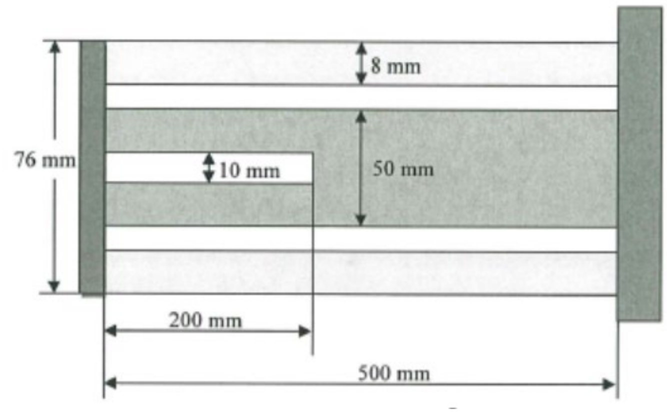
APPENDIX 1: SAMPLE PROBLEMS ON SELECTIVE TOPICS

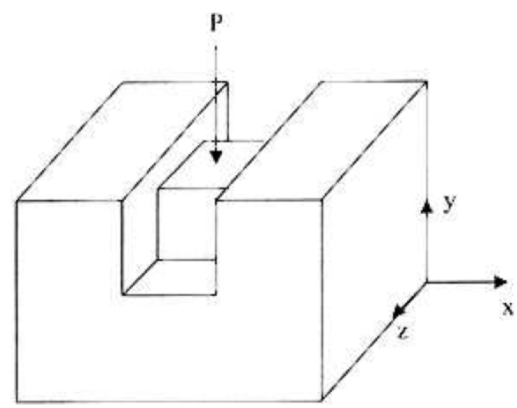
Sample Problem 1 (topic 1)

A cylindrical steel shaft and an aluminum tube are connected to a fixed support (right) and a rigid disk (left) as shown in the cross section. A 200-mm-long, 10-mm-diameter cavity has been drilled in the streel shaft from the left end. Determine the maximum torque T that can be applied to the rigid disk if the allowable shear stress is 120 MPa in the steel shaft and 70MPa in the aluminum tube. Use G=77GPa for steel and G=27GPa for aluminum.



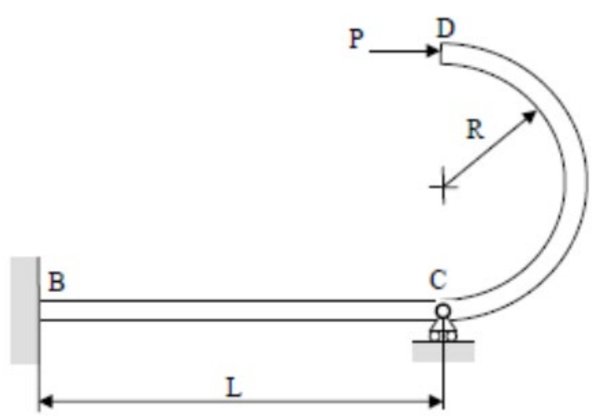
Sample Problem 2 (topic 3)

An aluminum cube with the dimensions of 10mm x 10mm x 10mm is inserted into the rectangular notch of a steel block as shown in the figure. The width of the notch is 10mm. The top surface of the cube is subjected to a uniform pressure P=60N/mm2. Young's modulus and Poisson's ratio of the cube are 7e4MPa and 0.3, respectively. Assuming the steel block is a rigid body and the fit between the cube and the notch is perfect and frictionless, determine all stress and strain components of the cube.



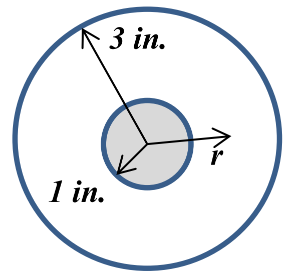
Sample Problem 3 (topic 4)

Member BCD shown in the figure has a uniform square cross-section of area A. the members is subjected to a horizontal force P. Assuming the effect of shear is negligible, determine the support reaction at C and the horizontal displacement at D in terms of applied force P, modulus of elasticity E, radius of curvature R, length L, cross-sectional area A and the moment of inertia of the cross-section I.



