A Bi-Objective Covering Location Model for EMS Systems

Sunarin Chanta[a], Maria Mayorga[a], Laura McLay[b], and Margaret Wiecek[c]

[a]Department of Industrial Engineering
[c]Department of Mathematical Sciences
Clemson University, Clemson, SC 29634, USA

[b]Department of Statistical Sciences and Operations Research
Virginia Commonwealth University, Richmond, VA 23284, USA

Abstract

In this paper, we seek to optimally locate EMS vehicles at existing stations. To take into account issues of fairness, we propose a bi-objective model for locating EMS vehicles. The problem is formulated as an integer programming model. The objectives are to maximize the number of covered demand zones and to minimize the maximum distance between each uncovered demand zone and its closest opened station. The ε-constraint method is applied for solving this problem using real-world data collected from Hanover County, VA. The results are compared and discussed to provide a set of alternatives for decision makers.

Keywords

Covering location model, EMS, multi-objective optimization, ε-constraint approach

1. Introduction

Emergency medical service (EMS) systems are specially organized systems that provide emergency medical service within a particular area. The Hanover Fire/EMS department, which is located in Hanover, VA, responds to 911 calls 24 hours a day. Hanover department serves a county of 474 square miles, with a population near 100,000. The emergency medical services are varied depending on a call such as providing an emergency medical technician, a paramedic, or transportation. In this paper, we focus on locating vehicles in order to respond to the requested calls in time. We assume that a demand zone is "covered" when there exists an EMS vehicle that is able to respond within a fixed amount of time, and there is only one type of vehicle (i.e., ambulances). Generally, we would like to maximize the number of demand zones that can be covered. However, because of limited resources, there exist some demand zones that go uncovered. Based on preliminary results, we find that with a single objective that maximizes the number of covered demand zones, these uncovered demand zones tend to be located at the edges of the region. This results in an inequitable use of resources. To take into account issues of fairness, we propose a bi-objective model for locating EMS vehicles. The objectives are to maximize the number of covered demands and to minimize the maximum distance between each uncovered demand zone and its closest opened station. The problem is formulated as an integer programming model. The ε-constraint approach is selected to solve this problem.

2. The ε-Constraint Method

Several approaches exist for solving multi-objective problems such as weighted-sum, ε-constraint, and weighted-norm. The weighted-sum method, while popular, is not suitable for our problem because our solution space is integer and it is known that when the solution space is not convex, the weighted-sum method cannot find all solutions. However, both ε-constraint and weighted-norm approaches can find all solutions of integer problems. In this paper, we selected the ε-constraint method which was introduced by Haimes et al. [1], and an extensive discussion can be found in Chankong and Haimes [2]. The idea of this technique is to minimize or maximize one objective while the other objectives are bounded at acceptable fixed values. If we have a multi-objective problem, the formulation of the ε-constraint method is given as follows, refer to Ehrogtt [3].

The Multi-Objective Problem:

$$\text{Minimize} \quad [f_1(x), f_2(x), \ldots, f_p(x)]$$

Subject to $x \in X$.

The ε-Constraint Problem:

Minimize $f_j(x)$

Subject to $f_k(x) \le \varepsilon_k \;\; k = 1,\ldots,p \;\; k \ne j$

$x \in X$

where $\varepsilon \in R^P$.

We briefly discuss the concept of optimality as it relates to multi-objective problems. A feasible solution $\hat{x} \in X$ is called "efficient" or "Pareto optimal", if there is no other $x \in X$ such that $f(x) \le f(\hat{x})$. If $\hat{x}$ is efficient, the point $\hat{y} = f(\hat{x})$ is called non-dominated. A feasible solution $\hat{x} \in X$ is called weakly efficient or weakly Pareto optimal, if there is no other $x \in X$ such that $f(x) < f(\hat{x})$. If $\hat{x}$ is weakly efficient, the point $\hat{y} = f(\hat{x})$ is called weakly non-dominated, refer to Ehrogtt [3]. Thus solving a multi-objective problem will result in a set of solutions, the decision-maker should be interested in the Pareto set because it represents a solution that is better than any other with respect to at-least one of the criteria of interest.

