Impulsive actuation has been researched in the past two decades as an inexpensive alternative to multi-degree-of-freedom precision positioning systems. The position of a sliding workpiece can be controlled by a 2-degree-of-freedom actuation system through simple pushing path planning. However, the final part position as a result of the last touch of the actuator is subject to uncertainty in the friction model used for actuation planning, particularly the free-sliding distance undergone by the workpiece after losing contact with the actuator.

This paper first reviews an impact planning method, then augments it using a restitution-based model that results in an explicit actuator velocity function. Results are given for positioning of a continually rotating workpiece that show improvement over constant-velocity pushing actuation. Such a positioning system is applicable to dynamic positioning for precision metrology or positioning prior to manufacturing operations (e.g., magnetic chuck grinding with part being moved while the table is rotating).

Precision positioning is a necessary practice in manufacturing, both from the standpoint of machine element actuation and workpiece positioning prior to processing. Particularly for workpiece actuation, research in precision positioning by pushing, sliding or tapping has been recently considered as an inexpensive alternative to more complex pick-and-place or vision / fiducial systems. Additionally in some applications, higher precision can be achieved by this method through avoidance of relative movement errors associated with gripper release.

However, positioning by pushing is subject to the nonlinear and time-variant effects of friction at the sliding interface. This effect, particularly the stick-slip effect (stiction) must be accounted for in planning.
for in motion planning to ensure precision in sliding positioning.

The case of pushing (i.e., constant contact) rather than impacting (i.e., brief energetic contact) is considered. This method would greatly simplify planning and mitigate the effects of uncertainty in the friction model, but is difficult to implement in a dynamic application where there is lateral motion between the pusher and part, such as actuating a part to center of rotation while the support surface (e.g., spindle base) is constantly rotating. However, pushing serves as a basis for the described method.

ACTUATION BY PUSHING

In the past 20 years, there have been numerous research efforts in the field of precision positioning by sliding the target object across a surface. Peshkin and Sanderson describe the motion of a sliding workpiece for all possible pressure distributions on the support surface [Peshkin & Sanderson]. Zesch and Fearing explore force-controlled pushing for microparts with positional results in the 1µm range [Zesch & Fearing]. Lynch and Mason have done extensive work on planning and control for stable pushing in the application of robotic manipulation as an alternative to pick-and-place positioning, including feasibility studies through both kinematic and force analyses [Lynch & Mason 1995a; Lynch & Mason 1995b, 1996]. Lynch also explores friction estimation for pushed objects and open-loop control for pushing the general polygonal shape, characterized by the “maneuverability” property [Lynch 1993, 1999].

IMPULSIVE ACTUATION

Huang thoroughly examined manipulation by impulse for robotic applications, including path step planning, object translation and rotation modeling. Huang and Mason break the impulsive positioning problems into two subparts: the Inverse Sliding Problem and the Impact Problem [Huang & Mason].

The Inverse Sliding Problem

Given an initial position and orientation and a desired final position and orientation for the target object, what initial translational and rotational velocities need to be imparted to the object? Given the strongly coupled generalized equations of motion in one dimension (ignoring the viscous frictional effect at low velocity),

$$m\ddot{v} = -F_f(v, \omega), \quad v(0) = v_0$$

$$F_f \equiv \text{force due to friction}$$

$$J\dot{\omega} = -T_f(v, \omega), \quad \omega(0) = \omega_0$$

$$T_f \equiv \text{torque due to friction}$$

The final object positions are given by

$$x_f(v_0, \omega_0) = \int_0^{t_f} v(t) \, dt$$

$$\theta_f(v_0, \omega_0) = \int_0^{t_f} \omega(t) \, dt$$

where $t_f = $ time object rests and $v(t), \omega(t)$ are solutions to the equations of motion.

Lee addresses frictional energy of contact with respect to the hot rolling process [Lee et al.]. Tao also addresses friction modeling in manufacturing through material removal process modeling [Tao & Lovell]. Additionally, new models of friction are being developed that lend themselves well to control due to their continuously differentiable nature [Canudas-de-Wit et al.; Makkar et al.].

The Impact Problem

Given the required initial translational and rotational velocities $v_0^*$ and $\omega_0^*$, how should these be generated by impact? Huang addresses this question by considering a free mass striker and the friction cone of possible impact vectors, then searching the boundary of the object for a valid impact point. An analytic search form exists for primitives such as a square cylindrical form.

Actuator tip friction limits the available velocity ratio, so in some cases multiple tap planning is required. Huang also addresses these methods.
Additional treatments of Impulsive Actuation

Yamagata and Higuchi treat impact using piezoelectric elements in the application of micropositioning [Higuchi et al. 1990; Yamagata & Higuchi 1995]. Huang and Mason study manipulation of sliding objects by imparting a momentum through impulsive actuation, then allowing the object to come to rest [Huang & Mason]. Analysis of such actuation requires separate analysis of energy transfer during impact, then analysis of the free sliding motion with friction. Huang et al. gave a general solution to these problems (first the inverse sliding problem, then the impact problem) to a rotationally symmetric class of objects, and present limiting cases of this application in Huang and Mason [Huang et al. 1995; Huang & Mason 1996]. Yao has recently explored an energy-based coefficient of restitution for the planar impact problem to better describe the dynamics of impact [Yao et al.]. Mirtich and Canny take a novel approach to impulsive actuation treatment by creating a dynamic simulation environment completely based on the impulse contact model, where all forms of actuation (pushing, sliding, and impact) are modeled by a series of collisions [Mirtich & Canny]. This has led to treatment of frictional analyses through time-stepping methods, whereby the integrals of modeled forces are applied over each time step, somewhat blurring the boundary between finite forces and impulses [Stewart & Trinkle].

