi}{T} t \right)}{4\pi} \right]_{0}^{T} = V_m \sqrt{\frac{1}{T} \left[ \frac{1}{2} T + 0 \right] = \frac{V_m}{\sqrt{2}} \right} \] (2.3)

Square-Wave Voltage:

\[ V_{rms} = \sqrt{\frac{1}{T} \int_{0}^{T} v^2(t) dt} = \sqrt{\frac{1}{T} \int_{0}^{T/2} (V_m)^2 dt} + \int_{T/2}^{T} (-V_m)^2 dt = \sqrt{\frac{1}{T} \left[ (V_m)^2 t \right]_{0}^{T/2} + (V_m)^2 t \right]_{T/2}^{T} \]

\[ = \sqrt{\frac{1}{T} \left[ \frac{(V_m)^2}{2} t \right]_{0}^{T} + (-V_m)^2 t \right]_{T/2}^{T} = \sqrt{\frac{1}{T} \left[ \frac{(V_m)^2}{2} T \right] = V_m \right} \] (2.4)

Triangular-Wave Voltage:

\[ V_{rms} = \sqrt{\frac{1}{T} \int_{0}^{T} v^2(t) dt} = \sqrt{\frac{1}{T} \left[ \frac{4V_m^2 t}{T} \right]_{0}^{T/4} + \int_{T/4}^{3T/4} \left( 2V_m - \frac{4V_m^2}{T} t \right)^2 dt} + \int_{3T/4}^{T} \left( -4V_m + \frac{4V_m^2}{T} t \right)^2 dt \]

\[ = \sqrt{\frac{1}{T} \left[ \frac{16V_m^2 t}{3T} \right]_{0}^{T/4} + \left( 4V_m^2 t - \frac{16V_m^2 t^2}{2T} + \frac{16V_m^2 t^3}{3T^2} \right)_{T/4}^{3T/4} + \left( 16V_m^2 t - \frac{32V_m^2 t^2}{2T} + \frac{16V_m^2 t^3}{3T^2} \right)_{3T/4}^{T} \right] \]

\[ = \sqrt{\frac{1}{T} \left[ \frac{V_m^2 T}{12} + \frac{2V_m^2 T}{12} + \frac{V_m^2 T}{12} \right] = \frac{V_m}{\sqrt{3}} \right} \] (2.5)

If we use what is called a “true rms” voltage meter, then the relationship between the magnitude of the waveform and the measured value would be given by the three equations given above.

However, many of the meters available are not “true rms” meters, and as a result are only designed for measurements in circuits with either DC voltages or sinusoidal AC voltages. For ob-
taining the AC voltage, most digital meters effectively perform a full-wave rectification of the waveform and compute the average absolute value. A constant factor is then applied to compute an RMS value. Often the constant factor is chosen to give a correct result for a sinusoidal waveform.

For a sinusoidal waveform, the average value for the rectified voltage is calculated as follows:

\[
V_{\text{ave rect}} = \frac{1}{T} \int_0^T |v(t)| \, dt = \frac{1}{T} \int_0^T V_m \sin \left( \frac{2\pi}{T} t \right) \, dt = \frac{V_m}{T} \left[ \int_0^{T/2} V_m \sin \left( \frac{2\pi}{T} t \right) \, dt + \int_{T/2}^{T} -V_m \sin \left( \frac{2\pi}{T} t \right) \, dt \right]
\]

\[
= \frac{2V_m}{T} \int_0^{T/2} V_m \sin \left( \frac{2\pi}{T} t \right) \, dt = \frac{2V_m}{T} \left[ -\frac{T}{2\pi} \cos \left( \frac{2\pi}{T} t \right) \right]_{t=0}^{T/2} = \frac{2V_m}{\pi}
\]

(2.6)

Since the rms value for a sinusoidal waveform should be related to the amplitude by \( \sqrt{2} \), we need to apply a conversion factor to get the correct rms value on the meter readout. In this case the conversion factor would be:

\[
V_{\text{rms}} = \frac{1}{\sqrt{2}} \left( \frac{\pi}{2} V_{\text{avg rectified}} \right) = 1.111*V_{\text{avg rectified}}
\]

(2.7)

Now what happens when we measure a square wave with 50% duty cycle? To find out, we compute the average of the rectified square-wave waveform:

\[
V_{\text{avg rectified}} = \frac{1}{T} \int_0^T |v(t)| \, dt = \frac{1}{T} \left[ \int_0^{T/4} \left( \frac{4V_m}{T} t \right) \, dt + \int_{T/4}^{T/2} \left( 2V_m - \frac{4V_m}{T} t \right) \, dt + \int_{T/2}^{3T/4} \left( -2V_m + \frac{4V_m}{T} t \right) \, dt + \int_{3T/4}^T \left( 4V_m - \frac{4V_m}{T} t \right) \, dt \right] = V_m
\]

(2.8)

The non-true-rms meter will apply the same conversion factor it applied to the sine wave. Hence, what we will see on the meter readout for a square wave is:

\[
V_{\text{meter}} = 1.111*V_m
\]

(2.9)

This meter will report a rms value that is 11% higher than the actual rms value we should have for a square wave.

Finally, for a triangular waveform, the average rectified voltage is:

\[
V_{\text{avg rectified}} = \frac{1}{T} \int_0^T |v(t)| \, dt = \frac{1}{T} \left[ \int_0^{T/4} \left( \frac{4V_m}{T} t^2 \right) \, dt + \int_{T/4}^{T/2} \left( 2V_m - \frac{4V_m}{T} t^2 \right) \, dt + \int_{T/2}^{3T/4} \left( -2V_m + \frac{4V_m}{T} t^2 \right) \, dt + \int_{3T/4}^T \left( 4V_m - \frac{4V_m}{T} t^2 \right) \, dt \right]
\]

\[
= \frac{1}{T} \left[ \frac{4V_m}{2T} \int_0^{T/4} t^2 \, dt + \left( 2V_m \int_0^{T/4} t^2 \, dt \right) + \left( -2V_m \int_{T/2}^{3T/4} t^2 \, dt \right) + \left( 4V_m \int_{3T/4}^T t^2 \, dt \right) \right] = \frac{V_m}{2}
\]

(2.10)

So for a triangular waveform the rms voltage indicated on the non-true-rms meter will be
\[ V_{\text{meter}} = 1.111 \times \frac{V_m}{2} = 0.555 \times V_m \] (2.11)

while it should be registering \( V_m / \sqrt{3} = 0.577 \times V_m \). This meter gives a reading only 96\% of what we should actually get, and so will under-report the rms voltage for a triangular wave.

In practice you will not get the exact results predicted by the equations, due to a number of errors, such as inability to set the peak voltage to the exact value, slight errors in the meter reading, and inaccuracies in the shape of the waveform produced by the function generator. Also, we have assumed that the duty cycle of the square wave is exactly 50\% in these calculations, which might not actually be the case for your waveform generator.

**PROCEDURE**

1. Connect the output terminals of the Function Generator to a Resistance Decade Box.

![Circuit Diagram]

2. Set the decade box to 1 kΩ resistance as the load on the output of the Function Generator.

3. Turn on the Function Generator and use the oscilloscope to observe the voltage drop across the 1 kΩ resistance. *Remember that the oscilloscope’s ground is the same as the Function Generator’s ground.*

4. Select the sine wave output option of the Function Generator
   
   a. Set the frequency of the function generator to 1 kHz.
   
   b. Observe the waveform on the oscilloscope using the CH0 BNC connector.
   
   c. Connect the SYNC signal (located directly below the FGEN signal row) to the Trigger BNC input on the left side on the workstation. Change the Trigger type to "Digital".
   
   d. Adjust the DC offset control of the Function Generator to obtain a zero DC level in the output.
e. Use the Function Generator’s output level control (Amplitude control knob/numerical input) to obtain a 2.0 Volt peak value (4.0 Vpp) on the scope measurement.

f. Use the digital multimeter (DMM) AC Voltage [V~] function to measure the Function Generator [FGEN] voltage by connecting the VΩ and COM banana jacks to FGEN and GND respectively. Make sure to include DC offset values in the RMS calculation.

5. Repeat the steps from 4a to 4f using a triangular wave output from the Function Generator.

6. Repeat the steps from 4a to 4f using a square wave (50% duty factor) output from the Function Generator.

7. Adjust the DC offset control to add a 2 Volt DC offset to the original 1 kHz sine wave. Now the waveform should vary from 0 to 4 Volts. Sketch this waveform in your lab notebook. Use the DMM to measure the AC RMS voltage and the DC voltage across the resistance. Record the values. Repeat this same step for triangular wave and square wave.

PROBING FURTHER

1. Compare the RMS voltage value calculations you made for the Pre-Lab to the AC voltage measurements made in 4th, 5th, and 6th steps in the Procedure. Do your measured values agree with the calculated values in all cases? If not, why?

2. Determine the theoretical RMS voltage value and DC voltage value you should have measured in Step 7 of the Procedure and compare this to what you actually measured. Show all of your math calculations. Do your measured values agree with the calculated values? If not, why?

3. Summarize your comparisons: Does the digital multimeter (DMM) in the NI-ELVIS provide true RMS measurements of voltage for all three waveforms? Justify your conclusion.

REPORT: Your Laboratory Teaching Assistant will inform you when a report is due, and on which experiments you will report. Make sure that you have recorded all necessary information and data in your laboratory notebook to enable you to prepare a report on this experiment, if so directed, at some time in the future.
Laboratory 3

Capacitors and Series RC Circuits

INTRODUCTION: Linear circuit elements — resistors, capacitors, and inductors — are the backbone of electrical and electronic circuits. These three types of elements respond to electrical voltages in different ways, variously consuming, storing, or supplying electrical energy. Understanding these behaviors and learning to calculate the result of combining elements is critical for designing and working with electric circuits. While a resistor consumes electrical energy, converting it to heat, capacitors and inductors vary their responses according to the frequency of the voltage or current applied to them. This laboratory will explore those responses for series-connected capacitors.

EDUCATIONAL OBJECTIVES:
(1) Learn to avoid oscilloscope grounding errors when measuring voltages. Learn the two-channel difference method and the interchanging-components method (Appendix C).
(2) Learn to measure capacitive reactance.
(3) Learn to measure phase angles between voltages.
(4) Learn to draw impedance and voltage phasor diagrams for resistors and capacitors in series.
(5) Understand how impedance and voltage phasors add (i.e., like vectors).
(6) Learn to simulate AC series circuit in B2-SPICE.

EXPERIMENTAL OBJECTIVES:
(1) Confirm how capacitances add when two capacitors are connected in parallel; in series.
(2) Determine the reactance of a capacitor in a series RC circuit by measuring voltages.
(3) Draw impedance and voltage phasor diagrams for a series RC circuit.
(4) Explain the effect of frequency on the impedance and voltage phasors for a series RC circuit.

PRE-LAB:
Reading: (1) Read and study the Background section of this Laboratory. (2) Read Appendix C, especially Avoiding Grounding Errors with Oscilloscope, Voltage Measurement, and Phase-Angle Measurement for an oscilloscope with two vertical inputs.
Written: (1) In your lab notebook sketch the circuit diagram to be used in the procedure and prepare Tables 3.1 and 3.2 to record data. (2) Sketch the impedance and voltage phasors diagrams (as in Figure 3.2) you would get at 500 Hz for the circuit in Figure 3.3. (3) Make a table of the magnitudes and phase angles. You will use this to check that your experimental setup is correct.

EQUIPMENT NEEDED:
• NI-ELVIS II
  • Function generator
  • Oscilloscope
  • Digital Multimeter
• Resistor decade box
• Capacitors, 0.01 µf, qty 2 (discrete capacitors)
BACKGROUND

A capacitor is formed whenever two conductors are separated by an insulating material. Consider the simple example of two parallel conducting plates separated by a small gap that is filled with an insulating material (vacuum, air, glass, or other dielectric). If a potential difference exists between the two plates, then an electric field exists between them, and opposite electric charges will be attracted to the two plates. The ability to store that electric charge is a fundamental property of capacitors. The larger the plates, the more charge can be stored. The closer the plates, the more charge can be stored…at least until the charges leap the gap and the dielectric breaks down.

If a voltage source is connected across a capacitor, charge will flow in the external circuit until the voltage across the capacitor is equal to the applied voltage. The charge that flows is proportional to the size of the capacitor (its “capacitance”) and to the applied voltage. The relationship is given by the equation

\[ Q = CV \]

where \( Q \) is the charge in coulombs, \( C \) is the capacitance in farads, and \( V \) is the applied voltage in volts.

**Capacitors in Series**

Electric current \( I \) is the amount of charge that flows per unit time; that is, \( I = Q/T \). Thus, the total charge that flows through a circuit (or a capacitor) is \( Q = IT \). So, if two capacitors are connected in series and a voltage is applied across the pair, the same current, and therefore the same charge, must flow through both capacitors, and the total voltage \( V_T \) must be divided across both capacitors:

\[ V_T = V_1 + V_2 = \frac{Q}{C_1} + \frac{Q}{C_2} = Q \left( \frac{1}{C_1} + \frac{1}{C_2} \right) = \frac{Q}{C_T} \]

where \( V_1 \) and \( V_2 \) are the voltages across the capacitors with capacitances \( C_1 \) and \( C_2 \). Thus, \( C_T \), the total capacitance of two capacitors in series, is found by

\[ \frac{1}{C_T} = \frac{1}{C_1} + \frac{1}{C_2} \quad \text{or} \quad C_T = \frac{C_1 C_2}{C_1 + C_2} \]

**Capacitors in Parallel**

Connecting capacitors in parallel is effectively the same as making a single capacitor’s plates larger, and therefore able to hold more charge for a given applied voltage. This simple view is borne out if one analyzes the flow of charge through a parallel array of capacitors connected to a voltage source. The result of such analysis is that capacitances in parallel add directly:

\[ C_T = C_1 + C_2 + C_3 + \ldots \]
**Time Constant**

If a voltage $V_0$ is applied to a capacitor $C$ connected in series with a resistor $R$, the voltage across the capacitor gradually increases. The rate at which the capacitor’s voltage changes is characterized by a “time constant”, $\tau$:

$$\tau = RC$$

$\tau$ is the time required for the voltage on the capacitor to rise from 0 to 0.632 $V_0$.

$\tau$ is also the time required for the voltage of a fully charged capacitor to fall from $V_0$ to 0.368 $V_0$. The number 0.368 = $e^{-1}$ and the number 0.632 = $(1-e^{-1})$.

**Capacitive Reactance**

Reactance is a characteristic exhibited by capacitors and inductors in circuits with time-varying voltages and currents, such as common sinusoidal AC circuits. Like resistance, reactance opposes the flow of electric current and is measured in ohms. Capacitive reactance $X_C$ can be found by the equation:

$$X_C = \frac{1}{2\pi fC}$$

where $f$ is the frequency of the applied voltage or current and $C$ is the capacitance in farads. As with resistance, reactance obeys Ohm’s law:

$$V_C = I_C X_C \quad \text{or} \quad X_C = \frac{V_C}{I_C}$$

If a sinusoidal voltage is applied across a resistor, the current through the resistor is in phase with the voltage. That is not true for a capacitor. If we connect a capacitor across a sinusoidal voltage, the maximum current flows through the capacitor when the voltage’s rate of change is maximum (i.e., at $V=0$), and diminishes as the voltage on the capacitor increases, until finally the current is zero when the voltage is at maximum and its derivative is zero. At that instant, the maximum possible charge for the applied voltage is stored in the capacitor, and so the flow of charge (i.e., the current) stops.

The current and the voltage have exactly the same frequency, but the current through the capacitor is leading the voltage by $\frac{1}{4}$ cycle — 90° or $\pi/2$ radians. Figure 3.1 illustrates this relationship.

![Figure 3.1](image-url)
Phasors
When a sinusoidal voltage at frequency $f$ drives a circuit that contains only linear elements, the waveforms throughout the circuit are also sinusoidal and at the same frequency. To understand the relationships among the sinusoidal voltages, currents, and impedances, we represent the various waveforms as two-dimensional vectors called phasors. A phasor is a complex number used to represent a sinusoidal wave, taking into account both its amplitude and phase angle. As a complex number, a phasor has “real” and “imaginary” components, but like any two-dimensional vector, it can be drawn simply on ordinary XY axes, with the “real” axis in the usual X direction and the “imaginary” axis in the usual Y direction. Such phasor drawings are very helpful in analyzing circuits and understanding the relationships of the various voltages and currents. The algebra of complex numbers can then be used to perform arithmetic operations on the sinusoidal waves. Make no mistake: adding voltages or currents in an AC circuit without taking account of phase angles will lead to confusing and wrong results.

A series RC circuit is illustrated in Figure 3.2(a) and a phasor diagram of its impedances is shown in Figure 3.2(b). The vector for resistance $R$ is shown along the Real (X) axis, while the reactance $X_C$ is shown in the negative Imaginary (Y) axis, since its voltage trails its current by $90^\circ$. The vector sum of $R$ and $X_C$ is labeled $Z$ and has magnitude 5 kΩ. Therefore, the magnitude of the current through the circuit is $V_S / Z = 1.0$ mA.

