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Magnetic Fields and Forces in a Three Phase Caternary Power Line System

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Magnetic Fields and Forces in a Three Phase Catenary Power Line System

I. Abstract

The study of the secondary fault of a power line system is very important to understand the causes and the possible prevention of these system faults. A number of calculations have been performed to quantify the forces applied on these power lines and the changes that occur within the system. A number of these previous studies have been based on the assumption of an infinitely long straight power line, which is not the case in the modern power line system. The weight of the lines and the spacing of the support poles result in a periodic catenary shape, which increasingly complicates the calculation of the fields and forces to which the lines are subjected. This study is dedicated to accurately modeling the catenary shape and determining the resulting magnetic fields and forces. The study was conducted as follows and all data was recorded and graphed if appropriate:

i. The catenary equation was solved and plotted. This equation was also used to derive an equation for the magnetic field based on Biot-Savart's Law.

ii. An approximation for the catenary equation was tested and validated for use in the calculations. This approximation simplified the catenary plots and was also manipulated for an approximation magnetic field equation. With the validation of the approximation, the derivation of the approximation field equation was used for the remainder of the study.

iii. The equation for the magnetic field was expanded to account for the effect of the adjacent sections of the line and the other two phases. This resulted in an equation that calculates the electric field near one of the catenary sections as produced by the entire power line system. The results of these calculations were also plotted to examine the direction and magnitude of the field along the catenary section.

iv. The information that was gained from the study of the magnetic field was used to determine the magnetic force applied on one catenary section by the entire system. The force was evaluated for a number of intervals along the section and the total force was determined. This was also graphed along the catenary to determine the dispersion of the force.

II. Background

A. Problem Statement

The fault of a three-phase power line is a dreaded event for all those involved. A fallen tree or some other unforeseen event may bring on a preliminary fault of the system. This fault is nearly unavoidable and must be accepted with any power line system. There is, however, a secondary fault in many power line systems that can be avoided if properly designed.

To better understand the secondary fault of the power line, consider the primary fault to be caused by a fallen tree that has landed across two of the three phases of the line. This connection will change the magnitude, phase, and possibly direction of current in the two phases that are now under new circumstances. This change in the system is the primary fault and can lead to more drastic failures in the system.

With the major change that occurs in the current of two phases of the system, the third phase is subjected to a major change in the magnetic field that it experiences. This change in magnetic field is in accordance with Biot-Savart's Law, as given below:

$$B_o = \mu_o \int_l \frac{\mathbf{I}(l)d\mathbf{l} \times \mathbf{a}_o(l)}{4\pi|\mathbf{r}_o(l)|^2}$$

As seen by the equation, the current and the distance between the phases will be the largest contributing factors to the change in the magnetic field. If the change in current is great enough and the phases are not placed far enough apart, the third phase may be pulled towards the other faulted phases. This is known as a secondary fault, and can lead to greater consequences than the primary fault. This is the reason why it is necessary to determine the magnetic force that will be applied to a power line in the occurrence of a primary fault. If the system is designed properly, the secondary fault will be much less likely to occur and the damage will be minimized.

B. Field and Force Determination

The determination of a safe system is rooted in a very simple electromagnetics problem. To design the system, the magnetic force must be known, and to obtain this the magnetic field must be known. This is an effortless problem when the lines are considered to be infinitely long straight wires. This is done by superposition of a very basic equation that has been derived from Biot-Savart's Law to be applied to infinitely long straight wires. This equation can be solved for each of the three phases and in a matter of minutes the field and force are apparent.

The problem arises in the assumption of an infinitely long and straight system. The assumption of infinite length is not harmful assuming that the equations are applied to sections far from the end of the power line. However, the assumption of a straight wire simply is not true. The power line system presents itself as a periodic catenary due to the weight of the cable and the spacing of the supporting poles. This incredibly complicates the computation of the fields and forces that are applied to and created by the power line. As a matter of fact, the computations become so complicated that a great deal of this study was spent on deriving and validating approximations that could more easily be used to obtain an accurate result. When a correct mathematical model of the periodic catenary system is determined, then the equations for the fields and forces will be much more accurate in the study of the secondary fault. This study is dedicated to determining a reliable approximation for the power line configuration and determining the resulting fields and forces that are applied within this system.