1-D VELOCITY PLANNING

The prescribed actuation velocity \( v_s \) is first explored through the balance of part kinetic energy with the dissipative work of the frictional force. As the analysis will arrive at an initial sliding velocity, a strictly dynamic friction model is employed:

\[
E_{\text{kinetic}} = E_{\text{friction}}
\]

\[
m v_0^2 = F_f d
\]

where \( d \) is the sliding distance before rest (free-sliding distance) and \( F_f \) is the dynamic friction force defined as

\[
F_f = \mu_k W = \mu_k mg
\]

The required initial workpiece velocity to travel a distance \( d \) is therefore

\[
v_0 = \sqrt{2 \mu_s g d}
\]

To impart such an initial velocity to the part, a slide velocity to strike the part is determined by analysis of free impact. An expression for the slide velocity after impact is determined from Newton’s one-dimensional Kinematic Impact Law:

\[
-\varepsilon (v_{\text{slide},b} - v_{\text{part},b}) = v_{\text{slide},a} - v_{\text{part},a}
\]

where \( \varepsilon \) is the coefficient of restitution, \( v_{\text{part},b} \) is the part velocity before impact, \( v_{\text{part},a} \) is the part velocity after impact, \( v_{\text{slide},b} \) is the actuator velocity before impact, \( v_{\text{slide},a} \) is the actuator velocity after impact. Assuming no initial part velocity, this is simplified to

\[
v_{\text{slide},a} = v_{\text{part},a} - \varepsilon v_{\text{slide},b}
\]

The coefficient of restitution is determined through free impact experiment, with the slide accelerated freely by hand and released before impact, with velocities of the slide and part measured directly before and after contact. The data are given as

\[
\begin{align*}
v_{\text{slide},b} &= 139 \ \mu m/s \\
v_{\text{slide},a} &= 104 \ \mu m/s \\
v_{\text{part},a} &= 222 \ \mu m/s
\end{align*}
\]

resulting in a restitution coefficient for the impact of 0.85 as determined by (8). This value is confirmed over a range of initial slide velocities to within 5%.

Momentum balance before and after impact is considered to determine a required slide velocity before impact given a desired initial part velocity:
\[ m_{\text{slide}}v_{\text{slide},b} + m_{\text{part}}v_{\text{part},b} = m_{\text{slide}}v_{\text{slide},a} + m_{\text{part}}v_{\text{part},a} \quad (11) \]

\[ v_{\text{slide},b} = v_{\text{slide},a} + \frac{m_{\text{part}}}{m_{\text{slide}}}v_{\text{part},a} \]

Substituting from (9):

\[ v_{\text{slide},b} = v_{\text{part},a} - \varepsilon v_{\text{slide},b} + \frac{m_{\text{part}}}{m_{\text{slide}}}v_{\text{part},a} \]

\[ v_{\text{slide},b} \left(1 + \varepsilon\right) = v_{\text{part},a} \left(1 + \frac{m_{\text{part}}}{m_{\text{slide}}}\right) \quad (12) \]

\[ v_{\text{slide},b} = v_{\text{part},a} \left(\frac{m_{\text{slide}} + m_{\text{part}}}{m_{\text{slide}} \left(1 + \varepsilon\right)}\right) \]

Substituting from (7),

\[ v_{\text{slide},b} = \left(\frac{m_{\text{slide}} + m_{\text{part}}}{m_{\text{slide}} \left(1 + \varepsilon\right)}\right) \sqrt{2\mu_k gd} \quad (13) \]

Applying this rule in a pushing system with feedback control violates the free impact assumption of the model since the actuator is driven with a constant velocity command regardless of resisting force. The proportional feedback gain of the controller is set high enough to allow the slider to act as a very large free impact mass \(i.e., v_{\text{slide},a} \approx v_{\text{slide},b}\), resulting in the simplification assumption:

\[ m_{\text{slide}} \gg m_{\text{part}} \quad (14) \]

This assumption reduces (13) to

\[ v_{\text{slide},b} = \frac{\sqrt{2\mu_k gd}}{1 + \varepsilon} \quad (15) \]

The previous model is used to generate the constant slide velocity required to actuate a stationary part over a distance \(d\). The assumptions implicit in this model are

- The coefficient of restitution is independent of the contact velocity
- The static friction coefficient is equal to the kinetic friction coefficient (part is assumed to have a negligible presliding velocity)
- The slider deceleration is begun after contact is broken \(i.e., \text{impact is complete}\), and the slider is able to completely stop its forward motion in a shorter distance than the desired actuation distance.
- The theoretical velocity cannot exceed that which would cause a following error in the motion system.