Notice that the current phasor is in the same as the direction of the voltage across the resistor, because voltage and current are in phase for a resistor. The current is the same throughout the series circuit, including the capacitor, and the voltage across the capacitor trails the current by $90^\circ$. The source voltage is the vector sum of the voltages across the resistor and the capacitor, as is illustrated in the phasor diagram, Figure 3.2(c).

![Figure 3.2](image)

SAFETY PRECAUTIONS
1. Always turn off power to the circuit when changing the circuit.
2. Failure to turn off power when making circuit changes is a major reason for blowing fuses in the equipment, thereby rendering the equipment unusable and wasting your time and that of others. Please carefully check circuit wiring, resistor settings, and voltage settings before applying power to the circuits.
3. Only reapply power after verifying that the circuit is properly wired and that the voltage to be applied is at or below the required value.
PROCEDURE

The measurement instruments indicated in the following procedures are built into the NI-ELVIS system.

1. Set 10 kΩ on the resistor decade box and measure its resistance using the digital multimeter (DMM). Record the value in Table 3.1.

2. Select two 0.01 µf capacitors. Identify them as C₁ and C₂.
   a. Measure the capacitances using the DMM by connecting the capacitor leads to the DUT+ and DUT– pin sockets. (DUT means “device under test”.) Record the values in Table 3.1.
   b. Connect the capacitors in parallel and measure the combined capacitance using the DMM and record the value in Table 3.1. Wait for the measurement to stabilize before recording. Calculate the theoretical prediction for the parallel capacitance and record it in Table 3.1. Compare the theoretical and measured values. Does the measurement tend to confirm the theoretical prediction? Explain.
   c. Connect the capacitors in series and measure the combined capacitance using the DMM and record the value in Table 3.1. Wait for the measurement to stabilize before recording. Calculate the theoretical prediction for the series capacitance and record it in Table 3.1. Compare the theoretical and measured values. Does the measurement tend to confirm the theoretical prediction? Explain.
   d. Select capacitor C₁. Compute its theoretical reactance $X_{C1}$ at a frequency $f = 500$ Hz. Record the value in Table 3.1

<table>
<thead>
<tr>
<th></th>
<th>Listed or Theoretical value</th>
<th>Measured value</th>
</tr>
</thead>
<tbody>
<tr>
<td>R</td>
<td></td>
<td></td>
</tr>
<tr>
<td>C₁</td>
<td></td>
<td></td>
</tr>
<tr>
<td>C₂</td>
<td></td>
<td></td>
</tr>
<tr>
<td>C₁-C₂ parallel</td>
<td></td>
<td></td>
</tr>
<tr>
<td>C₁-C₂ series</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$X_{C1}(500$ Hz)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
For steps 3 through 6, record the results of the measurements in Table 3.2. Throughout the experiment measure all voltages as peak-to-peak values.

### Table 3.2

<table>
<thead>
<tr>
<th>Frequency, Nominal</th>
<th>Frequency, Actual</th>
<th>( V_S ) Actual</th>
<th>( V_{Cl} )</th>
<th>( V_R )</th>
<th>( I_{pp} )</th>
<th>( \phi )</th>
<th>( X_{CI} )</th>
<th>( C_I )</th>
<th>( Z_{TOTAL} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>500 Hz</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1000 Hz</td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>2000 Hz</td>
<td></td>
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<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4000 Hz</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8000 Hz</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

3. Using the resistor \( R \) and capacitor \( C_I \), connect the series RC circuit shown in Figure 3.3. With the circuit connected, adjust the supply voltage \( V_S \) to 2.0 V peak-to-peak at 500 Hz. Check the voltage \( V_S \) on CHANNEL 0 of the oscilloscope and record its value in table 3.2.

4. Connect CHANNEL 1 of the oscilloscope across capacitor \( C_I \). Be sure that the ground leads of CHANNEL 1 and CHANNEL 0 are connected to ground. Measure \( V_{Cl} \). Record the value in table 3.2.

5. **Interchange** \( R \) and \( C_I \). [This is required because the ground connection of the NI-ELVIS oscilloscope input must be connected to circuit ground, and therefore, one of the leads of the device to be tested must also be connected to ground. See “Avoiding Grounding Errors with Oscilloscope” in Appendix C.]

6. Connect CHANNEL 1 of the oscilloscope across resistor \( R \) and measure \( V_R \).

7. Measure the phase angle \( \phi \) between \( V_R \) and \( V_S \). The method is described in Appendix C, section “Phase Angle Measurement”, subsection “For an oscilloscope with two vertical inputs”. Set the oscilloscope’s TRIGGER to EDGE and trigger on CHANNEL 0. You may use the AUTOSCALE button to ensure the waveforms are approximately the same height and press STOP to improve accuracy when taking cursor measurements.

8. Compute the peak-to-peak current \( I_{pp} \) from \( I_{pp} = V_{pp}/R \). Remember, the current is the same throughout the circuit, so this current also flows through the capacitor.
9. Compute the capacitor’s reactance $X_{C1}$ from $X_{C1} = V_{C1}/I_{PP}$. Compute $C_1$ from the measured $X_{C1}$ and compare to your earlier measurement.

10. Compute the total impedance $Z_{Total}$ by applying Ohm’s law to the circuit. Use the supply voltage set in step 3 and the current found in step 4. Remember, the impedance has both a magnitude and a phase angle (measured relative to the resistor).

11. Repeat steps 3 to 10 (resetting $V_S$ if necessary) for the following frequencies: 1000, 2000, 4000, 8000 Hz.

12. Draw impedance and voltage phasors (as in Figure 3.2) for frequency $f = 1000$ Hz.

13. Draw impedance and voltage phasors for frequency $f = 4000$ Hz.

14. The phasor diagrams at various frequencies show how the impedances, and therefore the voltages, change with frequency. To better see the net effect on the circuit, graph $V_{C1}$ and $V_R$ versus frequency for the values in Table 3.2. Label the curves.

PROBING FURTHER

1. Describe what happens to the current in this RC series circuit as the frequency increases. Explain in general terms why the observed change should occur.

2. Simulate the circuit in B2-SPICE and graph $V_{C1}$ and $V_R$ versus frequency over the range of values in Table 3.2. Compare to your manually drawn curves. [Instructions for setting up the necessary AC simulation in B2-SPICE are shown in Appendix D.]

3. In this experiment it was shown that the voltage phasor diagram can be obtained by multiplying each of the impedance phasors by the current in the circuit. If each of the voltage phasors in the voltage phasor diagram is again multiplied by the current, the resulting diagram is the power phasor diagram. Using the data in Table 3.2 convert the current $I$ and source voltage $V_S$ to RMS values. Then draw a plot of the power phasor diagrams at a frequency of 1000 Hz and another at a frequency of 4000 Hz. Determine the real power, the reactive power, and the apparent power in the RC circuit at those frequencies.

REPORT: Your Laboratory Teaching Assistant will inform you when a report is due, and on which experiment you will report. Make sure that you have recorded all necessary information and data in your laboratory notebook to enable you to prepare a report on this experiment, if so directed, at some time in the future.
Laboratory 4

Inductors and Series RL Circuits

INTRODUCTION: This laboratory continues the study of series linear circuits, this time looking at the effect of inductors in series linear circuits. Besides studying the behavior of inductors, you will use measurements of magnitude and phase to construct phasor diagrams for sinusoidal voltages in a series circuit, and thereby validate Kirchhoff’s Voltage Law even when the current varies with time.

EDUCATIONAL OBJECTIVES:
(1) Learn to predict and to measure inductive reactance.
(2) Learn to apply Ohm’s law to reactances of impedances.
(3) Learn to measure phase angles between voltages.
(4) Learn to draw impedance and voltage phasor diagrams for resistors and inductors in series.
(5) Gain experience in the construction and use of phasor diagrams.

EXPERIMENTAL OBJECTIVES:
(1) Determine the reactance of an inductor in a series RL circuit by measuring voltages.
(2) Draw impedance and voltage phasor diagrams for a series RL circuit.
(3) Determine the real, reactive, and apparent power for a series RL circuit.
(4) Explain the effect of frequency on the impedance and voltage phasors for a series RL circuit.

PRE-LAB:

Reading:
(1) Study the Background section of this Laboratory experiment.
(2) Review Appendix C, especially Avoiding Grounding Errors with Oscilloscope, and Phase-Angle Measurement for an oscilloscope with two vertical inputs.

Written:
(1) In your lab notebook sketch the circuit diagram to be used in the procedure.
(2) Prepare tables to record the data.
(3) Sketch the impedance and voltage phasors diagrams (as in Figure 4.2) you would get at 10 kHz for the circuit in Figure 4.3, using nominal values for the components. Make a table of the magnitudes and phase angles. You will use this to check that your experimental setup is correct.

EQUIPMENT NEEDED:
- NI-ELVIS II
  - Function generator
  - Oscilloscope
  - Digital Multimeter
- Resistor, 5 kΩ
- Inductor, 50 mH
Components may be discrete or via decade substitution boxes.
BACKGROUND

When a current passes through a wire, a magnetic field is generated around the wire. The magnetic field results from the movement of electric charge and is proportional to the magnitude of the current. Turning the wire into a coil concentrates that magnetic field, with the field of one turn of the coil reinforcing another. Such a coil is called an inductor. From an electrical point of view the especially interesting fact is that if the current in the wire changes, the magnetic field will react to try to keep the current constant. This property of inductors is described by Lenz’s law.

An inductor’s response to changes of current is called inductance. Inductance opposes changes in current, just as capacitance opposes changes in voltage. Inductance is the electric current equivalent of inertia in mechanical systems.

Inductance is measured in henries. One henry is the amount of inductance present when one volt is generated as a result of a current changing at the rate of one ampere per second.

When inductors are connected in series, the total inductance is the sum of the individual inductors. This is similar to the way resistors in series add. When inductors are connected in parallel, the inductances add in the same way that parallel resistors add. However, an additional effect can appear in inductance circuits that is not present with resistors. This effect is called mutual inductance and is caused by the interaction of the magnetic fields of neighboring inductors. Mutual inductance can either increase or decrease the total inductance, depending on the orientation of the interacting inductors.

Inductance circuits have a time constant associated with them, just as do capacitance circuits, except that the curve of interest for inductors is the current, rather than the voltage. The time constant \( \tau \) for inductors is

\[
\tau = \frac{L}{R},
\]

where \( \tau \) is in seconds; \( L \) is in henries; \( R \) is in ohms.

The voltage induced across the inductor is a maximum when the change in current is a maximum. When a sinusoidal current is applied to an inductor, the largest induced voltage appears across the inductor when the current is passing through zero. At the peaks of the applied current, the slope is zero and the current is not changing, so the induced voltage is zero. Therefore, the voltage that appears across an inductor leads the current in the inductor by 1/4 cycle.

![Figure 4.1 Voltage leads current in inductor.](image-url)
Inductive reactance

As the frequency of the sine wave increases, the rate of change of the current also increases, and so the induced (reacting) voltage across the inductor increases. As a result, the net current through the inductor decreases. That means, the inductor’s reactance increases with frequency. The inductive reactance is given by

$$X_L = 2\pi f L$$

As with capacitors and resistors, Ohm’s law can be applied to inductive circuits:

$$X_L = \frac{V_L}{I_L}.$$  

Series RL circuits

When a sine wave is applied to a series circuit of linear components (resistors, capacitors, and inductors), the phase relationships between current and voltage depend on the types of components. The current and voltage are always in phase across an ideal resistor. The current through a capacitor leads the voltage across the capacitor by 90°. The voltage across an ideal inductor leads the current through the inductor by 90°.

A common memory aid for these relationships is “ELI the ICE man”, where E represents voltage (E is short for “electromotive force”, which is another term for voltage), I represents current, L represents inductance, and C represents capacitance.

From Kirchhoff’s current law (KCL), we know that the current is the same throughout a series circuit. Since the current and voltage are in phase for a resistor, we can determine the phase of the current by measuring the phase of the voltage across the resistor. This is commonly done by using an oscilloscope to compare the voltage from the source to the voltage across the resistor.

Consider the series RL circuit shown in Figure 4.2(a). Graphical representations for the impedance phasors are shown in Figure 4.2(b). As with the series RC circuit, the total impedance of the series RL circuit is obtained by the vector sum of the impedance phasors. (Because the vectors are in the complex plane, this summation can also be done equivalently by summing the complex numbers.)

![Figure 4.2](image-url)

In this example, 5V are applied and the total impedance is 5 kΩ, so the total current is 1.0 mA. Because the current is the same in all components of the series circuit, we use its direction...
through the resistor for the phase reference direction. Multiplying the impedance phasors by the current gives the voltage phasors, as shown in Figure 4.2(c).

In this experiment you will use an oscilloscope to measure the phase angles in the circuit. Because of the resistance of the wires in an inductor, actual inductors may have enough resistance to affect the phase angle. To minimize that effect in this experiment, we will use a series resistor that is large compared to the inductor’s resistance.

SAFETY PRECAUTIONS

1. Always **turn off power to the circuit when changing the circuit.**
2. Only reapply power after verifying that the circuit is properly wired and that the voltage to be applied is at or below the required value.
3. Failure to turn off power when making circuit changes is a major reason for blowing fuses in the equipment, thereby rendering the equipment unusable and wasting your time and that of others. Please carefully check circuit wiring, resistor settings, and voltage settings before applying power to the circuits.

PROCEDURE

The measurement instruments indicated in the following procedures are built into the NI-ELVIS system. Use peak-to-peak readings for all voltage and current measurements in this experiment.

1. Construct a table for recording experimental data:

<table>
<thead>
<tr>
<th>R</th>
<th>L</th>
<th>( f )</th>
<th>( V_{S(\text{gen})} )</th>
<th>( V_{S(\text{osc})} )</th>
<th>( V_R )</th>
<th>( V_L )</th>
<th>I</th>
<th>( X_L )</th>
<th>( Z_T )</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>( \phi_{\text{meas}} )</th>
<th>( \phi_{\text{calc}} )</th>
</tr>
</thead>
</table>

2. Select a 5 kΩ resistor or set that value on the resistor decade box. Measure its resistance using the digital multimeter (DMM) and record the measured value.
3. Select a 50 mH inductor or set that value on the inductor decade box. Measure its inductance \( L \) and winding resistance \( R_W \) using the DMM and record the value. To measure inductance requires connecting the leads to DUT+ and DUT−, as you did for the capacitor.
4. Construct the circuit shown in Figure 4.3. With the circuit connected, adjust the function generator’s frequency \( f \) to 10 kHz with an output voltage of 4.0 \( V_{PP} \). Record the actual values of \( f \) and \( V_S(\text{generator}) \).
5. Using the 2-channel oscilloscope, connect CHANNEL 0 to measure $V_S$, the voltage across the source (nodes A and G). Connect CHANNEL 1 to measure $V_R$, the voltage across the resistor (nodes B and G). Be sure that the ground leads are both connected to the same point (node G). Record $V_S$ (Oscilloscope) and $V_R$.

6. With the oscilloscope still connected as in step 5, measure the phase angle $\phi$ between $V_R$ and $V_S$. The method is described in Appendix C, section “Phase Angle Measurement”, subsection “For an oscilloscope with two vertical inputs”. Set the TRIGGER to EDGE and trigger on CHANNEL 0. You may use the AUTOSCALE button to ensure the waveforms are approximately the same height and press STOP to improve accuracy when taking cursor measurements. Record this value as $\phi_{meas}$ in the table.

7. **Interchange** $R$ and $L$. [This is required because the ground connection of the NI-ELVIS oscilloscope input must be connected to circuit ground, and therefore, one of the leads of the device to be tested must also be connected to ground.] Connect CHANNEL 1 of the oscilloscope across the inductor $L$ and measure $V_L$. Record the value.

8. Calculate the peak-to-peak current in the circuit by applying Ohm’s law to the resistor. That is, $I = \frac{V}{R}$. Record the calculated current $I$ in the table.

9. Calculate the inductive reactance $X_L$ by applying Ohm’s law to the inductor. That is, $X_L = \frac{V_L}{I}$.

   Record the calculated reactance in the table.

10. Calculate the total impedance $Z_T$ by applying Ohm’s law to the entire circuit. $Z_T = \frac{V_S}{I}$.

    Record the value of $Z_T$ in the table.

11. Using the values listed in the table, draw a diagram of the impedance phasors and a second diagram of the voltage phasors, as illustrated in Figure 4.2 of the Background section of this laboratory experiment.
12. Compute the phase angle $\varphi$ between $V_S$ and $V_R$. Recall that

$$\varphi = \arctan\left(\frac{V_L}{V_R}\right).$$

Record this value as $\varphi_{\text{calc}}$ in the table, and compare it to the value measured on the oscilloscope.

13. Using the data in the table, convert the current $I$ and source voltage $V_S$ to RMS values. Then draw plots of the power phasor diagrams at the frequency of 10k Hz. Determine the real power, the reactive power, and the apparent power in the RL circuit at that frequency.

