FLUID MECHANICS
PhD Qualifying Exam
Spring 2010

ID: ________________

Instructions
Each problem is worth 25 points credit.

Do all of your work on the attached sheets. Work on the back side of the paper if needed.

Setup the problem! Clearly state any and all assumptions used. The Committee is interested in your ability to identify, formulate, simplify, and approach problems correctly. **Justify your assumptions.** The Committee is interested in how you select and apply assumptions in the formulation of the problem. Most credit is awarded to these aspects of a problem.

Include freebody diagrams and control volumes. A problem cannot be formulated without such bases and all equations relate back to the system as defined by you. Hence: no system definitions, no credit.

Please keep your work organized. If we cannot figure it out, you will not receive credit.

Problem:

1 ____________

2 ____________

3 ____________

4 ____________

Total ____________

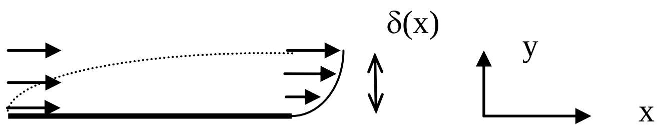
1. A long, thin 2-D flat plate is pulled at a constant velocity through an otherwise static (non-moving) fluid. Starting with the full equations of motion, develop an expression for the subsequent steady flow field developed. **You will earn credit through your explanations of how appropriate assumptions are used and how they affect the problem formulation.** Solve for the velocity field. Most credit will be for problem formulation with proper application of any assumptions made and application of boundary conditions.

1. (continued)

2. Consider the viscous flow over a flat plate of length c. At the leading edge of the plate, the velocity profile in the vertical direction can be considered as uniformly distributed with velocity of strength U_0 aligned parallel with the plate. A boundary layer of thickness $\delta(x)$ develops and grows (dotted line) along the plate. At the trailing edge,

$$\frac{u(y)}{U_0} = (\frac{y}{\delta})^{1/7}$$

(i) Estimate the mass flow displaced by the boundary layer . Be sure to clearly define any control volumes or differential elements and how assumptions are used.



2. (cont)

3. The torque (T) generated by a wind mill depends on its diameter, D, the rotational speed, ω, the free-stream velocity, U, the blade pitch angle, α, the air density, ρ, the air kinematic viscosity, ν, and the speed of sound, a. Put this relationship into a non-dimensional form.

3.(cont)

4. A free vortex has a velocity profile, $v = \frac{a}{r}$, where r is the distance from the center. Use the radial component of the following form of the steady Euler's equation,

$$\frac{1}{2}\nabla \underset{\rightarrow}{v}^2 - \underset{\rightarrow}{v} \times (\nabla \times \underset{\rightarrow}{v}) = -\frac{1}{\rho}\nabla p \,,$$

to determine the pressure, p, as a function of r. Here ρ and v are the fluid density and velocity respectively. Use the pressure at any point, r_0, as the reference pressure. Hint: A free vortex is irrotational.

(continued)

Equations of Motion (Incompressible formulation)

Cartesian coordinates

$$\rho\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}\right)=-\frac{\partial p}{\partial x}+\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right)+\rho g_x$$

$$\rho\left(\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}\right)=-\frac{\partial p}{\partial y}+\mu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial z^2}\right)+\rho g_y$$

$$\rho\left(\frac{\partial w}{\partial t}+u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}\right)=-\frac{\partial p}{\partial z}+\mu\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}+\frac{\partial^2 w}{\partial z^2}\right)+\rho g_z$$

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0$$

Cylindrical coordinates (r: radial, θ: azimuthal, z: cylinder axis)

$$\rho\left(\frac{\partial u_r}{\partial t}+u_r\frac{\partial u_r}{\partial r}+\frac{u_\theta}{r}\frac{\partial u_r}{\partial \theta}+u_z\frac{\partial u_r}{\partial z}-\frac{u_\theta^2}{r}\right)=-\frac{\partial p}{\partial r}+\mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u_r}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 u_r}{\partial \theta^2}+\frac{\partial^2 u_r}{\partial z^2}-\frac{u_r}{r^2}-\frac{2}{r^2}\frac{\partial u_\theta}{\partial \theta}\right]+\rho g_r$$

$$\rho\left(\frac{\partial u_z}{\partial t}+u_r\frac{\partial u_z}{\partial r}+\frac{u_\theta}{r}\frac{\partial u_z}{\partial \theta}+u_z\frac{\partial u_z}{\partial z}\right)=-\frac{\partial p}{\partial z}+\mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u_z}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 u_z}{\partial \theta^2}+\frac{\partial^2 u_z}{\partial z^2}\right]+\rho g_z$$

$$\rho\left(\frac{\partial u_\theta}{\partial t}+u_r\frac{\partial u_\theta}{\partial r}+\frac{u_\theta}{r}\frac{\partial u_\theta}{\partial \theta}+u_z\frac{\partial u_\theta}{\partial z}+\frac{u_r u_\theta}{r}\right)=-\frac{1}{r}\frac{\partial p}{\partial \theta}+\mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u_\theta}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 u_\theta}{\partial \theta^2}+\frac{\partial^2 u_\theta}{\partial z^2}+\frac{2}{r^2}\frac{\partial u_r}{\partial \theta}-\frac{u_\theta}{r^2}\right]+\rho g_\theta$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r u_r\right)+\frac{1}{r}\frac{\partial u_\theta}{\partial \theta}+\frac{\partial u_z}{\partial z}=0.$$