FLUID MECHANICS
PhD Qualifying Exam
SPRING 2009

ID: ________________

Instructions
Each problem is worth 25 points credit.

Do all of your work on the attached sheets. Work on the back side of the paper if needed.

Setup the problem! Clearly state any and all assumptions used. The Committee is interested in your ability to identify, formulate, simplify, and approach problems correctly. **Justify your assumptions.** The Committee is interested in how you select and apply assumptions in the formulation of the problem. Most credit is awarded to these aspects of a problem.

Include freebody diagrams and control volumes. A problem cannot be formulated without such bases and all equations relate back to the system as defined by you. Hence: no system definitions, no credit.

Please keep your work organized. If we cannot figure it out, you will not receive credit.

Problem:

1 ______________

2 ______________

3 ______________

4 ______________

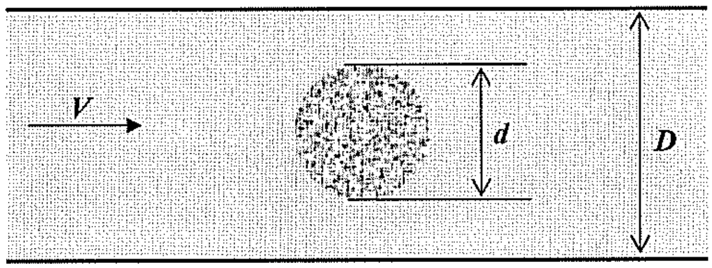
Total ______________

1. The drag F_D acting on a sphere located in a pipe is a function of the sphere diameter d, pipe diameter D, fluid velocity V and fluid density ρ.

a. *What is the number of dimensionless groups for this problem?*

b. *Derive the dimensionless group(s) for this problem. Show your work.*

c. *Can you conclude whether the flow about the sphere has a laminar or a turbulent character? What would govern your decision? Justify your conclusion.*



1. (continued)

2. A cylinder of radius R is rotating at a constant angular velocity ω in an infinite domain of viscous fluid. *Starting with the Navier-Stokes equations in cylindrical coordinates, show that the steady-state solution is also an invicid (potential) flow solution. Determine this solution.*

2. (continued)

3. A thin aircraft wing moves through the air at speed U_0. The decision as to whether a gaseous flow can be treated as incompressible (negligible relative density change, $d\rho/\rho$) or compressible can be related back to the Mach number, M (speed divided by local speed of sound). Using the basic property describing fluid compressibility, known as the Bulk Modulus, $E = \rho \, {}^{\partial p}\!/_{\partial \rho}$, *come up with an expression that relates the relative fluid compressibility to the flow Mach number.* Then *discuss the conditions under which an incompressible flow assumption might be reasonable and why it is reasonable.*
Treat the air as a perfect gas and imagine the process to be isentropic.

3. (continued)

4. Consider steady, viscous, laminar flow in a pipe. At the entrance to the pipe, the *pressure driven* entrance flow is uniform, $u(r) = U_o$, whereas the developed flow downstream at position 'x' becomes parabolic, $u(r) = C(1 - r^2/R^2)$, where R is the pipe radius and C is a coefficient related to the pressure gradient and equal to one-half the centerline velocity. The pressure drop along the pipe is linear. Derive the expression for the viscous drag on the pipe walls occurring between the entrance to the circular pipe and downstream location 'x'.

4. (continued)

Equations of Motion (Incompressible formulation)

Cartesian coordinates

$$\rho\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}\right)=-\frac{\partial p}{\partial x}+\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right)+\rho g_x$$

$$\rho\left(\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}\right)=-\frac{\partial p}{\partial y}+\mu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial z^2}\right)+\rho g_y$$

$$\rho\left(\frac{\partial w}{\partial t}+u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}\right)=-\frac{\partial p}{\partial z}+\mu\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}+\frac{\partial^2 w}{\partial z^2}\right)+\rho g_z$$

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0$$

Cylindrical coordinates (r: radial, θ: azimuthal, z: cylinder axis)

$$\rho\left(\frac{\partial u_r}{\partial t}+u_r\frac{\partial u_r}{\partial r}+\frac{u_\theta}{r}\frac{\partial u_r}{\partial \theta}+u_z\frac{\partial u_r}{\partial z}-\frac{u_\theta^2}{r}\right)=-\frac{\partial p}{\partial r}+\mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u_r}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 u_r}{\partial \theta^2}+\frac{\partial^2 u_r}{\partial z^2}-\frac{u_r}{r^2}-\frac{2}{r^2}\frac{\partial u_\theta}{\partial \theta}\right]+\rho g_r$$

$$\rho\left(\frac{\partial u_z}{\partial t}+u_r\frac{\partial u_z}{\partial r}+\frac{u_\theta}{r}\frac{\partial u_z}{\partial \theta}+u_z\frac{\partial u_z}{\partial z}\right)=-\frac{\partial p}{\partial z}+\mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u_z}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 u_z}{\partial \theta^2}+\frac{\partial^2 u_z}{\partial z^2}\right]+\rho g_z$$

$$\rho\left(\frac{\partial u_\theta}{\partial t}+u_r\frac{\partial u_\theta}{\partial r}+\frac{u_\theta}{r}\frac{\partial u_\theta}{\partial \theta}+u_z\frac{\partial u_\theta}{\partial z}+\frac{u_r u_\theta}{r}\right)=-\frac{1}{r}\frac{\partial p}{\partial \theta}+\mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u_\theta}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 u_\theta}{\partial \theta^2}+\frac{\partial^2 u_\theta}{\partial z^2}+\frac{2}{r^2}\frac{\partial u_r}{\partial \theta}-\frac{u_\theta}{r^2}\right]+\rho g_\theta$$

$$\frac{1}{r}\frac{\partial}{\partial r}(r u_r)+\frac{1}{r}\frac{\partial u_\theta}{\partial \theta}+\frac{\partial u_z}{\partial z}=0.$$