PROBING FURTHER

1. What you think would happen to the current in this RL series circuit if the frequency were decreased, say, to 2000 Hz? Why?

2. Simulate the circuit in B2-SPICE and graph $V_L$ and $V_R$ versus frequency from 1000 Hz to 20,000 Hz. Does the simulation support your prediction in question 1?

REPORT: Your Laboratory Teaching Assistant will inform you when a report is due, and on which experiment you will report. Make sure that you have recorded all necessary information and data in your laboratory notebook to enable you to prepare a report on this experiment, if so directed, at some time in the future.
INTRODUCTION: This laboratory explores the behavior of parallel RC and RL circuits, and the application of Kirchhoff’s current and voltage laws to such circuits. For series RC and RL circuits, we saw that Kirchhoff’s voltage law applies, but that the voltages must be added as phasors. Similarly, in parallel circuits, Kirchhoff’s current law applies to any junction, but again, the currents must be added as phasors.

EDUCATIONAL OBJECTIVES:
1. Learn to apply Kirchhoff’s voltage law (KVL) and Kirchhoff’s current law (KCL) in parallel circuits.
2. Learn to draw current phasor diagrams for parallel circuits.
4. Gain experience in calculating real, reactive, and apparent power.

EXPERIMENTAL OBJECTIVES:
1. Confirm Kirchhoff’s current law (KCL) in parallel circuits.
2. Draw current phasor diagrams for parallel circuits.
3. Determine the real, reactive, and apparent power for a parallel RC circuit.

PRE-LAB:
Reading:
1. Study the Background section of this Laboratory.

Written:
1. In your lab notebook sketch the circuit diagrams to be used in the procedure.
2. Prepare tables to record data.

EQUIPMENT NEEDED:
- NI-ELVIS II
  - Function generator
  - Oscilloscope
  - Digital Multimeter
- Resistor, 10 kΩ
- Resistor, 2.2 kΩ
- Resistor, 100 Ω, Qty. 2 (discrete)
- Resistor, 22 Ω, Qty. 2 (discrete)
- Inductor, 100 mH
- Capacitor, 0.01 µF

Components may be discrete or via decade substitution boxes, unless otherwise indicated.
BACKGROUND

As was seen in prior experiments, in a series circuit the same current is in all components, and so current is generally used as a reference in series circuits. However, in parallel circuits, the same voltage is across all components, so voltage is the logical and appropriate reference. The current in each branch then evolves from the circuit voltage.

For series RC and RL circuits, we saw that Kirchhoff's voltage law applies, but that the voltages must be added as phasors. Similarly, in parallel circuits, Kirchhoff's current law applies to any junction, but again, the currents must be added as phasors. The current entering a junction is always equal to the current leaving the junction.

In a parallel circuit, if the impedance of each branch is known, then the current in that branch can be determined directly from the applied voltage and Ohm's law. The current phasor diagram can then be constructed, and the total current can be found as the phasor sum of the currents in each branch.

Consider the current phasor diagram for the parallel RC circuit shown in Figure 5.1. The current in the capacitor is shown at +90° from the voltage reference because the current leads the voltage in a capacitor. The current in the resistor is along the x-axis because current and voltage are in phase in a resistor.

Figure 5.1: Parallel RC circuit

Figure 5.2 shows a parallel RL circuit and its current phasor diagram. Here we have assumed all components are “ideal”. The current in an ideal inductor is at -90° from the voltage reference, because the current lags the voltage in an inductor. However, practical inductors contain resistance that often is large enough to affect the phasor. The effect of the inductor’s resistance on the phasor diagram would be to reduce the angle between the $I_L$ and $I_R$. In a practical circuit, this an-
gle will be slightly less than the -90° of a pure inductor. This experiment will illustrate this difference between the approximation of circuit performance based on ideal components and the actual measured values.

For both RC and RL circuits, the Pythagorean theorem and ordinary vector addition can be applied to the current phasors to determine the magnitude of the total current, $I_T$.

$$I_T = \sqrt{I_R^2 + I_C^2} \quad \text{and} \quad I_T = \sqrt{I_R^2 + I_L^2}.$$  

Recall that in series circuits, the phase angle was measured between the source voltage $V_S$ and the resistor voltage $V_R$ using the oscilloscope. The oscilloscope is a voltage-sensitive device, so examining those voltages and phase angles is straightforward. But in parallel circuits, the phase angle of interest usually is between the total current, $I_T$, and one of the branch currents. To use the oscilloscope to measure the phase angle in a parallel circuit, we must convert the current to a voltage. This is commonly done by inserting a small resistor (a "sense" resistor) in the branch with the current to be measured. Such a sense resistor makes it easy to determine the magnitude and the phase of the current in that branch, but the resistor must be small enough not to have a significant effect on the values to be measured.

**PROCEDURE**

**Parallel RC Circuit**

This part of the experiment will give you experience making the measurements with the digital multimeter (DMM). Because the electrodes of this device are isolated from the circuit ground and the grounds in NI-ELVIS, you may make voltage measurements directly across any of the components, whether or not they are grounded.

1. Create a table similar to Table 5.1 for recording experimental data: $R_1$, $R_{S1}$, $R_{S2}$, $C$, $f$, $V_S$, $V_{RS1}$, $V_{RS2}$, $V_{R1}$, $I_T$, $I_C$, $I_{R1}$.

**Table 5.1**

<table>
<thead>
<tr>
<th></th>
<th>Nominal value</th>
<th>Measured value</th>
<th>Voltage across R</th>
<th>Current calculated</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_{S1}$</td>
<td>100 Ω</td>
<td></td>
<td>$V_{RS1}$</td>
<td>$I_T$</td>
</tr>
<tr>
<td>$R_{S2}$</td>
<td>100 Ω</td>
<td></td>
<td>$V_{RS2}$</td>
<td>$I_C$</td>
</tr>
<tr>
<td>$R_1$</td>
<td>10 kΩ</td>
<td></td>
<td>$V_{R1}$</td>
<td>$I_{R1}$</td>
</tr>
<tr>
<td>$C$</td>
<td>0.01 μF</td>
<td></td>
<td></td>
<td>$I_C$</td>
</tr>
<tr>
<td>$V_S$</td>
<td>2.0 Vrms</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$f$</td>
<td>1.0 kHz</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
2. Measure circuit components using the DMM: one 10 kΩ resistor, two 100 Ω sense resistors, and one 0.01 µF capacitor. (Be sure to wait until the capacitance measurement stabilizes.) Record the measured values in Table 5.1.

3. Construct the circuit shown in Figure 5.3. Set the function generator to provide a sine wave with a voltage of 2.0 V\text{rms} at 1.0 kHz. Verify the voltage and frequency with your oscilloscope while the circuit is connected and operating; adjust if necessary.

![Figure 5.3](image)

4. Using the DMM voltmeter in AC mode [V~], measure the voltage drop across each resistor. The voltage drops are small, so measure as accurately as possible and keep three significant figures in your measurement. Record the voltage drops in Table 5.1.

5. Compute the current in each resistor using Ohm's law, and record the calculated currents in Table 5.1.

6. Draw the current phasors $I_{R1}$, $I_C$, and the total current $I_T$ in a plot similar to that of Figure 5.1. The total current is through sense resistor $R_{S1}$. The current $I_C$ is through sense resistor $R_{S2}$. Ignore the small effect of the sense resistors on the phasor diagram. Note carefully the direction of the phasors. Label each of the current phasors.

7. Compute $X_C$ for the 1.0 kHz frequency and record the value in Table 5.2. Using this value and that of the sense resistor, calculate the expected current, $I_C$, through the capacitor. Record the value in the available space in Table 5.1 on the line listing the capacitor’s value. How does this value compare to that found from the sense resistor?

8. Using the value of $X_C$ for the 1.0 kHz frequency and the measured resistance of $R_1$, find the total impedance, $Z_T$, of the circuit. Remember that these impedances add like parallel resistors. Ignore the sense resistors for this calculation. Show your work.

9. Using $Z_T$ and the applied voltage, $V_S$, compute the total current, $I_T$. Show your work. The total current should reasonably agree with the value determined in step 5.

Table 5.2

<table>
<thead>
<tr>
<th>$X_C$</th>
<th>$Z_T$</th>
<th>$V_S$</th>
<th>$I_{T,\text{computed}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
10. Change the frequency to 2.0 kHz at 2.0 V_{rms}. Repeat steps 4, 5, and 6, recording the results in a separate table.

**PROBING FURTHER**

1. Ignoring the effect of the sense resistors, compute the real, reactive, and apparent power of the circuit in Figure 5.3 at 1.0 kHz and at 2.0 kHz.

2. Explain how increasing the frequency affected (a) the total impedance of the circuit, and (b) the phase angle between the function generator's voltage and the function generator's current.

3. If the capacitor C were made smaller, what would happen to the current phasor diagrams?

**Parallel RL Circuit**

This part of the experiment will give you experience making similar measurements using the oscilloscope. Remember that the ground lead of the oscilloscope must be connected to circuit ground, that is, the ground of the function generator, in this case. **Do not try to make measurements directly across ungrounded components.**

Also, in this part of the experiment, record all voltages and currents as peak-to-peak values.

1. Create a table similar to Table 5.3 for recording experimental data: \(R_1, R_{S1}, R_{S2}, L, f, V_S, V_{RS1}, V_{RS2}, V_{RI}, I_T, I_L, I_{R1} \).

<table>
<thead>
<tr>
<th></th>
<th>Nominal value</th>
<th>Measured value</th>
<th>Voltage across R</th>
<th>Current calculated</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R_{S1} )</td>
<td>22 ( \Omega )</td>
<td></td>
<td>( V_{RS1} )</td>
<td>( I_T )</td>
</tr>
<tr>
<td>( R_{S2} )</td>
<td>22 ( \Omega )</td>
<td></td>
<td>( V_{RS2} )</td>
<td>( I_T )</td>
</tr>
<tr>
<td>( R_I )</td>
<td>2.2 k( \Omega )</td>
<td></td>
<td>( V_{RI} )</td>
<td>( I_{R1} )</td>
</tr>
<tr>
<td>( L )</td>
<td>100 mH</td>
<td></td>
<td></td>
<td>( I_L )</td>
</tr>
<tr>
<td>( V_S )</td>
<td>4.0 V_{PP}</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( f )</td>
<td>4.0 kHz</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2. Using the DMM, measure the actual values of a 2.2 k\( \Omega \) resistor, two 22 ohm resistors, and a 100 mH inductor.

3. Measure the winding resistance of the inductor, \( R_W \), with an ohmmeter.
4. Construct the circuit shown in Figure 5.4. Using the oscilloscope’s CHANNEL 0 to monitor the function generator, set the source voltage to a sine wave with a voltage of 4.0 V<sub>pp</sub> at 4.0 kHz. Verify both the voltage and the frequency with the oscilloscope.

5. Using oscilloscope CHANNEL 1, measure the peak-to-peak voltage across R<sub>S1</sub>. Apply Ohm’s law to calculate the peak-to-peak value of the total current, I<sub>T</sub>.

6. Because R<sub>S1</sub> is small compared to R<sub>1</sub>, the source voltage V<sub>S</sub> is very nearly the voltage across the resistor R<sub>1</sub>. Furthermore, since only resistors are in that loop, the phase of the voltage across R<sub>1</sub> — and current through R<sub>1</sub> — is the same as the phase of the source voltage V<sub>S</sub>.

With CHANNEL 0 of the oscilloscope displaying the voltage across the function generator (V<sub>S</sub>), and CHANNEL 1 displaying the voltage across R<sub>S1</sub>, measure the phase angle between the generator voltage, V<sub>S</sub>, and the generator current, I<sub>T</sub>, (i.e., the current flowing through R<sub>S1</sub>). This is equivalently the phase angle between I<sub>R1</sub> and I<sub>T</sub>.

To make the measurement, set the scope’s TRIGGER to EDGE and trigger on CHANNEL 0. Use the AUTOSCALE button to ensure the waveforms are approximately the same height. You may press STOP to improve accuracy when taking cursor measurements. Record the measured phase angle between I<sub>R1</sub> and I<sub>T</sub>.

7. Replace the sense resistor R<sub>S1</sub> with a jumper wire. CHANNEL 0 is still measuring the output voltage of the function generator, which now is also the voltage across R<sub>1</sub>. Connect CHANNEL 1 of the oscilloscope across sense resistor R<sub>S2</sub>. Record the voltages across R<sub>1</sub> and across R<sub>S2</sub>. Apply Ohm’s law to calculate the current in each branch of the circuit, and record the currents. Since L is in series with R<sub>S2</sub>, enter the same current for both in Table 5.3.

8. Using the computed peak-to-peak currents from Table 5.3, draw the current phasor diagram for the circuit. Ignore the effects of the sense resistors. Your diagram should look somewhat similar to Figure 5.2 (b).

9. The phasor diagram depicts visually the relationship between the total current and the currents in each branch of the circuit. From the currents in the phasor diagram, compute the phase angle between the total current (I<sub>T</sub>) and the current in R<sub>1</sub> (I<sub>R1</sub>). Then compute the phase angle between the total current (I<sub>T</sub>), and the current in L (I<sub>L</sub>). Ideally, what should be the sum of these angles?
10. On the oscilloscope, measure the angle between $I_L$ and $I_{R1}$. (The oscilloscope’s leads were already connected to do this as a result of step 7.) Ideally, this measurement should be 90°, but resistance in the inductor, may reduce the angle. If necessary, adjust both scope channels to have the same apparent height on the oscilloscope screen in order to make the measurement.

11. In step 6 you measured the phase angle $\Phi_{RT}$ between $I_{R1}$ and $I_T$. In step 10 you measured the phase angle $\Phi_{RL}$ between $I_{R1}$ and $I_L$. Compute the phase angle $\Phi_{TL}$ between $I_T$ and $I_L$ by subtracting $\Phi_{RT}$ from $\Phi_{RL}$. That is, $\Phi_{TL} = \Phi_{RL} - \Phi_{RT}$.

12. Construct a table that will allow easy comparison of the computed and measured phase angles from steps 6, 9, 10, and 11. Compare the measured phase angles versus the computed phase angles. Discuss likely causes for any discrepancies.

**PROBING FURTHER**

1. The currents $I_{R1}$ and $I_L$ were measured in step 7. If those currents are 90° apart, we can calculate the total current $I_T$ using the Pythagorean theorem: $I_T = \sqrt{I_{R1}^2 + I_L^2}$.
   
a. Compare this calculated total current to the total current measured in step 5.
   
b. What factors might cause any discrepancies observed between the values?

2. What effect does the inductor’s coil resistance have on the phase angle between the currents in the resistor and the inductor?

**REPORT**: Your Laboratory Teaching Assistant will inform you when a report is due, and on which experiment you will report. Make sure that you have recorded all necessary information and data in your laboratory notebook to enable you to prepare a report on this experiment, if so directed, at some time in the future.
Laboratory 6

Circuit Resonance

INTRODUCTION: The response of a circuit containing both inductors and capacitors in series or in parallel depends on the frequency of the driving voltage or current. This laboratory will explore one of the more dramatic effects of the interplay of capacitance and inductance, namely, resonance, when the inductive and capacitive reactances cancel each other. Resonance is the fundamental principle upon which most filters are based — filters that allow us to tune radios, televisions, cell phones, and a myriad of other devices deemed essential for modern living.

EDUCATIONAL OBJECTIVES:
(1) Learn the definition of resonance in AC circuits.
(2) Learn to calculate resonant frequencies, band widths, and quality factors for series and parallel resonant circuits.
(3) Learn to use NI-ELVIS’s Bode plot function to view circuit response.

EXPERIMENTAL OBJECTIVES:
(1) Predict resonant frequencies, band widths, and quality factors for series resonant circuits.
(2) Measure resonant frequencies, band widths, and quality factors for series resonant circuits, and compare them to predicted value.
(3) Determine what effect changing the series resistance has on the resonance.

PRE-LAB:
Reading:
(1) Study the Background section of this Laboratory exercise.

Written:
(1) In your lab notebook sketch the circuit diagram to be used in the procedure.
(2) Prepare tables to record data.

EQUIPMENT NEEDED:
- NI-ELVIS II
  - Function generator
  - Oscilloscope
  - Digital Multimeter
  - Bode analyzer
- Resistor, 100 Ω
- Resistor, 10 Ω
- Inductor, 100 mH
- Capacitor, 0.01 µF
Components may be discrete or via decade substitution boxes, unless otherwise indicated.
BACKGROUND

The reactance of inductors increases with frequency: $X_L = 2\pi f L$.

The reactance of capacitors decreases with frequency: $X_C = \frac{1}{2\pi f C}$.

In an LC circuit, whether series or parallel, there is some frequency at which the magnitudes of these two reactances are equal. That point is called resonance.