Sample Problem 4 (topic 9)

A composite circular shaft is constructed of two materials as shown. The inner cylinder has a shear modulus of 12×10^3 ksi and yield strength of 10 ksi, while the outer annulus has a shear modulus of 8×10^3 ksi and yield strength of 20 ksi. Both materials are assumed to have elastic–perfectly plastic stress-strain behavior in shear ($\tau = G\gamma$ when $\text{abs}(\tau) < \tau_{ys}$, otherwise $\tau = \pm\tau_{ys}$). Assuming no slip at the interface, determine



a) the largest value of the applied torque that can be supported by the shaft,

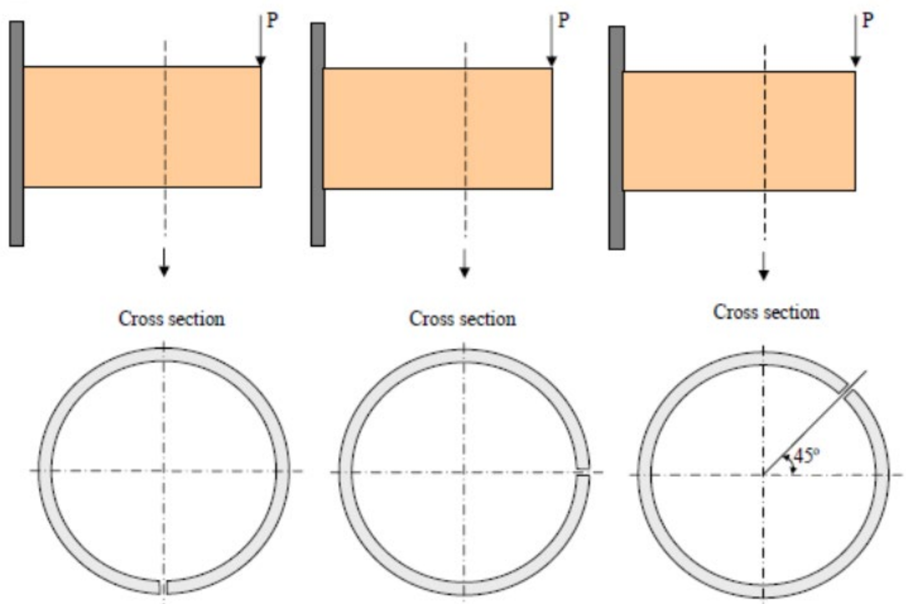
b) the largest value of the applied torque that will not yield either material,

c) the stress state in the two materials as a function of the radial coordinate, ***r***, for an applied torque that is the average of the torques from parts "a" and "b."

Sample Problem 5 (topic 10)

Transverse shear force P in the vertical direction is applied to three cantilever beams with thin walled open circular cross sections shown in the figure. The beams have identical length and identical cross sections except the position of the open slit.

(1) Draw the shear flow on the cross-sections
(2) On each cross section, approximately mark the location where the magnitude of the transverse shear stress is the maximum, explain your answer
(3) Are the magnitude of the three maximum shear stresses the same? If not, which one is the largest and which one is the smallest? Explain your answer.
(4) Will the cantilever beams be twisted by the loading? If yes, which beam has the largest angle of twist, and which one has the smallest? Explain your answer.



APPENDIX II EQUATION SHEET

Fundamental Equations of Mechanics of Materials

Axial Load

Normal Stress

$$\sigma = \frac{P}{A}$$

Displacement

$$\delta = \int_0^L \frac{P(x)dx}{A(x)E}$$

$$\delta = \Sigma \frac{PL}{AE}$$

$$\delta_T = \alpha \, \Delta T L$$

Torsion

Shear stress in circular shaft

$$\tau = \frac{T\rho}{J}$$

where

$$J = \frac{\pi}{2} c^4 \text{ solid cross section}$$

$$J = \frac{\pi}{2} (c_o^4 - c_i^4) \text{ tubular cross section}$$

Power

$$P = T\omega = 2\pi f T$$

Angle of twist

$$\phi = \int_0^L \frac{T(x)dx}{J(x)G}$$

$$\phi = \Sigma \frac{TL}{JG}$$

Average shear stress in a thin-walled tube

(closed section)