3. Covering Location Model

The covering problem is the problem in which facilities are located at existing stations on the network so as to cover all the demand zones while minimizing the number of facilities. The basic coverage model was developed by Toregas et al. [4] with the objective of minimizing the number of vehicles needed to cover all demand nodes. Church and ReVelle [5] extended the basic coverage model to deal with the situation in which the number of vehicles available is less than the number needed to cover all demand zones, called a maximal covering location problem (MCLP). Hogan and ReVelle [6] considered backup coverage which is the secondary coverage of a demand zone that is required in high-demand areas to maintain a uniform level of service when vehicles can respond to only one call at a time. Daskin et al. [7] integrated different covering models such as multiple, excess, backup and expected covering models. Pirkul and Schilling [8] modeled the objective of maximizing covered calls while considering workload capacities and backup service. Pirkul and Schilling [9] extended the model with the addition of workload limits on the facilities and the quality of service delivered to the uncovered demand zones. Narasimhan et al. [10] extended the model with multiple levels of backup. ReVelle et al. [11] considered extensions to the maximal conditional covering problem. In their models, the facility locations are supposed to be covered by other facilities and may not be used to cover their own zones. Berman and Krass [12] presented the generalized maximal cover location problem which allows for partial coverage. The degree of coverage is defined as a decreasing step function of the distance to the closest facility. Karasakal and Karasakal [13] introduced a notion of partial coverage, defined as a function of distance of the demand point to the facility. Araz et al. [14] developed a fuzzy multi-objective covering location model by considering the population covered by one vehicle, backup coverage, and the total distance from locations at a distance bigger that a specified distance standard for all zones. In this covering problem, we want to cover as much demand as possible but at the same time we do not want to leave rural populations uncovered. So, we directly factor in issues of fairness, such as underserving a certain demographic. Therefore, we have two objectives which are 1) to maximize the number of requested calls in the demand zones that can be covered, 2) to minimize the maximum distance between the uncovered demand zones and opened stations. Assume that each uncovered zone is always assigned to its closest opened station. The objectives can be formulated as in equations (1) and (2), respectively. There are three constraints which are shown in equations (3) to (5). The first constraint is the limitation of the number of vehicles available to locate at all stations. The second constraint is the limitation of the number of vehicles located at each station. In the third constraint, a demand zone can receive service from a station that has at least one vehicle located there. Equations (6) and (7) represent non-negativity and integrality constraints.

$$\text{Maximize} \quad Z_1 = \sum_{i=1}^{n} \sum_{k=1}^{kA} h_i w_k y_{ik} \tag{1}$$

$$\text{Minimize} \quad Z_2 = \underset{i \in U, j \in O}{Max} \{(d_{ij}, x_j)\} \tag{2}$$

Subject to:

$$\sum_{j=1}^{m} x_j \le NA \tag{3}$$

$$x_j \le NU, \qquad j=1, \dots, m \tag{4}$$

$$\sum_{k=1}^{kA} y_{ik} \le \sum_{j \in J_i} x_j, \qquad i=1, \dots, n \tag{5}$$

$$x_j \in \{0, 1, \dots, NU\} \qquad \text{for all } j \tag{6}$$

$$y_{ik} \in \{0, 1\} \qquad \text{for all } i \text{ and } k \tag{7}$$

Where: y_{ik} = 1 if demand zone *i* is covered by vehicle *k*

0 otherwise

w_k = the probability that the vehicle *k* is available

h_i = the population size in demands zone *i*

x_j = the number of vehicles located at station *j*

d_{ij} = the distance from station *j* to demand zone *i*

(d_{ij}, x_j) = the distance from demand zone *i* to its closest opened station

NA = the total number of vehicles to be located

NU = the maximum number of vehicles that are allowed to be located at each station

J_i = $\{j \mid d_{ij} \le D\}$ = set of stations that can covered demand zone *i*

D = the maximum distance that can be reached within 9 minutes (4 miles)

U = set of uncovered demand zones

O = set of opened stations

kA = min$\{NA, NU\}$ = upper bound on the number of vehicles that can cover a demand zone

n = the number of demand zones

m = the number of stations

To apply the ε-constraint approach to solve this problem, we have to reformulate the problem in the ε-constraint form. In this case, we select to maximize the second objective while the first objective is bounded at an acceptable value, ε_1. So the first objective in equation (1) now becomes the new constraint in the ε-constraint form as shown in equation (9). In order to make the second objective be convenient to solve, we rewrite the second objective in an alternative form. Equations (8) and (10) represent the second objective. The entire ε-constraint problem is represented as equations (8) - (10) and equations (3) - (7).

$$\text{Minimize} \quad Z_2 \tag{8}$$

Subject to: (3) - (7)

$$\sum_{i=1}^{n} \sum_{k=1}^{kA} h_i w_k y_{ik} \ge \varepsilon_1 \tag{9}$$

$$Z_2 \ge (d_{ij}, x_j) \tag{10}$$

Where: ε_1 = the acceptable bound of objective 1

4. Data

The data are collected from the Fire/EMS department at Hanover County, VA. The responsibility area is divided into 30 demand zones and there are 16 existing station locations for locating EMS vehicles. The number of requested calls in each demand zone is different. A demand zone is covered when there exists an EMS vehicle that is able to respond within 9 minutes. Using empirical evidence, we assume that the maximum distance that can be reached in 9 minutes is 4 miles. In this case, we have constructed a set of locations that are able to respond calls within 9 minutes for each demand zone. To set up the location of the station and demand zone, we drew grid lines over the area or interest, with one block representing 2 miles. The coordinate (*a, b*) of the stations and demand zones are used to calculate the distance between each demand zone and each station. Distance between two points can be measured in many ways [15], the most familiar two are rectilinear distance and Euclidean distance. In this case we use the Euclidean distance because approximately 70% of the Hanover County area is rural. Given the demand zone i at (a_i, b_i) and the station location j at (a_j, b_j), the distance (d_{ij}) between demand zone i and station j are calculated as follows.