Setting $X_L = X_C$, and solving for $f$, we find that the resonant frequency $f_o$ of an LC circuit is

$$f_o = \frac{1}{2\pi \sqrt{LC}}.$$

The frequency $f$ has units cycles/second or $\text{sec}^{-1}$. The frequency may also be expressed as angular frequency, $\omega$, where $\omega = 2\pi f$ and has units radians/sec. Thus, the resonant frequency may also be written as

$$\omega_o = 2\pi f_o = \frac{1}{\sqrt{LC}}.$$

The resonant frequency is generally the highest point of a peak (or the deepest point of a valley) with bandwidth $\text{BW}$ (cycles/sec) or $\beta$ (radians/sec). The resonant frequency is also called the center frequency, because it is at the mid-point of the peak frequency response.

The lowest frequency ($f_1$ or $\omega_1$) and the highest frequency ($f_2$ or $\omega_2$) of the band are the “half-power points” at which the power is $\frac{1}{2}$ that at the peak frequency. Since power goes like the square of the current, the current at the half-power points is $\frac{1}{\sqrt{2}} (= 0.707)$ times the current at the maximum. Thus, the bandwidth of a resonant circuit is the frequency range over which the current is at least 70.7% of the maximum.

$$\text{BW} = f_2 - f_1 \quad \text{or} \quad \beta = \omega_2 - \omega_1.$$

As the bandwidth narrows, the circuit becomes more highly selective, responding to a narrow range of frequencies close to the center frequency. The sharpness (narrowness) of that resonant peak is measured by the quality factor $Q$. The quality factor is a unitless quantity that is defined as

$$Q \equiv 2\pi \left( \frac{\text{maximum energy stored}}{\text{energy dissipated per cycle}} \right).$$

In more practical terms,

$$Q = \frac{f_o}{\text{BW}} \quad \text{or} \quad Q = \frac{\omega_o}{\beta}.$$
**Series Resonance**

For a series LC circuit, the current is the same throughout. What about the voltages? To visualize the concept of resonance, consider the simple series RLC circuit in Figure 6.1 operating at resonance, and its associated reactance diagram.

![Figure 6.1](image)

The phase shift caused by the capacitor is directly opposite the phase shift caused by the inductor; that is, they are 180° out of phase. Therefore, in the reactance phasor diagram (b) for the circuit, the two phasors point in opposite directions. At resonance, the magnitudes of the capacitor reactance and the inductor reactance are equal, so the sum of the two phasors is zero, and the only remaining impedance is due to the resistor. Notice in the voltage phasor diagram (c) that the voltage drop across the inductor and the capacitor may be quite large — bigger even than the source voltage — but those voltages are opposite in phase and so cancel each other out as voltages are summed around the circuit. Kirchhoff’s voltage law remains valid, and the generator’s voltage output is dropped entirely over the resistor R.

Since at resonance the only impedance is the resistance R, the impedance of the series circuit is at a minimum, and so the current is a maximum. That current is \( V_s / R \). The source voltage and the current are in phase with each other, so the power factor = 1, and maximum power is delivered to the resistor.

But what happens at neighboring frequencies? At lower frequencies, the inductor’s reactance decreases, and the capacitor has greater effect. At higher frequencies, the inductor dominates, and the circuit will take on inductive characteristics.

How sharply defined is the resonance? How selective is it? We have said that for a resonant circuit, the quality factor Q is the ratio of the resonant frequency to the bandwidth. Thus, Q gives a measure of the bandwidth normalized to the frequency, thereby describing the shape of the circuit’s response independent of the actual resonant frequency.

\[
Q = \frac{f_o}{BW}
\]

We list here two other useful relationships for Q in a series resonant circuit. The first relates Q to the circuit’s capacitance, inductance, and total series resistance.
\[ Q = \frac{1}{R} \sqrt{\frac{L}{C}}. \]

The value of R in this equation is the **total equivalent series resistance** in the circuit. This form of the equation makes it easy to see ways to optimize the Q for the desired circuit. Decreasing R, increasing inductance, or decreasing capacitance will all tend to make Q larger and increase the circuit's selectivity.

The second useful relationship for Q can be derived from the previous equation. Recall that \( X_L = 2\pi f L \) and \( X_C = \frac{1}{2\pi f C} \). Then the previous equation can be rewritten as

\[ Q = \frac{1}{R} \sqrt{X_L \cdot X_C} \]

Since at resonance the inductive and capacitive reactances are equal, this equation can be reduced to

\[ Q = \frac{X_L}{R} \quad \text{or} \quad Q = \frac{X_C}{R} \]

where R is again the total equivalent series resistance of the circuit. Usually the \( X_L \) form is used because the resistance of the inductor frequently is the dominant resistance in the circuit.

An equivalent form of this last equation is

\[ Q = \frac{2\pi f L}{R} \quad \text{or} \quad Q = \frac{1}{2\pi f C R}. \]

**Parallel Resonance**

For a parallel RLC circuit, the same voltage is applied across all the branches. The current in each branch is determined by the voltage applied to the branch and the impedance in that branch.

A parallel RLC circuit with ideal components is illustrated below, along with its current phasor diagram. Notice that the total source current at resonance is the current in the resistor. The currents in the inductor and capacitor cancel out because of their opposite phase shifts. The net impedance of the circuit at resonance is solely determined by R, since the inductor and the capacitor appear to be open.

![Figure 6.2](image)

In an *ideal* two-branch LC circuit including an inductor and capacitor (i.e., R is removed or equivalently R=\( \infty \)), the source current would be zero. In a *practical* two-branch LC circuit, the only significant resistance is the inductor’s winding resistance, so that resistance often plays an important role in the circuit’s behavior.
In series resonant circuits, we defined the bandwidth by observing the response of the current. In parallel resonant circuits, the corresponding response to observe is the impedance. The bandwidth of a parallel resonant circuit is the frequency range over which the circuit impedance is at least 70.7% of the maximum impedance. The sharpness of the response is again measured as Q. Note that the Q of the circuit will be different from the Q of the inductor if other resistance is in parallel with L and C. If there is no additional resistance (i.e., \( R = \infty \)), then the circuit Q is the same as the inductor’s Q.

**Table 6.1: Summary of the characteristics of resonant RLC circuits**

<table>
<thead>
<tr>
<th>Characteristic</th>
<th>Series circuit</th>
<th>Parallel circuit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Resonant Frequency, ( f_o )</td>
<td>( \frac{1}{2\pi \sqrt{LC}} )</td>
<td>( \frac{1}{2\pi \sqrt{LC}} )</td>
</tr>
<tr>
<td>Quality factor, Q</td>
<td>( \frac{2\pi f_o L}{R} ) or ( \frac{1}{2\pi f_o RC} )</td>
<td>( \frac{R}{2\pi f_o L} ) or ( 2\pi f_o RC )</td>
</tr>
<tr>
<td>Bandwidth, BW</td>
<td>( \frac{f_o}{Q} )</td>
<td>( \frac{f_o}{Q} )</td>
</tr>
<tr>
<td>Half-power frequencies ( f_1, f_2 )</td>
<td>( f_o \sqrt{1 + \left( \frac{1}{2Q} \right)^2 \pm \frac{f_o}{2Q}} )</td>
<td>( f_o \sqrt{1 + \left( \frac{1}{2Q} \right)^2 \pm \frac{f_o}{2Q}} )</td>
</tr>
<tr>
<td>For Q ( \geq 10 ), ( f_1, f_2 )</td>
<td>( f_o \pm \frac{BW}{2} )</td>
<td>( f_o \pm \frac{BW}{2} )</td>
</tr>
</tbody>
</table>

**PROCEDURE**

1. Using the NI-ELVIS’s DMM, measure the values of the following components: 100mH inductor; 0.01µF capacitor; 100 \( \Omega \) resistor; 10 \( \Omega \) resistor. Also measure the winding resistance \( R_W \) of the 100 mH inductor. Record the nominal and measured values in Table 6.1.

**Table 6.1: Component values**

<table>
<thead>
<tr>
<th></th>
<th>Nominal</th>
<th>Actual</th>
</tr>
</thead>
<tbody>
<tr>
<td>( L )</td>
<td>100 mH</td>
<td></td>
</tr>
<tr>
<td>( C )</td>
<td>0.01 µF</td>
<td></td>
</tr>
<tr>
<td>( R_W )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( R_1 )</td>
<td>100 ( \Omega )</td>
<td></td>
</tr>
<tr>
<td>( R_2 )</td>
<td>10 ( \Omega )</td>
<td></td>
</tr>
</tbody>
</table>

2. For the circuit shown in Figure 6.3, calculate predictions for \( f_0, Q, BW, f_1, \) and \( f_2 \). Don’t forget to include the impedance of the function generator (\( R_S \approx 50 \Omega \)) and \( R_W \) as part of
the total resistance in the circuit. Record the results in the first “predicted” column in a table such as Table 6.2.

![Circuit Diagram](image)

**Figure 6.3**

**Table 6.2: Experimental values**

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$L$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$C$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$R_W$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$R$</td>
<td>100 Ω</td>
<td>10 Ω</td>
<td></td>
</tr>
<tr>
<td>$R_{total}$</td>
<td></td>
<td>Predicted</td>
<td>Measured</td>
</tr>
<tr>
<td>$f_o$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$Q$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$BW$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$f_1$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$f_2$</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

3. Construct the circuit shown in Figure 6.3. Adjust the function generator to generate a sine wave with voltage 1.0 $V_{pp}$. Initially set the frequency to 1 kHz.

4. Connect oscilloscope CHANNEL 0 across the function generator (FGEN and GND) and confirm that the voltage is 1.0 $V_{pp}$.

5. Connect oscilloscope CHANNEL 1 across the resistor R and observe the voltage.

6. Using your predicted values as a guide, adjust the frequency of the function generator to tune for resonance, as observed on CHANNEL 1 of the oscilloscope. Measure the resonant frequency $f_o$ on the oscilloscope, and record the value in the first “measured” column of Table 6.2.

7. Confirm that the voltage on CHANNEL 0 of the scope is 1.0 $V_{pp}$, and adjust it if necessary. The current through the circuit and resistor R is proportional to the voltage across R. Record the voltage across resistor R.
For steps 8 and 9, DO NOT adjust the voltage output of the function generator.

8. Reduce the frequency on the function generator until the voltage across R is 70.7% of the initial value. This is the lower half-power point $f_1$. Record the measured frequency $f_1$ in the first “measured” column of Table 6.2.

9. Increase the frequency through resonance and continue to increase it until the voltage across R is 70.7% of the value at resonance. This is the upper half-power point $f_2$. Record the measured frequency $f_2$ in the first “measured” column of Table 6.2.

10. Calculate the bandwidth $BW = f_2 - f_1$. Record the result in the first “measured” column of Table 6.2.

11. Stop the function generator. Remove the 100 Ω resistor from the circuit and replace it with the 10 Ω resistor measured earlier.

12. Calculate predictions for $f_0$, $Q$, $BW$, $f_1$, and $f_2$ and record the results in the second “predicted” column in Table 6.2.

13. Start the function generator and, as before, adjust the function generator to create a sine wave with voltage 1.0 V\textsubscript{PP}.

14. Repeat steps 5 through 10, recording the measured values in the second “measured” column.

15. Fill out Table 6.3, calculating the percent differences between predicted and measured values.

### Table 6.3: Percent difference

<table>
<thead>
<tr>
<th>% difference [=100% \left( \frac{\text{measured} - \text{predicted}}{\text{predicted}} \right) ]</th>
<th>R = 100 Ω</th>
<th>R = 10 Ω</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_0$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$Q$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$BW$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$f_1$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$f_2$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

16. Use the Bode (pronounced Bō´·dē) analyzer of the NI-ELVIS II to view the plot of the last circuit’s response. To do so:

[a] connect jumpers from the function generator to the Analog Inputs, i.e., FGEN (point A) to AI1+ and GND (point G) to AI1–;

[b] connect a jumper from the high side of the resistor (point B) to A0+;

[c] connect the a jumper from the low side of the resistor (point G) to A0–. Then start the Bode Analyzer. Set the frequency to cover the range of interest. Be sure to sample enough points for an adequate view of the circuit’s response. Turn on CURSORS to be able to locate specific values. The NI-ELVIS’s Help function can aid you in learning to use the analyzer to explore the plot.
PROBING FURTHER

1. From the measured \( L, C, f_0, \) and BW, compute the total series resistance of the circuit. Suggest and explain likely causes for any discrepancy; what might not have been taken into account for the predictions?

2. Discuss the effect of changing the resistor \( R \) from 100 \( \Omega \) to 10 \( \Omega \). How dramatic was the impact?

3. What would happen to the resonant frequency if the inductance were doubled and the capacitance cut in half? What would happen to the bandwidth? What would happen to the quality factor?

4. Simulate the second circuit (with \( R=10\Omega \)) in B2-SPICE, using the AC Frequency Sweep (see Appendix D). In the simulation, set up a voltmeter across \( R \) and an ammeter to measure the total current. Be sure to have B2-SPICE display both graphs and a table of the results. Select an appropriate range of frequencies to sweep and sample at least 100 steps per interval. When the graph is displayed, right-click the graph and toggle the X-axis to be logarithmic. Do you see a sharp peak in the magnitude (dB) plots of voltage and current at the resonant frequency you predicted? Notice that the phase plot for the circuit rapidly changes from positive (capacitive) to negative (inductive) as the frequency passes through resonance. Record the resonant frequency indicated by the graph, and compare it to your calculated and measured values.

REPORT: Your Laboratory Teaching Assistant will inform you when a report is due, and on which experiment you will report. Make sure that you have recorded all necessary information and data in your laboratory notebook to enable you to prepare a report on this experiment, if so directed, at some time in the future.
Laboratory 7

Filters: High-pass, Low-pass, Bandpass, and Notch

INTRODUCTION: This laboratory studies the use of passive components to create filters to separate portions of time-dependant waveforms. Filters are an essential tool in our complex world of mixed signals — both electronic and otherwise. Passive components (resistors, capacitors, and inductors) have long served as filter components for everything from selecting radio stations to filtering out electrical noise.

EDUCATIONAL OBJECTIVES:
(1) Learn the four general filter types: High-pass, Low-pass, Bandpass, and Notch
(2) Learn to alter filter type by changing contacts for output voltage.
(3) Learn phase angle at cutoff for simple RC and RL filters.
(4) Design simple filter.
(5) Gain experience with Bode Analyzer and Bode Plots.

EXPERIMENTAL OBJECTIVES:
(1) Calculate and measure cutoff frequency for series RC and RL filters.
(2) Design simple RL low-pass filter.
(3) Generate and interpret Bode plots for series filters.
(4) Observe behavior of a single circuit used as a bandpass filter and as a notch filter.

PRE-LAB:
Reading:
(1) Study the Background section of this Laboratory.
(2) Study textbook regarding origin and meaning of “decibels”.

Written:
(1) In your lab notebook sketch the circuit diagrams to be used in the procedure.
(2) Perform the design calculations for the filter for Procedure 2.
(3) Construct tables suitable for recording data where necessary, especially in Procedure 1.

EQUIPMENT NEEDED:
• NI-ELVIS II
  • Function generator
  • Oscilloscope
  • Bode Analyzer
  • Digital Multimeter

Note: Suitable Decade Substitution boxes may be used for the following components:
• Resistors: 10 kΩ, 100 Ω, and a resistor to be determined
• Inductor: 100 mH
• Capacitor: 0.005 μF, 0.01 μF

BACKGROUND
In many circuits, a wide range of different frequencies are present, some of which are desired, while others are not. The frequency response of capacitors and inductors allows us to construct
filters that will pass or reject certain ranges of the electrical frequencies that are applied to them. "Passive filters" created from “passive” components (inductors, capacitors, and resistors) have served us well for a long time for such purposes as selecting radio and television stations and filtering noise out of various signals. Indeed, much of the electronics we take for granted today would not be possible without the use of such filters.

The four typical types of filter behaviors are illustrated in Figure 7.1, along with schematics of simple filters that exhibit the indicated behavior.

The filter types are low-pass, high-pass, bandpass, and notch (or band-reject) filters. In Figure 7.1, the grayed area is the passband, that is, the part of the signal that is passed to the output of the filter. The rejected portions are called the stopband. The frequency that separates the passband from the stopband is called the cutoff frequency. The cutoff frequency is equivalent to the half-power points discussed in Laboratory 6. The cutoff frequency is also sometimes called the corner frequency.

A low-pass filter would allow extracting a low frequency, such as an audio signal, that is mixed with a high frequency radio wave. A high-pass filter would do the opposite. A resonant circuit can be tuned as a bandpass filter to retain signals in a narrow range of frequencies, while rejecting frequencies outside that range. Such is the case with a radio tuner. A notch filter generally keeps all frequencies except those in a narrow band. Notch filters are widely used to block interfering signals from noise sources. Bandpass and notch filters require resonant circuits, studied in Lab 6.

Notice that the components making the low-pass and high-pass filters in Figure 7.1 are the same. Whether the circuit is low-pass or high-pass depends only upon which voltage we look at: the voltage across the capacitor or the voltage across the resistor. (Equivalent circuits could have been made using an inductor and a resistor.) Similarly, the notch filter is identical to the RLC series resonant circuit we looked at in Lab 6, however in Lab 6 we looked at the voltage across the resistor, and so saw a bandpass filter. Caution: While one may be able to obtain the opposite response from the filter simply by putting the output terminals across a different filter component, one must be sure to stay within the power and current limitations of the circuit and its components.