$$\tau_{avg} = \frac{T}{2tA_m}$$

Shear Flow

$$q = \tau_{avg} t = \frac{T}{2A_m}$$

$$\tau = \frac{Tt}{\sum_{i=1}^{n} \frac{1}{3} b_i t_i^3}$$

(open section)

Bending

Normal stress

$$\sigma = \frac{My}{I}$$

Unsymmetric bending

$$\sigma = -\frac{M_z y}{I_z} + \frac{M_y z}{I_y}, \qquad \tan \alpha = \frac{I_z}{I_y} \tan \theta$$

Shear

Average direct shear stress

$$\tau_{avg} = \frac{V}{A}$$

Transverse shear stress

$$\tau = \frac{VQ}{It}$$

Shear flow

$$q = \tau t = \frac{VQ}{I}$$

Stress in Thin-Walled Pressure Vessel

Cylinder

$$\sigma_1 = \frac{pr}{t} \qquad \sigma_2 = \frac{pr}{2t}$$

Sphere

$$\sigma_1 = \sigma_2 = \frac{pr}{2t}$$

Stress Transformation Equations

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta$$

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2} \sin 2\theta + \tau_{xy} \cos 2\theta$$

Principal Stress

$$\tan 2\theta_p = \frac{\tau_{xy}}{(\sigma_x - \sigma_y)/2}$$

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

Maximum in-plane shear stress

$$\tan 2\theta_s = -\frac{(\sigma_x - \sigma_y)/2}{\tau_{xy}}$$

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2}$$

Absolute maximum shear stress

$$\tau_{\substack{abs \\ max}} = \frac{\sigma_{max} - \sigma_{min}}{2}$$

$$\sigma_{avg} = \frac{\sigma_{max} + \sigma_{min}}{2}$$

Material Property Relations

Poisson's ratio

$$\nu = -\frac{\epsilon_{\text{lat}}}{\epsilon_{\text{long}}}$$

Generalized Hooke's Law

$$\epsilon_x = \frac{1}{E}\left[\sigma_x - \nu(\sigma_y + \sigma_z)\right]$$

$$\epsilon_y = \frac{1}{E}\left[\sigma_y - \nu(\sigma_x + \sigma_z)\right]$$

$$\epsilon_z = \frac{1}{E}\left[\sigma_z - \nu(\sigma_x + \sigma_y)\right]$$

$$\gamma_{xy} = \frac{1}{G}\tau_{xy},\ \gamma_{yz} = \frac{1}{G}\tau_{yz},\ \gamma_{zx} = \frac{1}{G}\tau_{zx}$$

where

$$G = \frac{E}{2(1+\nu)}$$

Relations Between w, V, M

$$\frac{dV}{dx} = -w(x), \qquad \frac{dM}{dx} = V$$

Elastic Curve

$$\frac{1}{\rho} = \frac{M}{EI}$$

$$EI\frac{d^4v}{dx^4} = -w(x)$$

$$EI\frac{d^3v}{dx^3} = V(x)$$

$$EI\frac{d^2v}{dx^2} = M(x)$$

Buckling

Critical axial load

$$P_{\text{cr}} = \frac{\pi^2 EI}{(KL)^2}$$

Critical stress

$$\sigma_{\text{cr}} = \frac{\pi^2 E}{(KL/r)^2},\ r = \sqrt{I/A}$$

Secant formula

$$\sigma_{\max} = \frac{P}{A}\left[1 + \frac{ec}{r^2}\sec\left(\frac{L}{2r}\sqrt{\frac{P}{EA}}\right)\right]$$