$$d_{ij} = \sqrt{(a_i - a_j)^2 + (b_i - b_j)^2} \tag{11}$$

If there is one vehicle located at a station, the probability that this vehicle is busy is p and the probability that it is available is (1-p). The probability that a randomly selected vehicle will be busy, p, depends on the number of vehicles that are located in the station. From Daskin [16], one way to estimate p is using actual data of the system. The formula is shown in equation (12). Where, λ is the average number of calls per day, $1/\mu$ is the average service time per call (hours), and S is number of vehicles that are deployed. Based on the data during 2007, the average number of calls per day is 1.2 calls/hour or 28.8 calls/day. The average service time per call is 74 minutes or 1.2 hour. Therefore, knowing the number of vehicles that are deployed, we can estimate the probability p. If there are k vehicles located at a station, the probability that the vehicle k^{th} will be dispatched or it is available is calculated from the probability that k-1 vehicles are busy and the vehicle k^{th} is available. The probability the vehicle k^{th} is available (w_k) is shown in equation (13).

$$p = \lambda . \frac{1}{\mu} . \frac{1}{24} . \frac{1}{S} \tag{12}$$

$$w_k = (1-p)(p^{k-1}) \tag{13}$$

5. Computational results

We use the data from the Hanover Fire/EMS department with 30 demand zones and 16 station locations. The total number of vehicles located in all stations is varying from 5 to 15 and the maximum number of vehicles that are allowed at each station is 2. The results for using a single objective, which is maximizing number of demands that covered, are shown in Table 1. We see that when increasing the number of vehicles, the probability of a randomly selected vehicle being busy decreases and the number of demands that are covered increases. Stations 1, 7, 10, 13, and 14 are always selected to be solution set because they have high demand calls.

Table1: Results with single objective which is maximizing demands

Number of vehicles(S)	Probability vehicle will be busy	Expected demand that is covered (calls)	Opened stations	Number of vehicles at each stations	Coverage percentage
5	0.296	2321.4	{1,7,10,13,14}	{1,1,1,1,1}	61.51
6	0.247	2540.6	{1,5,7,10,13,14}	{1,1,1,1,1,1}	67.32
7	0.211	2756.6	{1,5,7,10,13,14}	{1,1,1,1,2,1}	73.04
8	0.185	2894.2	{1,5,7,10,11,13,14}	{1,1,1,1,1,2,1}	76.69
9	0.164	3003.0	{1,5,7,10,11,13,14}	{1,1,1,2,1,2,1}	79.57
10	0.148	3110.4	{1,5,7,10,11,13,14}	{1,1,1,2,1,2,2}	82.42
15	0.099	3466.8	{1,2,5,6,7,8,10,11,13,14}	{2,1,2,1,1,1,2,1,2,2}	91.86

To provide decision maker with more alternatives, we construct a bi-objective model in which we maximize number of covered calls and minimize the maximum distance between uncovered demand zones and opened stations. In order to solve this problem using the ε-constraint method, first of all, we formulate the problem in the ε-constraint form. Then, we find the ε_1 by solving the problem with one objective (Z_1) which is to maximize the covered demand, which yields and upper bound for ε_1 equal to 2321.4. In the same way, we find the ε_2 by solving the problem with respect to one objective (Z_2) which is to minimize the maximum distance between uncovered demand zones and opened stations, resulting in a lower bound for ε_2 of 6 miles. In this case, the number of vehicles is 5. There are two options to find the solution points. Firstly, we can maximize the first objective while varying the ε_2 value or, we can minimize the second objective while varying the ε_1 value. In this case, we chose the second option. By minimizing the second objective, we can find the solution points by changing the ε_1 value in the range [0, 2321.4]. By considering both objectives simultaneously, we can arrive at all the solution points. Figure 1 shows all the solution points that we found by minimizing the second objective and varying the ε_1 value in the range [1000, 2321.4]. A circle represents the solution point of the ε-constraint method. The blue circles represent non-dominated solutions (the points are numbered for later reference). From Figure 1, we see that the first objective values are between 1000 and 2321.4 and resulting in second objective values between 6 and 15. At the maximum number of requested calls, 2321.4, the maximum distance between uncovered demand zones and opened stations is 15 miles. If we deteriorate the value of the first objective by decreasing the ε_1 value, the value of the second objective will be improved. All the solution points that we found are weakly non-dominated points. The blue dots which are points 8, 12, and 14 are the non-dominated points.