The final assumption effectively limits the allowable velocity and prevents single actuation positioning at longer target distances. An example of application of this function is given in Figure 1. In this case, a sample part of 0.8 kg and a restitution coefficient of 0.85 is used to generate a velocity plan over a range of actuation distances.

This function is used to determine the prescribed slide velocity, given a required actuation distance and a kinetic friction coefficient determined from initial experimentation. In the future, it is anticipated that the friction model parameter and the coefficient of restitution will be determined in real time through system identification techniques. Initial results for a friction identification model are found in [Mears et al.].

The slide velocity is limited to 5000 mm/min to prevent following error in the prototype plant. The net effect of this limitation is that at large
required distances, actuation will take place as a series of impacts rather than a single impact.

Examining the slide deceleration assumption, given actuation at maximum velocity, the maximum stopping distance is given by

\[ d_{\text{max}} = \frac{v_{\text{max}}^2}{2a} \]  

(16)

The achievable slider acceleration is approximately \(2E5 \text{ mm/s}^2\), giving a maximum stopping distance of 17 µm at full velocity, well below the actuation distance at that speed. This is the limiting case.

**VALIDATION OF ENERGY BALANCE VELOCITY PLANNING**

The energy balance velocity planning method is employed in a precision positioning system with two degrees of freedom: a rotary table upon which the part slides and a single-axis linear motor actuator with 50 nm position feedback precision. Positioning takes place by aligning the center of a cylindrical primitive with the center of rotation of the polar axis. This allows any overshoot of the desired position to be actuated again by pushing through adjustment of the polar axis to realign centers before the next actuation.

The actuation velocity determined by energy balance is compared to position planning actuation at constant velocity. Data for a cycle of an 0.8 kg part at a constant actuation velocity level of 500 mm/min is shown on a logarithmic polar plot as Figure 5.6.

![Polar Plot](image)

**FIGURE 2. POLAR PLOT OF V=500MM/MIN ACTUATION.**

After approaching center, the off-center distance oscillates steadily across the tolerance zone, demonstrating a position limit cycle. As an alternative representation to show more data points, phase data are ignored and only absolute amplitudes are considered as the number of actuations increase. The absolute offset vs. number of actuations is shown in Figure 3.

![Off-Center Distance vs. Number of Actuations](image)

**FIGURE 3. OFF-CENTER DISTANCE OVER NUMBER OF ACTUATIONS, V=500 MM/MIN.**

The part approaches center, but oscillates near 70 µm absolute off-center distance, and is never able to converge below a chosen absolute precision tolerance limit of 2.5 µm.
Magnitude data for the cases of 1000 mm/min and 2000 mm/min are shown as Figure 4 and Figure 5 respectively.

The same effect is present as in the case for 500 mm/min, however more pronounced as the constant velocity level increases. For v=1000 mm/min, limit cycling is observed around 110 µm, and for v=2000 mm/min, limit cycling is observed around 1 mm. Total cycle magnitude data is shown for v=1000 and v=2000 mm/min is shown in Figure 6 and Figure 7 respectively.

FIGURE 4. POLAR PLOT OF V=1000 MM/MIN ACTUATION.

FIGURE 5. POLAR PLOT OF V=2000 MM/MIN ACTUATION.

FIGURE 6. OFF-CENTER DISTANCE OVER NUMBER OF ACTUATIONS, V=1000 MM/MIN.

FIGURE 7. OFF-CENTER DISTANCE OVER NUMBER OF ACTUATIONS, V=2000 MM/MIN.

Alternatively, data for a typical cycle using actuation velocity determined by the energy balance model is shown as Figure 8.
FIGURE 8. POLAR PLOT WITH ACTUATION VELOCITY DETERMINED BY ENERGY BALANCE METHOD.

Absolute offset distance magnitude vs. number of actuations is shown in Figure 9.

FIGURE 9. OFF-CENTER DISTANCE OVER NUMBER OF ACTUATIONS, V ACCORDING TO ENERGY BALANCE METHOD.

The velocity is adjusted for each actuation and decreases as the actuation distance decreases. The net effect is that the actuation distances are on the order of the desired distances, the offset distance steadily approaches zero, and although some overshoot occurs at larger actuation distances, no limit cycling is observed.

CONCLUSION

A method has been described to allow for balance of input energy and frictional dissipation energy in velocity planning for impulsive actuation of a sliding object. The method includes accounting for restitution, plant feedback control and limitation of actuation velocity to avoid following error.

Results show an improvement in limit cycling behavior from the constant-velocity actuation case to the velocity planned by energy balance. Additional limit cycling improvement is gained through augmentation of the initial planning algorithm to include a model of the free-sliding distance.

Additional work is planned to

- Include compensation for the free sliding distance that the part undergoes after the pushing actuator stops
- Include real-time system identification for friction model parameters and restitution
- Extend the friction models to include static and viscous effects in order to improve accuracy,
- Investigate the effect of control architecture and servo compliance on the velocity planning function

REFERENCES