RC and RL filters are simple, inexpensive, and often used effectively as filters. Their major problem is their generally slow (in frequency) transition from passband to stopband. The addition of a
few simple components in filter “stages” can increase the transition rate, giving the filter a sharper cutoff.

The ratio of an output response to an input signal is referred to as a transfer function. The input signal and the output response do not need to be the same entity type. For example, a transfer function may prescribe an output voltage resulting from an input current. Transfer functions are often used as a tool to characterize the effect of a filter regardless of the details of the filter’s structure. It can make the analysis of complex circuits easier. In this lab, however, we will mostly be studying the filter itself.

Cutoff Frequency for series RC and RL circuits
As mentioned, the cutoff frequency, sometimes called the corner frequency, is equivalent to the half-power points discussed in Laboratory 6. Since the power is half that at the peak, the voltage (or current) will be the peak voltage (or current) multiplied by $1/\sqrt{2} = 0.707$. For a simple 2-component RC or RL circuit, the half-power point will occur when half the power is dropped on the resistor and half on the capacitor or inductor. Thus, the cutoff frequency will occur when the reactance of the capacitor or inductor equals the total series resistance in the circuit. That is,

$$X_C = \frac{1}{2\pi f_c C} = R \quad \text{and} \quad X_L = 2\pi f_c L = R.$$ 

And so,

$$f_c = \frac{1}{2\pi RC} \quad \text{and} \quad f_c = \frac{R}{2\pi L}.$$ 

Resonant frequencies for RLC circuits were discussed in Laboratory 6.

decibels (dB)
As discussed in your textbook, the decibel (dB) is commonly used for the magnitude of voltages and currents in making Bode plots. Keep in mind that a decibel is a unit created to measure the transfer function for power gain (or loss) through a circuit module or stage:

$$\text{Number of decibels} = 10 \cdot \log_{10} \left( \frac{\text{power out}}{\text{power in}} \right).$$

Since power is proportional to the square of the voltage or the current, we have equivalently,

$$\text{Number of decibels} = 20 \cdot \log_{10} \left( \frac{V_{\text{out}}}{V_{\text{in}}} \right) \quad \text{and} \quad \text{Number of decibels} = 20 \cdot \log_{10} \left( \frac{I_{\text{out}}}{I_{\text{in}}} \right).$$

How many decibels correspond to a doubling of the power? To a halving of the power?

PROCEDURE
Note: In all cases below, you may use substitute a decade substitution box of the correct type and value for the specified resistor, capacitor, or inductor.

1. Low-pass filter.
   a. Obtain a 10 kΩ resistor and a 0.005 µF capacitor. Measure and record the actual values of the components.
b. Using the measured components, set up the circuit as shown in Figure 7.2. Use the function generator FGEN for the supply voltage.

![Figure 7.2 Low-pass filter](image)

Figure 7.2 Low-pass filter

c. Calculate the cutoff frequency for the circuit, assuming the output is at $V_o$. At the cutoff frequency, what, theoretically, will be the voltage $V_o$?

d. Connect CHANNEL 0 of the oscilloscope to measure the $V_{in}$ (i.e., FGEN).

e. Connect CHANNEL 1 of the oscilloscope to measure the filter’s output voltage $V_o$. On the oscilloscope, turn on the measurement functions for CH0 and CH1 (Channel 0 and Channel 1, respectively). Vary the frequency from 500 Hz to 10 kHz in steps indicated in Table 7.1.2, and record the indicated values. Set the TRIGGER to EDGE and trigger on CHANNEL 0. You may press STOP to freeze the display when taking cursor measurements. For the $V_{in}$ and $V_o$ measurements, be sure the time scale is sufficient to show at least a few cycles, or the instrument may not properly calculate the PP and RMS values.

f. Use the oscilloscope’s cursors C1 and C2 to measure the phase shift $\Delta t$ and then calculate the phase angle $\phi$ between $V_{in}$ and $V_o$ at 500 Hz, at 10000 Hz, and at the cutoff frequency.

g. Using the data of Table 7.1.2, sketch a Bode plot of the filter’s output voltage.

<table>
<thead>
<tr>
<th>Freq. kHz</th>
<th>Actual $f$ kHz</th>
<th>$V_{in}$ RMS</th>
<th>$V_{in}$ PP</th>
<th>$V_o$ RMS</th>
<th>$V_o$ PP</th>
<th>$\Delta t$ $\mu$s</th>
<th>$\phi$ degrees</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.500</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.000</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.000</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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</tr>
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<td>4.000</td>
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<td>6.000</td>
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<td></td>
<td></td>
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<td>8.000</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>10.000</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cutoff</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
2. **Design a low-pass inductive filter**

   a. Design a low-pass filter using a 100 mH inductor and a single resistor R to obtain a cutoff frequency of 7500 Hz ± 2%. In your notebook, show your design procedure, including your design calculations.

   b. Measure the actual value of the 100 mH inductor and record its value.

   c. Using the value of R that you calculated, construct an RL circuit (similar to the RC circuit used in Part 1) designed to be a low-pass filter with the specified cutoff frequency.

   d. Connect the output of the filter to the Bode analyzer: Connect the high side of the filter to AI0+ and connect the low side of the filter (GROUND) to AI0–.

   e. Connect FG GEN to AI1+. Connect GROUND to AI1–.

   f. Be sure NI-ELVIS is turned on, then launch the NI-ELVIS’s Bode Analyzer.

   g. Set the Bode Analyzer to scan from 1 kHz to 20 kHz. Set the “steps per decade” to at least 100, although you may need to increase this value to as much as 500 to get accurate readings of cutoff frequency and phase angle.

   h. Verify that the Graph mapping is “Logarithmic” and that “Autoscale” is selected. Turn on “Cursors”.

   i. Turn on power to the NI-ELVIS prototyping board.

   j. Start the Bode Analyzer by clicking on “Run”.

   k. When the analyzer has finished, use the cursors to locate the cutoff frequency by finding the half-power point. (What identifies the half-power point?) You may need to interpolate, if you did not use enough “steps per decade”. Record the cutoff frequency and the phase angle.

   l. Compare the experimentally determined cutoff frequency to the desired value.

   m. If the cutoff frequency is off by more than 5%, change the size of R as necessary to obtain the specified cutoff frequency, and test again for the cutoff frequency. Record the final R, cutoff frequency, and phase angle $\phi$.

   n. Draw a circuit diagram of the final circuit that accomplishes the design objective. Explain any differences between the final value of R and your originally calculated value.

   o. Sketch the Magnitude and Phase plots from the Bode Analyzer, marking key reference points.
3. RLC Circuit #1

![Figure 7.3 Series RLC circuit.](image)

a. Set up the series RLC circuit shown in Figure 7.3, using the function generator to provide the sinusoidal input voltage.

b. Calculate the resonant frequency \( f_0 \) of the circuit. (See Laboratory 6.)

c. As in part 2, connect the Bode Analyzer to test the response of the filter by connecting leads from \( V_o \) to AI0+ and AI0−. The FGEN and GROUND should be connected to AI1+ and AI1−, as before.

d. Set the Bode Analyzer to scan from 1 kHz to 20 kHz. Set the “steps per decade” to at least 100 (suggest 300 to 500). Set the “peak amplitude” to 1.0 V (that is, 2.0 \( V_{pp} \)).

e. What type of filter is this if the output voltage is \( V_o \)?

f. What is the measured resonant frequency? What is the measured bandwidth? What is the phase angle at resonance? (To measure these values from the Bode Analyzer, you may need to increase the “steps per decade” or interpolate between points.)

g. In your notebook, sketch the Magnitude and Phase plots from the Bode Analyzer, marking key reference points.

4. RLC Circuit #2

a. Switch the positions of the resistor with inductor and capacitor to get the series RLC circuit shown in Figure 7.4.
b. Calculate the resonant frequency $f_0$ of the circuit. (See Laboratory 6.)

c. As in part 3, connect the Bode Analyzer to test the response of the filter by connecting leads from $V_o$ to AI0+ and AI0−. The FGEN and GROUND should be connected to AI1+ and AI1−.

d. Set the Bode Analyzer to scan from 1 kHz to 20 kHz. Set the “steps per decade” to at least 100 (suggest 300 to 500). Set the “peak amplitude” to 1.0 V (that is, 2.0 VPP).

e. What type of filter is this if the output voltage is $V_o$?

f. From the Bode plots, what is the measured resonant frequency $f_c$? What are the lower and upper half-power points, $f_1$ and $f_2$? What is the measured bandwidth? What is the phase angle $\phi$ at resonance? (To measure these values from the Bode Analyzer, you may need to increase the “steps per decade” or interpolate between points.)

g. In your notebook, sketch the Magnitude and Phase plots from the Bode Analyzer, marking key reference points.

**PROBING FURTHER**

1. In Procedure parts 1 and 2 you measured the phase angle $\phi$ between $V_{in}$ and $V_o$ at the cut-off frequency. What values did you observe? What value would you expect? Why?

2. In Procedure parts 3 and 4 you measured the resonant frequencies and the bandwidths using the Bode Analyzer. Construct a table comparing the theoretical values (see Laboratory 6) and the values for $f_0$, $f_1$, $f_2$, bandwidth measured in Procedure parts 3 and 4. Calculate the Q for these circuits.

**REPORT**: Your Laboratory Teaching Assistant will inform you when a report is due, and on which experiment you will report. Make sure that you have recorded all necessary information and data in your laboratory notebook to enable you to prepare a report on this experiment, if so directed, at some time in the future.
Laboratory 8

Transformers

INTRODUCTION: Transformers are magnetic circuit elements used in AC circuits for a variety of applications. Transformers are often used to convert voltages from high to low values, or from low to high values. They are often used to isolate one AC line from another, for safety or for equipment isolation. And they are often used to connect two components with mismatched impedances, such as a high-impedance stereo amplifier and a low-impedance speaker. This laboratory exercise will introduce transformer modeling concepts and explore the characteristics of a common power transformer.

EDUCATIONAL OBJECTIVES: To teach the properties of common low-power transformers.

EXPERIMENTAL OBJECTIVES: To characterize the behavior of common power transformers, including concepts such as turns-ratios, impedance matching, and loading.

PRE-LAB:
Reading:
- Review the sections of your ECE 262 textbook regarding transformer modeling.

Written:
- In your laboratory notebook prepare for each circuit configuration (1) the circuit diagram you will use and (2) a table in which you can record the values measured. See for example Tables 8.1, 8.2, and 8.3.

EQUIPMENT NEEDED:
- Transformer Board with mounted 120V, center-tapped 12.6V transformer
- NI-ELVIS II
  - Function generator
  - Oscilloscope, dual-beam
  - Digital Multimeter
- Resistance Decade Substitution box

BACKGROUND
In its simplest form, a transformer consists of two closely coupled wire coils that share a common magnetic field. If a voltage is applied to one of the coils (called the primary), then a voltage will be induced on the second coil (called the secondary). The actual voltage induced in the secondary coil is proportional to the ratio of the number of turns in the two coils. That ratio is called the turns ratio, for which we will use the symbol \( a \). If the number of turns in the primary is \( N_1 \) and the number of turns in the secondary is \( N_2 \), the turns ratio \( a \) is

\[
a = \frac{N_2}{N_1}
\]  
(8.1)
and so,

\[ a = \frac{N_2}{N_1} = \frac{V_2}{V_1} \quad \text{and} \quad V_2 = a \cdot V_1. \] (8.2)

The two coils (also called windings) may be wound around air (an air core) or a dielectric, but more often they are wound around a ferromagnetic material, such as steel (an iron core), in order to concentrate the magnetic field and improve the coupling between the coils.

Transformers are most often used for three principal reasons: to change AC voltages; for electrical isolation; and for impedance matching.

Often we can assume that a transformer is "ideal", meaning that it has no internal power loss and all magnetic flux lines of the primary coil also pass through the secondary coil. The ideal model often fits large power transformers, where the losses are very small compared to the power transferred. An ideal transformer delivers all of the applied power to the load. That is,

\[ \text{Power} = I_1V_1 = I_2V_2 \] (8.3)

Rearranging this equation reveals an important relationship:

\[ I_1 = I_2 \frac{V_2}{V_1} = I_2a \quad \text{or} \quad I_2 = \frac{V_2}{V_1} = a. \] (8.4)

So, for example, if the secondary voltage is 1/10 the primary voltage, then the primary current will be 1/10 the secondary current. The equation further indicates that if there is no load current \( (I_2=0) \), then the primary current is also zero.

**Figure 8.1: Model for an ideal transformer**

**Impedance-Matching**

Besides converting voltages, transformers are often used to match impedances between two electrical components, such as an audio amplifier (typically having 600Ω output impedance) and a speaker system (typically 4Ω or 8Ω impedance). The right transformer can make the load appear to have the same impedance as the source, and thereby maximize the power transferred to the load — and minimize the power lost in the source. The transformer that can perform such magical matching will have the following property:
where \( a \) is the turns ratio, \( Z_{\text{load}} \) is the load impedance, and \( Z_{\text{source}} \) is the source impedance. Equation 8.5 is a direct result of Equation 8.3 after substituting \( V_1 = I_1 Z_1 \) and \( V_2 = I_2 Z_2 \).

In essence, \( Z_{\text{source}} \) is what the load impedance \( Z_{\text{load}} \) will look like from the primary side of the transformer. Expressed in other terms, \( Z_{\text{source}} \) is the effective load impedance reflected to the primary side of the transformer. That is,

\[
Z_{\text{source}} = \frac{1}{a^2} Z_{\text{load}}
\]  

(8.6)

Usually, matching transformers are designed differently from power transformers, because matching transformers must meet other requirements. For example, audio transformers must respond uniformly over a rather wide band of frequencies, and do so with very low distortion.

**Non-Ideal Transformer Model**

Real transformers do, in fact, have losses and do draw current even with no load connected. Real transformers have coil resistance; they require magnetizing currents to maintain the magnetic coupling between the coils; and there are eddy currents and other issues. And so, even though the ideal model works well most of the time, when required, we use other models suitable to the circumstances to represent non-ideal transformers. One such model is illustrated in Figure 8.2.

![Figure 8.2: Circuit model for non-ideal transformer](image)

For steady-state sinusoidal operation, the following two equations show the voltage sums in this circuit model:

\[
\tilde{V}_s = (R_1 + j\omega L_1)\tilde{I}_1 - j\omega M\tilde{I}_2
\]

\[
0 = -j\omega M\tilde{I}_1 + (R_2 + j\omega L_2 + R_{\text{load}} + j\omega L_{\text{load}})\tilde{I}_2
\]

(8.7)

where \( R_1 \) and \( R_2 \) are the winding resistances and \( M \) is the mutual inductance of the two windings. These equations can be used to compute the inductances of the transformer coils, as well as the mutual inductance between them.

**PROCEDURE**

For these experiments, use a 120V / 12.6V center-tapped transformer mounted on Plexiglas.
Construct in your notebook tables like Table 8.1, Table 8.2, and Table 8.3 to record your results. Throughout the experiment, set the TRIGGER to EDGE and use the AUTOSCALE button to ensure the waveforms are approximately the same height. You may press STOP to freeze the display when taking cursor measurements. You are encouraged to use the scope’s automatically calculated values for the peak-to-peak voltages displayed on the scope.

Note: Some of the resistances and voltages are small. The cables connecting the NI-ELVIS to the transformer must have a good connection at each end or the measurements will be unstable and significantly wrong.

1. **Resistance:** Using the DMM’s ohmmeter function, measure the transformer’s primary and secondary resistances. Record the values in Table 8.1.

2. **Nominal Turns Ratio:** Based on the nominal rating of the transformer’s primary and secondary voltages, calculate the nominal turns ratio \( a_{nom} \) for this transformer. Record this result in Table 8.1.

<table>
<thead>
<tr>
<th>Table 8.1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Primary resistance ( R_P )</td>
</tr>
<tr>
<td>Secondary resistance ( R_S )</td>
</tr>
<tr>
<td>Nominal turns ratio, ( a_{nom} )</td>
</tr>
<tr>
<td>( X_{13} ) turns ratio, ( a_{meas} )</td>
</tr>
<tr>
<td>% difference</td>
</tr>
<tr>
<td>( X_{12} ) turns ratio</td>
</tr>
<tr>
<td>( X_{23} ) turns ratio</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table 8.2</th>
</tr>
</thead>
<tbody>
<tr>
<td>( V_{PP-primary} )</td>
</tr>
<tr>
<td>( V_{PP-X1-X3} )</td>
</tr>
<tr>
<td>( V_{PP-X1-X2} )</td>
</tr>
<tr>
<td>( V_{PP-X2-X3} )</td>
</tr>
</tbody>
</table>

3. **Measured Turns Ratio:** Connect the NI-ELVIS’s function generator to the primary winding of the transformer, as illustrated in Figure 8.3. Select a sine wave output and set the generator’s voltage to \( 3 \ V_{PP} \). Set the frequency at 60 Hz, since that is the operating frequency for which this transformer is designed.