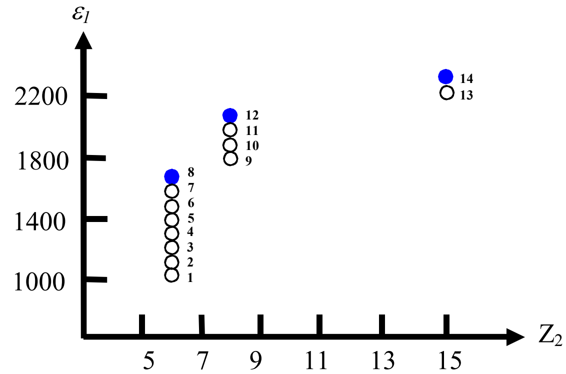

Figure 1: Solution points found by using ε-constraint method

The values of the objectives and the corresponding solution set are shown in Table 2. We see that as we require that more demand calls be covered, more stations will need to be opened. Note that station 13 is always in the solution set because it is located in an urban zone where we have the highest demand, and it also located in the middle of the area which close to all demand zones. From these results, the maximum distance between uncovered demand zones and opened stations is still big and the number of demands that can be covered is still small. With 5 vehicles, we only covered 2321.4 calls which are 61.51% of all demands and leave 12 zones to be uncovered. Each uncovered demand zone is located far from the opened stations, at most 15 miles. In order to decrease the maximum distance between uncovered demand zones and opened stations and increase the number of calls that can be covered, we should increase the number of the vehicles to be located. The results when number of vehicles is 10 and 15 are shown in Table 3.

Table 2: The objective values and the solutions when *NA*=5

Points	Objective 1 (Z_1) Maximize expected number of covered demands	Objective 2 (Z_2) Minimize maximum distance between uncovered demand zones and opened stations	Solutions	
			Opened Stations {rural; urban}	Number of uncovered zones
1	1000	6	{2,11,12; 1,13}	18
2	1100	6	{3,9,15; 1,13}	18
3	1200	6	{9,12,15; 1,13}	16
4	1300	6	{2,11,14; 1,13}	15
5,6	1400-1500	6	{11,14,15; 13,16}	16
7,8	1600-1700	6	{11,14,15; 1,13}	15
9	1800	8	{5,8; 7,10,13}	16
10	1900	8	{8,14; 1,13}	17
11,12	2000-2100	8	{8,14; 1,10,13}	14
13	2200	15	{5,14; 1,10,13}	13
14	2300-2354.4	15	{14; 1,7,10,13}	12

6. Conclusion and Discussion

In this paper, we applied the ε-constraint method to solve the bi-objective covering location problem. The first objective is to maximize the number of requested calls that can be covered by the vehicles within 9 minutes. The second objective is to minimize the maximum distance between each uncovered demand zone and its closest opened station. The results show that the maximum number of requested calls that can be covered are between 1000 and 2321.4, while the maximum distance between uncovered demand zones and opened stations are between 6 and 15 (when the number of vehicles is 5). With one objective, we can only get the solution at the maximum number of requested calls that can be covered which are 2321.4 or we can get the solution at the minimum maximum distance between uncovered demand zones and its closest opened station which is 6. While using bi-objective, we can find all the solution points in between the best value of first objective and the best value of the second objective. The solution points we found provide a set of efficient solutions, or alternatives, that are very useful for decision makers wishing to take into account issues of equitably locating EMS vehicles in rural areas.

Table 3: The objective values and the solutions when *NA*=10 and 15

Objective 1 (Z_1) Maximize expected number of covered demands	Objective 2 (Z_2) Minimize maximum distance between uncovered demand zones and opened stations	Solutions: Opened Stations {rural; urban}	Solutions: Number of uncovered zones
NA=10			
1000	5	{2,3,5,8,11,14; 1,4,6}	9
1500	5	{5,11,12,15; 1,4,6,7,13}	11
2000	5	{3,5,11,14,15; 1,4,6,7,13}	7
2500	5	{2,3,5,11,14; 1,4,10,13}	8
3000	6	{5,8,11,14; 1,7,10,13}	6
3110.4	14	{5,11,14; 1,7,10,13}	8
NA=15			
1000-3100	0	{2,3,5,8,9,11,12,14,15; 1,4,6,7,10,13}	0
3200	5	{2,3,5,8, 9,11,12,14,15; 1,4,7,10,13}	1
3300	5	{2,5,8,9,11,12,14,15; 1,4,7,10,13}	2
3400	5	{2,5,8,11,12,14,15; 1,4,6,7,10,13}	3
3466.8	6	{2,5,8,11,14; 1,6,7,10,13}	5

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