# Spring 2009 Qualifying tests

# Heat transfer

Closed book exam.
The three problems must be worked out.
Box all answers.

Alloted time: 2 hours

**Problem 1:**

Two-dimensional, steady-state conduction occurs in a hollow cylindrical solid of thermal conductivity $k$ = 16 W/(m·K), outer radius of $r_o$ = 1 m, and overall length $2z_o$ = 5 m, where the origin of the coordinate system is located at the midpoint of the centerline. The inner surface of the cylinder is insulated, and the temperature distribution within the cylinder has the form

$$T(r,z) = a + br^2 + c\ln(r) + dz^2$$

where $a$ = 20°C, $b$ = 150°C/m², $c$ = – 12°C, $d$ = – 300°C/m² and $r$ and $z$ are in meters.

1. Determine the inner radius of the cylinder, $r_i$.
2. Obtain an expression for the volumetric rate of heat generation, $\dot{q}$ (W/m³).
3. Determine the axial distribution of the heat flux at the outer surface, $q_r''(r_o,z)$. What is the heat rate at the outer surface? Is the heat rate *in* or *out* of the cylinder?
4. Determine the radial distribution of the heat flux at the end faces of the cylinder, $q_z''(r,+z_o)$ and $q_z''(r,-z_o)$, and the corresponding heat rates; are the heat rates *in* or *out* of the cylinder?
5. Verify that your results are consistent with an overall energy balance on the cylinder.

**Heat diffusion equation in Cartesian coordinates:**

$$\rho c_p \frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left(k\,\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k\,\frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(k\,\frac{\partial T}{\partial z}\right) + \dot{q}$$

**Heat diffusion equation in cylindrical coordinates:**

$$\rho c_p \frac{\partial T}{\partial t} = \frac{1}{r}\,\frac{\partial}{\partial r}\left(k\,r\,\frac{\partial T}{\partial r}\right) + \frac{1}{r^2}\,\frac{\partial}{\partial \phi}\left(k\,\frac{\partial T}{\partial \phi}\right) + \frac{\partial}{\partial z}\left(k\,\frac{\partial T}{\partial z}\right) + \dot{q}$$

**Heat diffusion equation in spherical coordinates:**

$$\rho c_p \frac{\partial T}{\partial t} = \frac{1}{r^2}\,\frac{\partial}{\partial r}\left(k\,r^2\,\frac{\partial T}{\partial r}\right) + \frac{1}{r^2\,\sin^2\theta}\,\frac{\partial}{\partial \phi}\left(k\,\frac{\partial T}{\partial \phi}\right) + \frac{1}{r^2\,\sin^2\theta}\,\frac{\partial}{\partial \theta}\left(k\,\sin\theta\,\frac{\partial T}{\partial \theta}\right) + \dot{q}$$

**Problem 2:**

A cross flow exchanger is used to heat water at a rate of $\dot{m}_c = 0.4$ kg/s from $T_{ci} = 15°C$ to $T_{co} = 85°C$ using hot gases at $T_{hi} = 240°C$. The water flows through $n = 60$ tubes each having an outside diameter $D = 3$ cm. The temperature of the gases leaving the exchanger is $T_{ho} = 120°C$.

The overall heat transfer coefficient is $U = 110$ W/(m$^2$·K). The water specific heat is $c_{pc} = 4182$ J/(kg·K). For a cross flow exchanger the log mean temperature difference $\Delta T_{lm}$ is given by:

$$\Delta T_{lm} = F \, \Delta T_{lmCF}$$

where $F$ is a correction factor equal to 0.92 and $\Delta T_{lmCF}$ is the LMTD for an equivalent single pass counterflow exchanger.

1. Determine the length $L$ of each tube using the LMTD method. List all the assumptions you have made to determine the length $L$.

2. What would be the length of a single tube cross flow heat exchanger?

**Problem 3:**

A **small**, solid metallic sphere has an opaque, diffuse coating for which the absorptivity is $\alpha_\lambda = 0.8$ for $\lambda \leq 5$ μm and $\alpha_\lambda = 0.1$ for $\lambda > 5$ μm. The sphere, which is initially at a uniform temperature of 300 K, is inserted into a **large** furnace whose walls are at 1200 K. Determine the total, hemispherical absorptivity and emissivity of the coating for the initial condition and for the final, steady-state condition.