![Figure 8.3: Testing Voltage Relationships](image)

- Connect the NI-ELVIS oscilloscope across the transformer as indicated in Figure 8.3: CHANNEL 0 across the transformer’s primary (\( H_1 - H_2 \)) with \( H_2 \) grounded; and CHANNEL 1 across the transformer’s secondary (\( X_1 - X_3 \)) with \( X_3 \) grounded.
Use the oscilloscope to measure the transformer voltages indicated below and record the results in Table 8.2.

- $V_{PP}$ across the primary ($H_1$ to $H_2$) (on CH0)
- $V_{PP}$ across the secondary ($X_1$ to $X_3$) (on CH1, as illustrated in Figure 8.3)
- $V_{PP}$ from the center tap (CT) to one side of the secondary ($X_1$ to $X_2$). (Move the CH1 ground lead to $X_2$ to make the measurement.)
- $V_{PP}$ from the center tap (CT) to the other side of the secondary ($X_2$ to $X_3$) (Move the CH1 leads to $X_2$ and $X_3$ to make the measurement.)

b. Do you see why the connection $X_2$ is referred to as the center tap?

c. Stop the function generator.

d. Calculate the turns ratio $a_{meas}$ between the primary winding and the secondary winding $X_1$–$X_3$. Record the result in Table 8.1.

Calculate the % Difference between $a_{nom}$ and $a_{meas}$ and record the result in Table 8.1:

$$\% \text{ Difference} = \frac{a_{meas} - a_{nom}}{a_{nom}} \times 100\%$$

Why are $a_{nom}$ and $a_{meas}$ different? Based on your measurements, estimate what the actual output voltage of the transformer ($X_1$–$X_3$) would be if the input voltage were 120V.

e. Calculate the turns ratio $a_{meas}$ between the primary winding and the secondary winding $X_1$–$X_2$. Record the result in Table 8.1.

f. Calculate the turns ratio $a_{meas}$ between the primary winding and the secondary winding $X_2$–$X_3$. Record the result in Table 8.1.

4. **Phase relationship between the primary and secondary voltages:**

a. Connect the NI-ELVIS oscilloscope across the transformer, as indicated in Figure 8.3: CHANNEL 0 across the transformer’s primary ($H_1$–$H_2$) with $H_2$ grounded; and CHANNEL 1 across the transformer’s secondary ($X_1$–$X_3$) with $X_3$ grounded.

b. Start the function generator (3 $V_{pp}$ at 60 Hz) and compare the phases of the two waveforms. Are the voltages in phase or out of phase? In your notebook, sketch the waveforms and record the phase shift, if any.

c. Reverse the scope leads on the secondary: that is, connect CHANNEL 1’s signal lead to $X_3$ and the ground lead to $X_1$. Describe the result and sketch the waveforms in your notebook. Record the phase shift, if any.

d. Connect CHANNEL 1 of the scope to observe the secondary-side voltage from $X_2$ (signal) to $X_1$ (ground). What is the difference from what you measured just previously in part c?

e. Stop the function generator.
5. **Secondary center-tap voltages**
   
a. Connect the NI-ELVIS oscilloscope across the transformer, as indicated in Figure 8.4: CHANNEL 0 across \(X_1-X_2\); and CHANNEL 1 across \(X_2-X_3\). In both cases, \(X_2\) must be connected to the scope’s ground lead.

![Figure 8.4: Testing Secondary Center Tap](image)

b. Start the function generator (3 \(V_{pp}\) at 60 Hz).

c. View the signals on each side of the center tap at the same time. Sketch the waveforms, showing the measured voltages. Describe your observations regarding voltages, phases, and anything else you observe about the signal.

d. Stop the function generator.

Notice that in Steps 3, 4, and 5 we could ground any terminal on the secondary. That is a demonstration of the third major use of the transformer: to isolate two circuits (especially the grounds) from one another. Attempting the same thing on the primary side (e.g., connecting the oscilloscope’s ground to \(H_1\)) could be disastrous because of the internal grounds on the function generator and the oscilloscope.

6. **The effect of loading**

a. Construct the circuit shown in Figure 8.5. Use a resistor substitution box for the resistor \(R\), and initially set the value to 1000 \(\Omega\).

![Figure 8.5](image)

b. Connect the NI-ELVIS oscilloscope across the transformer, as indicated in Figure 8.5: CHANNEL 0 across \(H_1-H_2\); and CHANNEL 1 across \(X_1-X_3\).
c. Set the function generator to 6 V<sub>pp</sub> at 60 Hz, and start the function generator and the oscilloscope.

d. For each of the resistor values in Table 8.3, record in turn the voltages <em>V</em><sub>1</sub> and <em>V</em><sub>2</sub>, and record the values in Table 8.3.

e. Calculate the apparent turns ratio for each of the loads and record the results in table 8.3.

f. Graph <em>V</em><sub>1</sub>, <em>V</em><sub>2</sub>, and <em>a</em> versus <em>R</em> to visualize the relationships. You may create the graph by hand or in a spreadsheet.

g. Compare these turns ratios with the <em>a</em><sub>nom</sub> and <em>a</em><sub>meas</sub> calculated in steps 2 and 3. How might these new values explain the difference between those earlier values?

<table>
<thead>
<tr>
<th>R (ohms)</th>
<th>&lt;em&gt;V&lt;/em&gt;&lt;sub&gt;1&lt;/sub&gt;</th>
<th>&lt;em&gt;V&lt;/em&gt;&lt;sub&gt;2&lt;/sub&gt;</th>
<th>&lt;em&gt;a&lt;/em&gt; = &lt;em&gt;V&lt;/em&gt;&lt;sub&gt;2&lt;/sub&gt;/&lt;em&gt;V&lt;/em&gt;&lt;sub&gt;1&lt;/sub&gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td>1000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>100</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>50</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>20</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**PROBING FURTHER**

1. Using the data from Procedure step 1, compute the resistance ratio between the secondary and primary windings by dividing the secondary resistance by the primary resistance. Record the result and compare it to the turns ratio. What factors might cause the resistance ratio to differ from the turns ratio?

2. Suppose that you had two of these 120 V transformers and you needed to convert 120 V to about 25 V. Show a sketch of how you could connect these two transformers to get this transformation. Make sure to show the winding connection labels for each transformer.

3. If an ideal transformer with 120 V across the primary and draws a current of 200 mA, what power is dissipated in the load? If the transformer’s secondary is 30 V, what is the secondary current? What is the turns ratio?

4. Using the specified Thévenin impedance of the function generator (50 Ω) and the specified impedance of an acoustic speaker (8 Ω), calculate the turns ratio required to match the speaker with the function generator.

**REPORT**: Your Laboratory Teaching Assistant will inform you when a report is due, and on which experiment you will report. Make sure that you have recorded all necessary information and data in your laboratory notebook to enable you to prepare a report on this experiment, if so directed, at some time in the future.
Laboratory 9

Two-Port Network Characterization

INTRODUCTION: You have now had experience with transfer functions and the concept of relating output quantities to input quantities. Many circuits can be treated as modules where an input is presented to two terminals (the "input port") and an output is taken from two terminals (the "output port"). Such circuits are termed two-port networks, and because they can be treated as a unit — a block — they allow great simplification of large-scale designs.

The last chapter in your textbook is an introduction to two-port circuits and to the methodology for developing the sets of parameters that may be used to relate the output variables of voltage and current, typically designated as $V_2$ and $I_2$, respectively, to the input variables of voltage and current, typically designated as $V_1$ and $I_1$, respectively. This approach is used to characterize a variety of components and circuit element combinations from filters, through transistors, to microwave circuits.

EDUCATIONAL OBJECTIVE: The objectives of this laboratory exercise are to gain familiarity with three alternative two-port network parameter sets, to learn to measure the parameter sets, and to demonstrate the operational definition of these parameters.

EXPERIMENTAL OBJECTIVE: To determine the impedance, admittance, and hybrid parameter sets for an unknown two-port network.

PRE-LAB:

Reading:
- Read the chapter in your textbook on two-port networks. Pay particular attention to the way in which the various parameters in each set (i.e., the $z$ parameters, the $y$ parameters, the $h$ parameters, etc.) are defined.
- Study the Background section below.

Written:
- Review the steps in the procedure below and plan how you will make your measurements to determine the required parameter values.
- Sketch the test circuits you will use to make the measurements.
- Construct suitable tables to record the measured values.

EQUIPMENT NEEDED:
- NI-ELVIS II
  - Function generator
  - Oscilloscope, dual-beam
  - Digital Multimeter
- Two-port Network Box
- Resistor decade substitution box (1)
BACKGROUND

A port consists of a pair of terminals; current enters through one of the terminals and the same current leaves through the other terminal. A resistor is a one-port network. In this lab we will study two-port networks with one input port and one output port. Such networks are often treated as “black boxes” or modules that may be plugged into a circuit to accomplish some task, such as filtering the signal or providing a controlled voltage. Engineers need a way to characterize the behavior of such a network and have developed several sets of parameters to do that. Each of these parameter sets relates the input (side 1) and output (side 2) voltages and currents.

In this lab we will use impedance, admittance, and hybrid parameter sets to characterize a simple circuit. Impedance and admittance parameters are commonly used to characterize filters, and are often useful in designing and characterizing impedance matching and power distribution networks. The term immittance is often applied to the use of either impedance or admittance parameters.

Be aware that because the voltages and currents are phasors with magnitude and phase angle, the parameters also have magnitude and phase angle. A simple RMS measurement will not suffice.

The impedance parameters (z parameters) relate the input and output voltages to the input and output currents by the following two equations:

\[ V_1 = z_{11} I_1 + z_{12} I_2 \]
\[ V_2 = z_{21} I_1 + z_{22} I_2 \]

or in matrix notation:

\[ \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \]

The z parameters have units of ohms and are most easily found by applying a set of open-circuit tests on the circuit. When we apply a voltage to the input with the output open-circuited, we can measure the input current and output voltage and find the first two z parameters as follows:

\[ z_{11} = \frac{V_1}{I_1} \bigg|_{I_2=0} ; \quad z_{21} = \frac{V_2}{I_1} \bigg|_{I_2=0} \]

(9.2)

We can determine the other two z parameters by applying a similar test to the output with the input open-circuited:

\[ z_{12} = \frac{V_1}{I_2} \bigg|_{I_1=0} ; \quad z_{22} = \frac{V_2}{I_2} \bigg|_{I_1=0} \]

(9.3)

Sometimes the impedance parameters do not exist because the voltages cannot be described by equation 9.1. Therefore, we need alternatives, such as the admittance parameters.
The admittance parameters (y parameters) relate the input and output currents to the input and output voltages by the following two equations:

\[ I_1 = y_{11}V_1 + y_{12}V_2 \]
\[ I_2 = y_{21}V_1 + y_{22}V_2 \]  

(9.4)

or in matrix notation:

\[
\begin{bmatrix}
I_1 \\
I_2
\end{bmatrix} =
\begin{bmatrix}
y_{11} & y_{12} \\
y_{21} & y_{22}
\end{bmatrix}
\begin{bmatrix}
V_1 \\
V_2
\end{bmatrix}
\]

The y parameters have units of siemens (or mhos) and are most easily found by applying a set of short-circuit tests on the circuit. When we apply a voltage to the inputs with the output short-circuited, we can measure the input current and output current to find the first two y parameters:

\[ y_{11} = \frac{I_1}{V_1} \bigg|_{V_2=0} ; \quad y_{21} = \frac{I_2}{V_1} \bigg|_{V_2=0} \]  

(9.5)

We can determine the other two y parameters by applying a similar test to the output with the input short-circuited:

\[ y_{12} = \frac{I_1}{V_2} \bigg|_{V_1=0} ; \quad y_{22} = \frac{I_2}{V_2} \bigg|_{V_1=0} \]  

(9.6)

There are occasions where neither the impedance nor the admittance parameters exist, so there is need for still another set of parameters.

The hybrid parameters (h parameters) are based on making \( V_1 \) and \( I_2 \) the dependent variables, and relating them to cross-variables \( V_2 \) and \( I_1 \).

. The h parameters satisfy the equations (9.7):

\[ V_1 = h_{11}I_1 + h_{12}V_2 \]
\[ I_2 = h_{21}I_1 + h_{22}V_2 \]  

(9.7)

or in matrix notation:

\[
\begin{bmatrix}
V_1 \\
I_2
\end{bmatrix} =
\begin{bmatrix}
h_{11} & h_{12} \\
h_{21} & h_{22}
\end{bmatrix}
\begin{bmatrix}
I_1 \\
V_2
\end{bmatrix}
\]

The h parameters are found using a mix of short- and open-circuit tests as follows:

Short-circuit tests

\[ h_{11} = \frac{V_1}{I_1} \bigg|_{V_2=0} ; \quad h_{21} = \frac{I_2}{I_1} \bigg|_{V_2=0} \]  

(9.8)
Open-circuit tests

\[ h_{12} = \frac{V_1}{V_2 | I_1 = 0}; \quad h_{22} = \frac{I_2}{V_2 | I_1 = 0} \quad (9.9) \]

**PROCEDURE**

**Warning:** Unless you are careful in your planning, in carrying out the experiment, and in recording your data, you are very likely to make errors. Here in this last laboratory you must figure out how to do most steps on your own.

**Hint:** Performing this lab is actually very straightforward, if you take it a step at a time. Think about what you have learned during this semester. If you need to measure the phase angle of a voltage or current, you will need to use the oscilloscope. When necessary, insert a sense resistor of about 10 ohms and measure the voltage across it, including its phase shift, then calculate the magnitude of the current flowing through the sense resistor. Be sure that you have accurately measured the resistor value, though, and not just relied on the nominal value. Use only one resistor decade box (it will make your life easier), and be careful where you put the resistor, so you don’t make a grounding error on the oscilloscope. Carefully record each of the measurements and calculations in tables that you have constructed for that purpose.

Think about what you are doing, sketch the circuits, work meticulously and methodically, and record your data carefully in tables.

1. **Z parameters:**
   - Use the function generator to apply a 1 kHz sine wave of about 10 \( V_{pp} \) to the input terminals of the Two-port Network Box given to you by your instructor.
   - Make the current and voltage measurements necessary to calculate the parameters \( z_{11} \) and \( z_{21} \) for the z-parameter set. Note that you must determine not only the magnitude but also the phase of the voltages and currents involved. How can you do that with the instruments at hand?
   - Now apply the 10 \( V_{pp} \), 1 kHz sine wave to the output terminals.
   - Make the current and voltage measurements necessary to calculate the parameters \( Z_{12} \) and \( Z_{22} \).
   - Describe your procedure in your laboratory notebook and record the data necessary to calculate the values for these parameters, including magnitude and phase.

2. Repeat Part 1 to determine the two-port parameter values for the \( y \)-parameter set.

3. Repeat Part 1, as needed, to determine the two-port parameter values for the \( h \)-parameter set.

**PROBING FURTHER**

1. Are the parameter values, which you have calculated for the various two-port network representations in this exercise valid for, say, 10 kHz? Why or why not?
2. The $z$ and $y$ parameters should be related by a matrix inversion. Invert the 2x2 $z$ parameter matrix and see if it matches with the $y$ parameters you calculated in lab.

**REPORT:** Your Laboratory Teaching Assistant will inform you when a report is due, and on which experiment you will report. Make sure that you have recorded all necessary information and data in your laboratory notebook to enable you to prepare a report on this experiment, if so directed, at some time in the future.
Laboratory 10

Final Exam

INTRODUCTION: This has been your second engineering circuits laboratory. Although you have been provided with a "cookbook" (this manual), hopefully you have not just blindly followed the instructions in order to get a good grade. If you have, you have cheated only yourself. The theory, which you have learned from your textbook and lectures, is a way of looking at reality and thinking about how to organize experience, i.e. dealing with real, physical things. It is only through combining practical experience with theory that you can begin to develop the necessary analytical skills to aid you in "taking things apart and putting them together in new ways" that is the essence of the practice of engineering.

EDUCATIONAL OBJECTIVE: This examination is designed to help you and your LTA determine how much you have developed your knowledge, skills, and self confidence.

PRE-LAB: Review your lab manual. Think carefully of the procedures that you have followed and what you have learned from them. How do you measure voltage, current, resistance, frequency, etc.? How accurate are your measurements? How are your measurements affected by the frequency response of your measuring instruments? Can you develop linear, semi-log, or log-log plots of your data to facilitate interpretation?

EQUIPMENT NEEDED:

Pencil
Calculator

PROCEDURE

The final exam will consist of a written exam and a practical exam. The practical exam involves performing a lab procedure to obtain a desired result. The written exam will cover the application of theory to understand circuit behavior, based upon your laboratory experiences in this course.

PROBING FURTHER

This has been the second in a series of four (4) laboratory courses. Regardless of the LTA who may teach your next course - what have you done to develop your expertise and skills? What will you do differently in the next course? What have you learned from your experience?
Appendix A

Safety

Electricity, when improperly used, is very dangerous to people and to equipment. This is especially true:

- in industrial environments, where large amounts of power are available, and high voltages are present;
- in environments where people are especially susceptible to electric shock, for example, where high-voltage systems must be maintained while in operation, or in hospitals where electronic equipment is used to test or control physiological functions; and
- in experimental or teaching laboratory environments, where inexperienced personnel may use electrical equipment in experimental or nonstandard configurations.