Energy Methods

Conservation of energy

$$U_e = U_i$$

Strain energy

$$U_i = \frac{N^2L}{2AE} \qquad \textit{constant axial load}$$

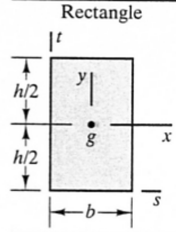
$$U_i = \int_0^L \frac{M^2dx}{EI} \qquad \textit{bending moment}$$

$$U_i = \int_0^L \frac{f_s V^2dx}{2GA} \qquad \textit{transverse shear}$$

$$U_i = \int_0^L \frac{T^2dx}{2GJ} \qquad \textit{torsional moment}$$

Properties of plane areas *Point g is the centroid*

Rectangle



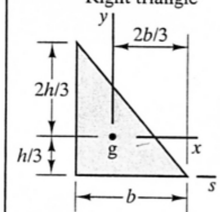
$A = bh$

$I_x = \frac{bh^3}{12}$

$I_s = \frac{bh^3}{3}$

$I_{st} = \frac{b^2h^2}{4}$

Right triangle



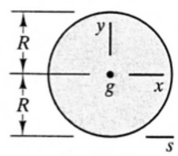
$A = \frac{bh}{2}$

$I_x = \frac{bh^3}{36}$

$I_s = \frac{bh^3}{12}$

$I_{xy} = -\frac{b^2h^2}{72}$

Complete circle



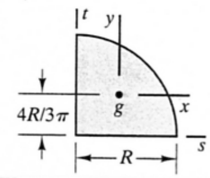
$A = \pi R^2$

$I_x = \frac{\pi R^4}{4}$

$I_s = \frac{5\pi R^4}{4}$

$J_g = \frac{\pi R^4}{2}$

Quarter circle



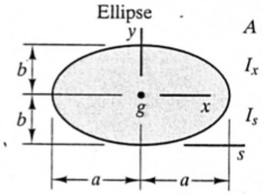
$A = \frac{\pi R^4}{4}$

$I_x = 0.0549\ R^4$

$I_s = \frac{\pi R^4}{16}$

$I_{st} = \frac{R^4}{8}$

Ellipse



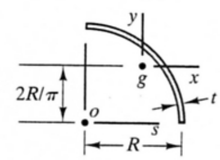
$A = \pi ab$

$I_x = \frac{\pi ab^3}{4}$

$I_s = \frac{5\pi ab^3}{4}$

$J_g = \frac{\pi ab}{4}(a^2 + b^2)$

Thin quarter-ring ($R \gg t$)