**Table 12.1** Blackbody Radiation Functions[a]

| $\lambda T$ ($\mu m \cdot K$) | $F_{(0\rightarrow\lambda)}$ | $I_{\lambda,b}(\lambda, T)/\sigma T^5$ ($\mu m \cdot K \cdot sr)^{-1}$ | $\dfrac{I_{\lambda,b}(\lambda, T)}{I_{\lambda,b}(\lambda_{max}, T)}$ |
|---|---|---|---|
| 200 | 0.000000 | $0.375034 \times 10^{-27}$ | 0.000000 |
| 400 | 0.000000 | $0.490335 \times 10^{-13}$ | 0.000000 |
| 600 | 0.000000 | $0.104046 \times 10^{-8}$ | 0.000014 |
| 800 | 0.000016 | $0.991126 \times 10^{-7}$ | 0.001372 |
| 1,000 | 0.000321 | $0.118505 \times 10^{-5}$ | 0.016406 |
| 1,200 | 0.002134 | $0.523927 \times 10^{-5}$ | 0.072534 |
| 1,400 | 0.007790 | $0.134411 \times 10^{-4}$ | 0.186082 |
| 1,600 | 0.019718 | 0.249130 | 0.344904 |
| 1,800 | 0.039341 | 0.375568 | 0.519949 |
| 2,000 | 0.066728 | 0.493432 | 0.683123 |
| 2,200 | 0.100888 | $0.589649 \times 10^{-4}$ | 0.816329 |
| 2,400 | 0.140256 | 0.658866 | 0.912155 |
| 2,600 | 0.183120 | 0.701292 | 0.970891 |
| 2,800 | 0.227897 | 0.720239 | 0.997123 |
| 2,898 | 0.250108 | $0.722318 \times 10^{-4}$ | 1.000000 |
| 3,000 | 0.273232 | $0.720254 \times 10^{-4}$ | 0.997143 |
| 3,200 | 0.318102 | 0.705974 | 0.977373 |
| 3,400 | 0.361735 | 0.681544 | 0.943551 |
| 3,600 | 0.403607 | 0.650396 | 0.900429 |
| 3,800 | 0.443382 | $0.615225 \times 10^{-4}$ | 0.851737 |
| 4,000 | 0.480877 | 0.578064 | 0.800291 |
| 4,200 | 0.516014 | 0.540394 | 0.748139 |
| 4,400 | 0.548796 | 0.503253 | 0.696720 |
| 4,600 | 0.579280 | 0.467343 | 0.647004 |
| 4,800 | 0.607559 | 0.433109 | 0.599610 |
| 5,000 | 0.633747 | 0.400813 | 0.554898 |
| 5,200 | 0.658970 | $0.370580 \times 10^{-4}$ | 0.513043 |
| 5,400 | 0.680360 | 0.342445 | 0.474092 |
| 5,600 | 0.701046 | 0.316376 | 0.438002 |
| 5,800 | 0.720158 | 0.292301 | 0.404671 |
| 6,000 | 0.737818 | 0.270121 | 0.373965 |
| 6,200 | 0.754140 | $0.249723 \times 10^{-4}$ | 0.345724 |
| 6,400 | 0.769234 | 0.230985 | 0.319783 |
| 6,600 | 0.783199 | 0.213786 | 0.295973 |
| 6,800 | 0.796129 | 0.198008 | 0.274128 |
| 7,000 | 0.808109 | 0.183534 | 0.254090 |
| 7,200 | 0.819217 | $0.170256 \times 10^{-4}$ | 0.235708 |
| 7,400 | 0.829527 | 0.158073 | 0.218842 |
| 7,600 | 0.839102 | 0.146891 | 0.203360 |
| 7,800 | 0.848005 | 0.136621 | 0.189143 |
| 8,000 | 0.856288 | 0.127185 | 0.176079 |
| 8,500 | 0.874608 | $0.106772 \times 10^{-4}$ | 0.147819 |
| 9,000 | 0.890029 | $0.901463 \times 10^{-5}$ | 0.124801 |
| 9,500 | 0.903085 | 0.765338 | 0.105956 |

**Table 12.1** *Continued*

| $\lambda T$ ($\mu m \cdot K$) | $F_{(0\rightarrow\lambda)}$ | $I_{\lambda,b}(\lambda, T)/\sigma T^5$ ($\mu m \cdot K \cdot sr)^{-1}$ | $\dfrac{I_{\lambda,b}(\lambda, T)}{I_{\lambda,b}(\lambda_{max}, T)}$ |
|---|---|---|---|
| 10,000 | 0.914199 | $0.653279 \times 10^{-5}$ | 0.090442 |
| 10,500 | 0.923710 | 0.560522 | 0.077600 |
| 11,000 | 0.931890 | 0.483321 | 0.066913 |
| 11,500 | 0.939959 | 0.418725 | 0.057970 |
| 12,000 | 0.945098 | $0.364394 \times 10^{-5}$ | 0.050448 |
| 13,000 | 0.955139 | 0.279457 | 0.038689 |
| 14,000 | 0.962898 | 0.217641 | 0.030131 |
| 15,000 | 0.969981 | $0.171866 \times 10^{-5}$ | 0.023794 |
| 16,000 | 0.973814 | 0.137429 | 0.019026 |
| 18,000 | 0.980860 | $0.908240 \times 10^{-6}$ | 0.012574 |
| 20,000 | 0.985602 | 0.623310 | 0.008629 |
| 25,000 | 0.992215 | 0.276474 | 0.003828 |
| 30,000 | 0.995340 | $0.140469 \times 10^{-6}$ | 0.001945 |
| 40,000 | 0.997967 | $0.473891 \times 10^{-7}$ | 0.000656 |
| 50,000 | 0.998953 | 0.201605 | 0.000279 |
| 75,000 | 0.999713 | $0.418597 \times 10^{-8}$ | 0.000058 |
| 100,000 | 0.999905 | 0.135752 | 0.000019 |

[a] The radiation constants used to generate these blackbody functions are:
$C_1 = 3.7420 \times 10^8$ W $\cdot \mu m^4/m^2$
$C_2 = 1.4388 \times 10^4$ $\mu m \cdot K$
$\sigma = 5.670 \times 10^{-8}$ $W/m^2 \cdot K^4$.