Engineers play a vital role in eliminating or alleviating the danger in all three environments. Where standard equipment is used in standard configurations, governmental agencies and insurance underwriters impose strict laws and regulations on the operation and use of electrical equipment, including switchgear, power lines, safety devices, etc. As a result, corporations and other organizations in turn impose strict rules and methods of operation on their employees and contractors. Engineers who are involved in using electrical equipment, in supervising others who use it, and in designing such systems, have a great responsibility to learn safety rules and practices, to observe them, and to see that a safe environment is maintained for those they supervise. In any working environment there is always pressure to “get the job done” and take short cuts. The engineer, as one who is capable of recognizing hazardous conditions, is in a responsible position both as an engineer and as a supervisor or manager and must maintain conditions to protect personnel and avoid damage to equipment.

Because of their non-standard activities, experimental laboratories are exempt from many of these rules and regulations. This puts more responsibility on the engineer in this environment to know and enforce the safest working procedures.

The learning necessary to develop the knowledge, the habit-forming experience to work safely around electrical equipment, and the ability to design safe electrical equipment begins with the first student laboratory experience and continues throughout life. This includes learning the types of electrical injuries and damage, the physiology of electrical injuries, ways to prevent such injuries, and steps to take when accidents occur.

Physiology of Electrical Injuries

Four main types of electrical injuries include the following:

- electric shock;
- electrical burns;
- injuries resulting from falls caused by electric shock;
- “sunburned” eyes from looking at a bright electric arc, such as that of an arc-welder.

While this is very painful and may cause loss of work time, it is usually temporary.
Other injuries may be indirectly caused by electrical accidents, such as burns from exploding oil-immersed switchgear or transformers.

Although electric shock is normally associated with contact with high-voltage alternating current (AC), under some circumstances death can occur from voltages substantially less than the nominal 120 volts AC found in residential systems. Electric shock is caused by an electric current passing through a part of the body. The human body normally has a high resistance to electric currents, so a high voltage is usually required to cause lethal currents. This resistance is almost all at the skin, however, and when the skin is wet its resistance is much lower. When a person is hot and sweaty or is standing in water, contact with 120 volts or less may well cause a fatal shock.

Electric shock is not a single phenomenon but is a disturbance of the nerves by an electric current. A current through a part of the body such as the arm or leg will cause pain and muscle contraction. If a victim receives an electric shock from grasping a live conductor, a current of greater than 15 to 30 milliamperes through the arm will cause muscle contractions so severe that the victim cannot let go. Similar currents through leg muscles may cause sudden contractions that make the victim jump or fall, possibly resulting in further injuries or death. Prolonged contact of more than a minute or so may cause chest muscles to contract, preventing breathing and resulting in suffocation or brain damage from lack of oxygen.

Death by electric shock is most often attributed to ventricular fibrillation, which is an uncontrolled twitching or beating of the heart that produces no pumping action and therefore no blood circulation. Unless corrective action is taken, death follows quickly from lack of oxygen to the brain. While the amount of current that will cause fibrillation depends on several variables, 0.5 to 5 amperes through the body will normally cause the very small current (approximately 1 mA) through the heart that is sufficient to cause fibrillation in most people. Larger currents than this through the heart cause contraction or clamping of the heart muscle, resulting in death unless corrective action is taken.

Electric burns may be caused by electric currents flowing in or near parts of the body. Such burns are similar to burns from ordinary heat sources, except that those caused by high-frequency currents are generally deeper and take longer to heal the other burns. Electrocution will often leave severe burns at the points where the current entered and left the body.

Source of Electric Shock

Since electric shock is caused by an electric current flowing through a part of the body, the problem may be prevented by not allowing the body to become part of any electric circuit. For this reason, one needs to understand how a person might inadvertently become part of an electric circuit, and the most likely way is through accidental grounding.

Electric circuits may be classified as either grounded or ungrounded. Grounded circuits are safer for most conditions, since the voltages at all points in the circuit are known and it is easier to protect against fault conditions. The disadvantage of a grounded circuit is that a person standing on a non-insulated floor can receive a shock by touching only one conductor.
Almost all electric power generation, transmission, and distribution systems are grounded to protect people and equipment against fault conditions caused by windstorms, lightning, etc. Residential, commercial, and industrial systems, such as lighting and heating, are always grounded for greater safety. Communication and computer systems, as well as general electronic equipment (e.g., DC power supplies, oscilloscopes, oscillators, and digital multimeters) are grounded for safety and to prevent or reduce electrical noise, crosstalk, and static.

Ungrounded circuits are used in systems where isolation from other systems is necessary, where low voltages and low power are used, and where obtaining a suitable ground connection is difficult or impractical. In an ungrounded circuit, contact with two points in the circuit that are at different potentials is required to produce a shock. The hazard is that, with no specific ground in such circuits, a hidden fault can cause some random point to be grounded, in which case, touching a supposedly safe conductor while standing on the ground could result in an electric shock.

Protecting People and Equipment in the Laboratory

Strict adherence to several common-sense rules, summarized below, can prevent electric shock to individuals and damage to laboratory equipment.

Protecting People

1. When building a circuit, connect the power source last, while the power is OFF.
2. Before changing a circuit, turn off or disconnect the power if possible.
3. Never work alone where the potential of electric shock exists.
4. When changing an energized connection, use only one hand. A common practice is to put one hand behind your back when touching a circuit that is possibly energized. Never touch two points in a circuit that are at different potentials.
5. Know and check that the circuit and connections are correct before applying power.
6. Avoid touching capacitors that may have a residual charge. Stored energy in a capacitor can cause severe shock even after a long period of time.
7. Insulate yourself from ground by standing on insulating mats where appropriate.

The above rules and the additional rules given below also serve to protect instruments and other circuits from damage.

Protecting Equipment

1. Set the scales of measurement instrument to the highest range before applying power.
2. When using an oscilloscope, do not leave a bright spot or trace on the screen for long periods of time. Doing so can burn the image into the screen.
3. Be sure instrument grounds are connected properly. Avoid ground loops and accidental grounding of “hot” leads.
4. Check polarity markings and connections of instruments carefully before connecting power.
5. Never connect an ammeter across a voltage source, but only in series with a load.
6. Do not exceed the voltage or current ratings of circuit elements or instruments. This particularly applies to wattmeters, since the current or voltage rating may be exceeded with the needle still reading on the scale.

7. Be sure any fuses and circuit breakers are of suitable value.

When connecting electrical elements to make up a network in the laboratory, it is easy to lose track of various points in the network and accidentally connect a wire to the wrong place. One procedure to help avoid this problem is to connect first the main series loop of the circuit, then go back and add the elements in parallel.

**Types of Equipment Damage**

Excessive currents and voltages can damage instruments and other circuit elements. A large over-current for a short time or a smaller over-current for a longer time will cause overheating, resulting in insulation scorching and equipment failure.

Blown fuses are the most common equipment failure mode in this laboratory. The principal causes for these failures include:

- incorrectly wired circuits;
- accidental shorts;
- switching resistance settings while power is applied to the circuit;
- changing the circuit while power is applied;
- using the wrong scale on a meter;
- connecting an ammeter across a voltage source;
- using a low-power resistor box (limit ½ amp) when high power is required;
- turning on an autotransformer at too high a setting.

All of these causes are the result of carelessness by the experimenter.

Some type of insulating material, such as paper, cloth, plastic, or ceramic, separates conductors that are at different potentials in electrical devices. The voltage difference that this material can withstand is determined by design (type, thickness, moisture content, temperature, etc.). Exceeding the voltage rating of a device by an appreciable amount can cause arcing or corona, resulting in insulation breakdown, and failure.

Some electrical devices can also be damaged mechanically by excessive currents. An example is the D’Arsonval meter, the indicator in most analog metering instruments. A large pulse of over-current will provide mechanical torque that can cause the needle to wrap around the pin at the top of the scale, thereby causing permanent damage even though the current may not have been on long enough to cause failure due to overheating.

**After-Accident Action**

Since accidents do happen despite efforts to prevent them, preparation for an accident can save valuable time and lives. This preparation should include immediate availability of first aid material suitable for minor injuries or for injuries that are likely because of the nature of the work.
Knowledge of how to obtain trained assistance such as Emergency Medical Service (EMS) should be readily available for everyone.

Treating victims for electrical shock includes four basic steps, shown below, that should be taken immediately. Step two requires qualification in CPR and step three requires knowledge of mouth-to-mouth resuscitation. Everyone who works around voltages that can cause dangerous electrical shock should take advantage of the many opportunities available to become qualified in CPR and artificial respiration.

**Immediate Steps After Electric Shock**

1. Shut off all power and remove victim from the electric circuit. If the power cannot be shut off immediately, use an insulator of some sort, such as a wooden pole, to remove the victim from the circuit. Attempts to pull the victim from the circuit with your hands will almost always result in your joining the victim in electric shock.

2. If you are qualified in CPR, check for ventricular fibrillation or cardiac arrest. If either is detected, external cardiac massage should be started at once. Whether you are qualified in CPR or not, notify the EMS and the ECE Department at once, using the telephone numbers listed below.

3. Check for respiratory failure and take appropriate action. This may have resulted from physical paralysis of respiratory muscles or from a head injury. Sometimes many hours pass before normal respiration returns. Artificial respiration should be continued until trained EMS assistance arrives.

4. Check for and treat other injuries such as fractures from a fall or burns from the current’s entry and exit sites.

Investigations are always made after accidents. As an engineer, you will be involved as a part of the investigating team or in providing information to an investigator. Information obtained and notes written immediately after the emergency will aid the investigation and assist in preventing future accidents of a similar nature.

**EMERGENCY NUMBERS**

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<td>All Emergencies</td>
<td>911</td>
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<td>Clemson Police</td>
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**References**

Appendix B

Instruments for Electrical Measurements

Electrical engineers measure and use a wide variety of electrical circuit variables, such as voltage, current, frequency, power, and energy, as well as electrical circuit parameters, such as resistance, capacitance, and inductance. Many instruments can be used to make such measurements, but the proper use of the instruments and interpretation of the measurements depend on a fundamental understanding of how the instruments work, their capabilities, and their limitations.

This appendix provides a brief overview of the fundamentals of the electrical equipment and instruments that you will use in this and other laboratory courses. As you encounter more and varied types of electrical equipment and instruments in this and subsequent courses, you will find several books, in addition to your textbook, useful in developing your understanding and measurement skills. In addition, many commercial instrument manufacturers publish handbooks and application notes that provide more information on specific measurement techniques.

Measurement of Current and Voltage

The basic electrical circuit variables of current and voltage are measured with ammeters (for current) and voltmeters (for voltage). These instruments may use either analog (continuous) or digital (numerical) indicators (“readouts”) to report the measurement results.

Basic facts you should keep in mind include the following:

- An ammeter is a low resistance instrument to measure current. It is inserted in series with the circuit branch of interest. If connected in parallel with a component, it will likely short out the component and blow the fuse or burn out the circuit or the instrument.

- A voltmeter is a high resistance instrument for measuring voltage. It is connected in parallel with the component(s) of interest. Voltage is also called “electromotive force” or EMF.

Analog Meter Instruments

Analog meter instruments were developed early in the history of electrical science and technology. Most are based on the d’Arsonval galvanometer movement. A brief description of this meter movement and its use in ammeters and voltmeters is given in the textbook Electric Circuits by J. W. Nilsson and S.A. Riedel [B-1], on pages 78 through 80 in the Seventh Edition. More information is available in Reference B-2.

In the d’Arsonval galvanometer, current through a coil of fine wire develops a magnetic field that opposes the field of a permanent magnet, and so rotates a needle across a scale that is marked off in units of the measured variable. This type of movement is used extensively in DC analog instruments. For AC measurement, however, the d’Arsonval movement is not sufficient by itself, but must be adapted in some way:
it must be used in conjunction with a rectifier diode that converts the AC into a waveform with a DC level to which the meter can respond, or

- it must use an electromagnet instead of the permanent magnet in the standard d’Arsonval movement, or
- it must use iron vanes [B-3].

Figure B.1 (a) shows an equivalent circuit representation of a dual-coil electrodynamometer wattmeter often used to measure 60 Hz AC power. It employs the electromagnet form of the galvanometer. Because the meter deflection is proportional to the product of the current through the current-sensing coil (in series with the load) and the current through the voltage sensing coil (across the load), the response is proportional to the product of the load current and the load voltage drop. The size of the resistance in series with the voltage coil determines the voltage range of the instrument.

Figure B.1 (b) shows the equivalent circuit representation of the analog voltmeter. The size of the resistor determines the voltmeter range.

Figure B.1 (c) shows the equivalent circuit of the analog ammeter. The size of the shunt (parallel) resistor determines the range of the ammeter.

Note that analog instruments often have non-linear scales.

![Equivalent circuit diagrams for (a) analog wattmeter, (b) analog voltmeter, (c) analog ammeter.](image)

**Digital Multimeter**

A multimeter is an electronic device that measures a multitude of electrical values, usually including at least AC and DC voltage and current, as well as resistance. Analog multimeters have an analog display and digital multimeter (DMM) have a digital display. The DMM in this laboratory is part of the NI-ELVIS II and is used to measure voltage (DC and AC), current (DC and AC), resistance, capacitance, inductance. Additionally it may be used for diode tests and audible continuity tests. For capacitance and inductance measurements you must make connections to
the DMM/Impedance Analyzer on the prototyping board. For all other measurements make con-
nections to the DMM banana jacks on the workstation.

The primary DMM instrument on NI ELVIS II Series is isolated and its terminals are the three
banana jacks on the side of the workstation. For DC Voltage, AC Voltage, Resistance, Diode,
and Continuity Test modes, use the VΩ and COM connectors. For DC Current and AC Current
modes, use the A and COM connectors. For easy access to circuits on the prototyping board, you
can use banana-to-banana cables to wrap the signals from the user-configurable banana jacks to
the DMM connectors on the workstation. For capacitance and inductance tests, the positive end
of the device under test is connected to the pin “DUT+” on the prototyping board, and the nega-
tive end of the device under test should be connected to the pin “DUT−” on the prototyping
board.

For details about the DMM and other NI-ELVIS instruments, refer to the ELVIS Operation Man-
ual at http://www.clemson.edu/ces/departments/ece/resources/lab_manuals.html.

![Figure B.11](image)

(a) Equivalent circuit for voltage and current measurement with the DMM.
(b) Equivalent circuit for resistance measurement with the DMM.

**Dual-Beam Oscilloscope**

The oscilloscope is a tool to allow engineers to look at the shape of an electrical voltage versus
time or versus a second signal. Until relatively recently, oscilloscopes used a cathode ray tube
(CRT) to draw the waveforms onto a screen, just like an image on a television. Televisions and
computer monitors also used cathode ray tubes until the advent of the new flat screens. Most en-
gineers refer to the instrument as a “scope”.

The Dual-Beam Oscilloscope has two vertical (“y”) input channels and one horizontal (“x”)
channel. The horizontal channel can be connected either to an external AC voltage signal or to an
internal time-base generator (x = time). You should become familiar with the scale options on
the y input channels, the x input channel, and the time base, since you will be using these to ob-
tain values for voltage and time. The y inputs can be either direct (1X) or through a 10X probe.
Figure B.12 (a) shows the equivalent input circuit for a direct input and Figure B.12 (b) shows an
equivalent circuit for the 10X probe input. Note that in both configurations one side of the input
is grounded, which means that care must be used in connecting the ground clip of the probe or
connector used to assure that these are not connected to a “hot” ($|V| > 0$) part of the circuit. (See the section “Oscilloscope Grounding Errors”.) The calibrated time base is useful when measuring the phase difference between two waveforms (on the $y_1$ and $y_2$ inputs) by carefully lining up the zero levels for both $y$ inputs and then using in the ac-couple mode to observe the time difference between zero crossings of the two waveforms.

![Diagram](image)

Figure B.12: (a) Equivalent circuit for oscilloscope input with 1X probe (direct input). (b) Equivalent circuit for oscilloscope input with 10X probe.

**Digital Storage Oscilloscope**

The digital storage oscilloscope (DSO) is now the preferred type of oscilloscope for most industrial applications, although analog oscilloscopes are still widely used. The DSO uses digital memory to store data as long as required without degradation. The digital storage allows use of an enormous array of sophisticated digital signal processing tools for the analysis of complex waveforms in today’s circuitry.

The digital storage oscilloscopes of the NI-ELVIS are dual-beam oscilloscopes with two vertical inputs, as described above. The vertical input on the oscilloscope, instead of driving a vertical amplifier, is digitized by an analog-to-digital (A-to-D) converter to create a data set that is stored in the memory of a microprocessor. The data set is processed and then sent to the display. The data set can be written to a flash drive or sent over a LAN or a WAN for processing or archiving. The screen image can be directly recorded on paper by means of an attached printer or plotter, without the need for an oscilloscope camera. The scope’s own signal analysis software can extract many useful time-domain features (e.g. rise time, pulse width, amplitude), frequency spectra, histograms and statistics, persistence maps, and a large number of parameters meaningful to engineers in specialized fields such as telecommunications, disk drive analysis, and power electronics.