R = mean radius

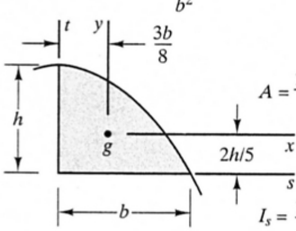
$A = \frac{\pi Rt}{2}$

$I_x \approx 0.149\ R^3t$

$I_s \approx \frac{\pi R^3t}{4}$

$J_o \approx \frac{\pi R^3t}{2}$

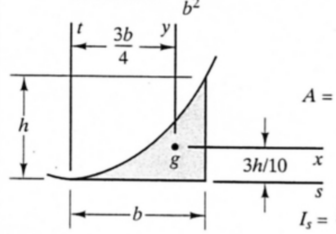
Parabola $t = h - \frac{hs^2}{b^2}$



$A = \frac{2bh}{3}$

$I_s = \frac{16bh^3}{105}$

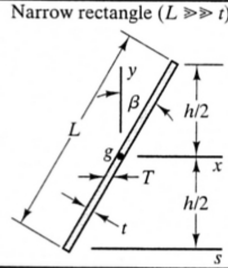
Parabola $t = \frac{hs^2}{b^2}$



$A = \frac{bh}{3}$

$I_s = \frac{bh^3}{21}$

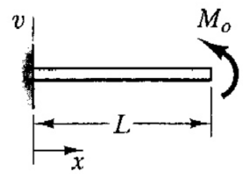
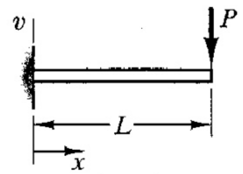
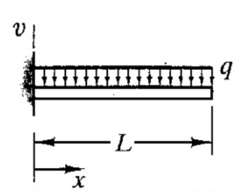
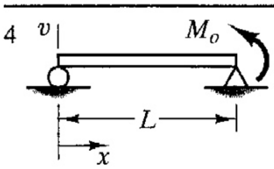
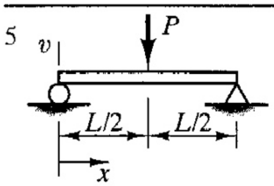
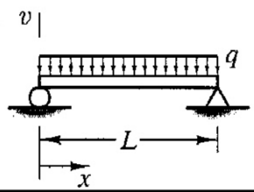
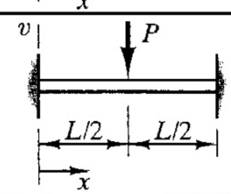
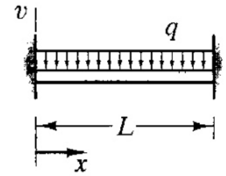
Narrow rectangle ($L \gg t$)



$T = \frac{t}{\cos\beta}$

$A = Lt$	or	$A = hT$
$I_x \approx \frac{tL^3}{12}\cos^2\beta$	or	$I_x \approx \frac{Th^3}{12}$
$I_s \approx \frac{tL^3}{3}\cos^2\beta$	or	$I_s \approx \frac{Th^3}{3}$

Deflections and slopes of uniform beams

Beam and loading	Deflection (+ up)	Slope (+ CCW)	Equations
1 	$v = \frac{M_o L^2}{2EI}$ at $x = L$	$\theta = \frac{M_o L}{EI}$ at $x = L$	$v = \frac{M_o}{2EI} x^2$ $M = M_o$
2 	$v = -\frac{PL^3}{3EI}$ at $x = L$	$\theta = -\frac{PL^2}{2EI}$ at $x = L$	$v = \frac{P}{6EI}(x^3 - 3Lx^2)$ $M = -P(L - x)$
3 	$v = -\frac{qL^4}{8EI}$ at $x = L$	$\theta = -\frac{qL^3}{6EI}$ at $x = L$	$v = -\frac{q}{24EI}(x^4 - 4Lx^3 + 6L^2x^2)$ $M = -\frac{q}{2}(L - x)^2$
4 	$v = -\frac{M_o L^2}{9\sqrt{3}EI}$ at $x = \frac{L}{\sqrt{3}}$	$\theta_o = -\frac{M_o L}{6EI}$ $\theta_L = \frac{M_o L}{3EI}$	$v = \frac{M_o}{6EIL}(x^3 - L^2x)$ $M = \frac{M_o x}{L}$
5 	$v = -\frac{PL^3}{48EI}$ at $x = \frac{L}{2}$	$\theta_o = -\frac{PL^2}{16EI}$ $\theta_L = +\frac{PL^2}{16EI}$	$v = \frac{P}{48EI}(4x^3 - 3L^2x)$ for $0 \leqslant x \leqslant \frac{L}{2}$
6 	$v = -\frac{5qL^4}{384EI}$ at $x = \frac{L}{2}$	$\theta_o = -\frac{qL^3}{24EI}$ $\theta_L = +\frac{qL^3}{24EI}$	$v = -\frac{q}{24EI}(x^4 - 2Lx^3 + L^3x)$ $M = \frac{q}{2}(L - x)x$
7 	$v = -\frac{PL^3}{192EI}$ at $x = \frac{L}{2}$	$\theta = \pm\frac{PL^2}{64EI}$ at quarter points	$v = \frac{P}{48EI}(4x^3 - 3Lx^2)$ for $0 \leqslant x \leqslant \frac{L}{2}$
8 	$v = -\frac{qL^4}{384EI}$ at $x = \frac{L}{2}$	$\theta = -\frac{qL^3}{124.7EI}$ at $x = 0.2113L$	$v = -\frac{qx^2}{24EI}(L - x)^2$ $M = \frac{q}{2}(L - x)x - \frac{qL^2}{12}$