Digital oscilloscopes are limited principally by the performance of the analog input circuitry and the sampling frequency. In general, the sampling frequency should be at least the Nyquist rate – double the frequency of the highest-frequency component of the observed signal – to avoid aliasing.
Appendix C

Operating Instructions for a Typical Oscilloscope

The oscilloscope is an instrument for the analysis of electrical circuits by observation of voltage and current waves. It may be used to study frequency, phase angle, and time, and to compare the relation between two variables directly on the display screen. Perhaps the greatest advantage of the oscilloscope is its ability to display the periodic waveforms being studied.

Until recently, oscilloscopes used a cathode-ray tube to display the signals of interest. A cathode-ray tube (CRT) contains an electron gun that directs a high-velocity beam of electrons onto a fluorescent screen. The beam is controlled by a pair of horizontal and a pair of vertical deflecting plates. When the voltage on the deflection plates is equal to zero, the beam produces a spot of light in the center of the screen. Any potential applied to the plates creates an electric field that deflects the electron beam proportionally to the applied voltage.

A simple block diagram of the cathode-ray oscilloscope is shown below.

The basic components of the traditional oscilloscope are cathode-ray tube, amplifiers, sweep or timing oscillator, and power supply. A voltage to be observed is applied to the vertical deflection plates. This signal may be amplified by the vertical amplifier in order to obtain a satisfactory vertical deflection. Meanwhile, a sweep oscillator moves the beam horizontally at a uniform rate. The simultaneous horizontal and vertical sweep of the beam across the CRT screen displays the waveform of the voltage applied to the vertical plates. The sweep oscillator blanks the CRT electron gun during its reverse sweep across the screen to switch off the electron beam.
If several voltage waveforms are to be studied and must maintain their relative phase positions, the sweep generator must be synchronized to the same voltage during the entire test. In this case, one voltage is applied to the oscilloscope as an external trigger.

An independent voltage may be applied to the horizontal input in place of the sweep oscillator voltage. In this case, two independent input voltages are displayed against one another. If the horizontal frequency is a submultiple of the vertical frequency, the trace will form a stationary pattern on the screen.

Today the traditional CRT oscilloscopes are rapidly being replaced with digital oscilloscopes that have flat-panel liquid crystal displays (LCDs), some with color displays. Instead of directly applying the incoming voltages to deflection plates, the digital oscilloscopes capture the voltage information and store it in computer memory as digital signals, which are then analyzed and displayed on the LCD. While the new digital scopes handle the incoming signal differently from the CRT-based scopes, the basic purpose and many of the operational controls remain the same. Therefore, the discussion that follows applies, for the most part, to both types of oscilloscopes.

**Avoiding Grounding Errors with Oscilloscope**

The shield or ground wire on the oscilloscope’s signal input is connected to the oscilloscope’s chassis ground, and therefore to the ground lead on the instrument’s electrical power connection. This fact creates the possibility of grounding errors when making connections to circuits, especially when trying to measure voltages across ungrounded components.

The problem is illustrated in the following diagram. Suppose one wants to measure the voltage across resistor R\(_1\). Because the circuit and the oscilloscope both have the same ground, connecting the oscilloscope’s input leads directly across R\(_1\), as indicated, will create a short across R\(_2\). Besides giving an incorrect reading, such a connection might damage either the oscilloscope or the circuit under test.

There are two solutions to the problem.

The first solution is simply to swap (interchange) the positions of R\(_1\) and R\(_2\), so that one end of R\(_1\) is at ground. Then there is no problem directly connecting the oscilloscope across R\(_1\), because both grounds are connected together.
The second solution is sometimes called the *two-channel difference method*, and is illustrated in the next figure. It requires an oscilloscope with two input channels and with the ability to subtract the signal on one channel from the other channel. As indicated in the illustration, the ground leads of both oscilloscope channels are connected to ground, so there is no grounding error. The signal leads are placed on both sides of R₁. Channel 1 will show the output voltage from the signal generator. Channel 2 will show the voltage across R₂. If the oscilloscope’s difference function is engaged so that the signal of Channel 2 is subtracted from Channel 1, the scope will display the voltage across R₁.

Unfortunately, the NI-ELVIS II oscilloscope does not have a difference function, and so it requires us to use the first solution; namely, to swap the positions of R₁ and R₂, and then measure the voltage across the now-grounded R₁.

**Preliminary Adjustment To Obtain a Trace**

To operate the oscilloscope, first turn on the power switch and allow the unit to warm up and initialize. Place the **HORIZONTAL CONTROL** in the sweep position. Adjust the **INTENSITY** and **FOCUS** controls on a CRT scope until the desired brightness and line width are obtained. Do not leave the spot stationary on the screen, as doing so may leave a permanent burn on the screen coating.

Once a trace is obtained, some oscilloscopes must be checked for calibration and balance. For the HP 120B CRT oscilloscope (a typical example), the procedure is as follows: Place the **TRIG-**
GER LEVEL on AUTO and the TRIGGER SOURCE to INT. Turn the horizontal and vertical VERNIERS to CAL, the VERTICAL SENSITIVITY to CAL, and the HORIZONTAL DISPLAY to 1 msec/cm. With the controls in these positions, an internal calibrating signal is produced on the screen. Turn the CAL adjustment until the upper and lower peaks of the square wave are 6 cm apart.

Digital oscilloscopes often have automated setup and calibration upon power-up.

**Waveform Observation**

After the initial adjustments are made, the oscilloscope is ready for operation. To observe the waveform of any periodic signal, apply the signal to the vertical input terminals. Use DC coupling if the input is DC or very low in frequency or if you want to capture any DC offset. Now the signal from an oscillator, signal generator, or some component of an electrical circuit may be observed on the screen. The best resolution of the waveform is obtained when the time scale is adjusted so one or two cycles appear on the screen and when the vertical scale is adjusted so the amplitude occupies most of the graticule. If the waveform will not stabilize, adjust the SYNC or TRIGGER just enough to cause the pattern to stop. Whenever possible, connect the oscilloscope ground to the common ground of the circuit. Exercise great care when making measurements with both terminals above ground potential, as there may be a difference in potential between two instrument cases, causing ground loop currents, faulty readings, and damaged equipment.

**Voltage Measurement (AC & DC)**

The oscilloscope has advantages as a voltmeter: a very high input impedance compared to an analog voltmeter; the ability to measure voltages over a very wide frequency range; and the ability to indicate magnitude regardless of waveform. Also, scopes measure peak-to-peak values of AC voltages, whereas standard AC voltmeters measure rms values of sine wave voltages. However, the oscilloscope only has an accuracy of 2% to 5%, while the AC voltmeter’s accuracy will be from 0.25% to 2%.

To use the oscilloscope as an AC voltmeter, apply the signal to the vertical input terminals, and adjust the calibrated VERTICAL SENSITIVITY (or VOLTS/DIV) so the amplitude is of suitable magnitude on the graticule. The peak-to-peak value is then the distance indicated multiplied by the vertical calibration. For example, assume that a sine wave generator is set to 1000 Hz and adjusted for maximum output voltage. A peak-to-peak value of 60V is observed on the oscilloscope. The output of the generator at 1000 Hz, therefore, is approximately 60V peak-to-peak, and 21.2 Vrms.

Note: \[ V_{rms} = \frac{1}{2\sqrt{2}} V_{p-p} \]
For DC measurements, apply the voltage to the vertical input terminals, again suitably adjusting the VERTICAL SENSITIVITY. A straight line is produced with the horizontal sweep functioning. With no horizontal voltage applied, a spot will appear on the screen. In measuring DC voltages, it is necessary to remember where the trace was with 0V applied to the vertical input.

**Frequency Measurement**

The frequency of an unknown signal may be calculated from the oscilloscope very easily. The period of the waveform is the product of the distance along the x-axis covered by one cycle and the horizontal sweep setting. As an example, a sine-wave generator is set to 1000 Hz with the voltage applied to the oscilloscope vertical. One cycle covers 9.95 cm, with a sweep speed of 100 µsec/cm. The period is \( T = (9.95) \times (100 \times 10^{-6}) \text{sec} \). The measured frequency is \( f = 1/T = 1005 \text{ Hz} \).

**Phase-Angle Measurement**

The difference in phase angle between two waveforms may be measured directly on the oscilloscope with little difficulty.

For an oscilloscope with only one vertical input:

One wave is chosen as the reverence and applied to the vertical input terminals. This same wave is applied to the external trigger input of the scope. Next, a convenient point on the wave is selected as a time reference, such as where the wave is zero and about to swing positive. Then, this waveform is removed from the vertical input and a second waveform is applied. The voltage of this wave at the time reference is observed. The ratio of voltage at the time reference to the maximum voltage is equal to the sine of the phase difference between the two waves. This relation is shown below.
For an oscilloscope with two vertical inputs:

Connect the reference voltage to Channel 1 of the oscilloscope and connect the second voltage to Channel 2. Adjust the amplitudes so the overlapping signals look something like the figure below, where the solid curve is Channel 1 (the reference) and the dashed curve is Channel 2. (In this figure, the dashed curve is lagging the solid curve; if the dashed curve were shifted to the left so it “started” before the solid curve, then the dashed curve would be leading the solid curve.) The phase shift $\phi$ in degrees can be calculated by the following formula:

$$\phi = \Delta t \cdot f \cdot 360^\circ$$

where $\phi$ is the phase shift, $f$ is the frequency, and $\Delta t$ is the time difference between the two waveforms. Many new digital scopes have cursors that allow directly marking, calculating, and displaying the time difference and perhaps even the phase shift.

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**Phase and Frequency Measurement by Lissajous Patterns**

The oscilloscope may be used to compare simultaneously two separate waveforms. When two voltages of the same frequency are impressed on the oscilloscope, one on the vertical and one on the horizontal plates, a straight line results on the screen if the voltages are in phase or $180^\circ$ out of phase. An ellipse is obtained for other phase angles. The pattern which results when two sine waves are applied to the oscilloscope in this way are called Lissajous patterns. They can take many forms, depending on the frequencies involved.

Phase shift (the phase angle between two waves of equal frequency) may be calculated from the pattern on the CRT screen. If the voltage applied to the vertical plates leads the voltage applied to the horizontal plates by an angle $\phi$, a pattern such as the one at right results. The phase differ-
ence can be found by taking the ratio of the horizontal intercept to the maximum horizontal deflection. However, twice the values provide more accuracy. The phase shift is now calculated $\phi = \arcsin(a/b)$.

Lissajous patterns may be used to compare the frequencies of voltages from two separate signal sources. If these are close to the same frequency, the pattern will slowly change back and forth from an ellipse to a straight line each half cycle of difference in frequency. For any stationary pattern, the ratio between the two frequencies may be determined by counting the number of pattern tangencies along the horizontal and vertical axes. The ratio of vertical to horizontal frequency ($Y:X$) equals the number of horizontal tangencies divided by the number of vertical tangencies. Lissajous patterns for three frequency ratios follow.

Be warned that many presentations of Lissajous patterns report $(X$ frequency) : $(Y$ frequency), which would be the inverse of the ratios shown here.

To measure frequency by Lissajous patterns, the unknown signal is applied to the vertical input terminals. Then, a variable signal of known frequency is applied to the horizontal input. The HORIZONTAL DISPLAY must be on external sensitivity. The known signal is varied until a stable pattern is obtained, and the unknown frequency is calculated. The simplest way to do this, however, is just to obtain a pattern in a $1:1$ ratio. Then the unknown can be read directly off the dial of the known source.
B2-SPICE is a powerful simulation program, but sometimes it is a little difficult to figure out how to get it to do what you want it to. A case in point is reporting voltages and currents in a simple AC circuit. To get you started, this appendix describes how to set up a simple series circuit and run B2-SPICE to report the current and voltages in that circuit.

Drawing circuits in B2-SPICE is straightforward, and becomes easy with a little practice. Start by opening B2-SPICE and creating the circuit shown below.

For this example, the power supply, $V_1$, is a function generator providing a $10 \, \text{V}_{\text{pp}}$ sinusoidal output. $R_1$ is a $1.0 \, \text{k}\Omega$ resistor, and $C_1$ is a $0.1 \, \mu\text{F}$ capacitor. It is asked that we calculate the current, the voltages, and the phase angles for frequencies from $500 \, \text{Hz}$ to $5000 \, \text{Hz}$.

Once the circuit is drawn, insert ammeters in any branches of interest and apply voltmeters across the devices of interest. For now, don't worry about the values indicated in the meters.

If necessary, change the values for the resistor and capacitor by double-clicking on the component and making the necessary changes to the name or the value. You must double-click on the component, not on the text describing it.
Set the power supply voltage by double-clicking on the round icon representing the voltage source. Change the part title to “V1” if necessary. Set the [DC VALUE] to 0.

Select the [SMALL SIGNAL AC AND DISTORTION] tab. Under [AC PROPERTIES FOR AC SMALL SIGNAL ANALYSIS ONLY], enter magnitude 10 V and phase 0. Check [USE]. Click [OK].

Set the range of frequencies to be simulated as follows: Click in the circuit window to be sure it is selected. Click on the menu item [SIMULATION] and then choose [SET UP SIMULATIONS]. Click on the button [AC (AC FREQUENCY SWEEP)]. Make the following entries:

- **Start Value**: 500 (the frequency at which you want to begin)
- **Stop Value**: 5000 (the frequency at which you want to stop)
- **Number of Each Steps per Interval**: 10 (the number of frequencies at which you want calculations, beginning at the Start Value and ending at the Stop Value.)

- **Stepping interval**: Linear
- **Check** (i.e., select) [SHOW AMPTITUDE PLOTS] and [SHOW PHASE PLOTS].
- **Select** [USE DEGREES FOR PLOTS] and [USE MAGNITUDE FOR PLOTS].
- **Check** [DISPLAY GRAPH].
- **Check** [DISPLAY TABLE].
- **Click** [OK].
- **Be sure** that [AC (AC FREQUENCY SWEEP)] is enabled.

Then click the button marked [RUN Now...].

Immediately a table of voltages, currents, and phase angles will be displayed from 500 Hz to 5000 Hz. The phase angles may be a bit misleading for our purposes, because they force the supply voltage to have a phase angle of zero (0), while our usual procedure is to require the voltage and current of the resistor to point along the real (X) axis in our phasor diagrams. Nonetheless, the simulation data show the relative phases of the voltages, and as expected, the current through the capacitor leads the voltage across the capacitor by 90°.

B2-SPICE will create a graph of the data, if you checked DISPLAY GRAPH, but the voltage and current scales are usually so small that they require magnification. One can overcome the display problem in the simulation by multiplying the simulation’s supply voltage by some constant so that it is near 100V. Be sure to correct for the multiplicative constant if you try to extract magnitude values from the graph later.

To graph the table of data in Excel, EXPORT the table to a text file (through the File menu), and then read that text file into Excel. Once the data is in Excel, use the search and replace function to change units multipliers:

- k to E3
- m to E-3
- u to E-6
- n to E-9
meg to E6

Excel recognizes the E# as equivalent to \(10^#\), where # is an appropriate number. Be sure you do not put any blank spaces in the substitution, or Excel will think it is all text.

After the substitutions are completed, use the standard Excel commands to graph the columns of interest. In this case you would probably choose an XY scatterplot \(V_C\) and \(V_R\) magnitudes versus frequency.

**Warning:** NEVER copy the simulated values and pretend that they are your calculations or your measurements. That is cheating. Whenever you report values from a simulation, be sure to state that they are from the simulation and what program you used to do that simulation. To do otherwise is to be dishonest.

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**Oscilloscope-like Display**

To have B2-SPICE show an oscilloscope-like display of the circuit’s voltages and currents, requires only a few changes.

Set up the circuit as before, and add meters for anything you want measured.

Double-click on the voltage source. In the [TRANSIENT PROPERTIES] section select [SINUSOIDAL]. To the right, click on the [PEAK AMPLITUDE OF VOLTAGE] and in the [VALUE] box, enter “10” and click [ACCEPT NEW VALUE]. Then click on [FREQUENCY] and enter “500” in the [VALUE] box and click [ACCEPT NEW VALUE]. Click [OK].

**Setting the simulation parameters**

Click on the circuit drawing to be sure it is selected. From the menu click on [Simulation] and choose [Set up Simulations]. Enable [Transient and Fourier Analyses] and then click that button. Input a reasonable STOP TIME that will show a few cycles. In this case the frequency is 500 Hz, and so a stop time of 0.01 seconds will display five cycles. Enable [Display Graph]. Click [OK].

Click the button marked [RUN NOW...]. Immediately B2-SPICE will generate a graph of whatever voltages and currents you had meters for on the circuit layout. If [AC (AC FREQUENCY SWEEP)] was also still enabled, then B2-SPICE would also generate that graph and table of data. The best of both worlds.