III. Project Objectives

A. Catenary Approximation

The preliminary objective of this study is to determine an accurate approximation of the catenary shape taken on by the power line. As mentioned in the Background section, the calculations of an exact catenary equation can be very tedious. To minimize computing time and complications, an approximation for the shape must be found to be less complex yet exceptionally accurate.

B. Magnetic Field

With a proper approximation validated, the magnetic field of the system is be calculated. This is to be done by employing Biot-Savart's Law and will be taken in two major steps: the calculation of the magnetic field of one period of a single phase, and finally calculation of the field as applied by the entire system of multiple periods and multiple phases. For each of these steps a relevant equation is to be derived and evaluated for a number of positions along the given phase.

C. Magnetic Force

The determination of the magnetic force will be the final step of this study. This will be evaluated as the force on a single-phase single period catenary as applied by the entire three-phase system.

IV. Project Goal

With the determination of the magnetic force applied on a catenary section of the three phase power system, it will then be possible to design a power system which will be much less likely to create a secondary fault within the system.

V. Methods

A. *Original Catenary and Catenary Approximation*

To discuss the exact catenary equations, it is necessary to define a number of terms that are used to represent the system. The following terms will be used for the study and can also be seen in Figure 1:

Table 1: Term Definitions

Term	Definition
L	Length of Catenary (Pole-Pole Distance)
H	Maximum Catenary Height (Pole Height)
h	Minimum Catenary Height (Lowest Wire Point)
s	Sag of Catenary (H-h)

Figure 1: Axes and Terms

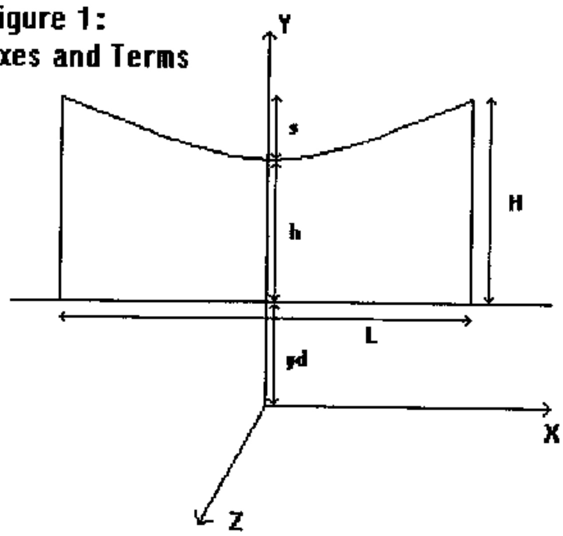


Figure 1 also displays the orientation of the axes that will be used. With these terms and orientations, the basic catenary centered on the y-axis is given by the following equations:

$$y' = \frac{1}{a}\cosh(ax) \qquad a = \frac{1}{h + y_d}$$

The term 'y_d' is the distance between the coordinates' origin and the ground. This is necessary because as the pitch of the catenary is changed, the origin must be adjusted with respect to the y-axis to accommodate for the height of 'H' and 'h'. This presents the first problem in the calculation of the original catenary equation. In this study the term

'y_d' was determined by an iteration method utilizing Matlab V5.1. The following equation was used to solve for the term:

$$\frac{H + y_d}{h + y_d} - \cosh\left(\frac{L}{2(h + y_d)}\right) = 0$$

With the knowledge of 'y_d', the equation given for the basic catenary can then be adapted to account for the distance from the origin to the ground. The following equation was used to graph the original equation version of catenaries for a number of different values:

$$y = \frac{1}{a}\cosh(ax) - y_d$$

The determination of y_d is a difficult task that complicates the use of the original equation. This is one reason why the approximation for the catenary is sought after. The approximation that was used and validated in this study takes into account the sag and the length rather than the origin of the axis. The following simple parabola was used as the approximation to be tested:

$$y = s\left(\frac{2x}{L}\right)^2 + h$$

In this study the approximation equation was plotted and compared with the original catenary equation to determine the validity of the parabolic approximation. Each of these plots was carried out in Microsoft Excel '97. This approximation, when deemed accurate, has already simplified the calculations by eliminating the need for the term 'y_d'.

B. Magnetic Fields

i. Single Period, Single Phase

With the validation of the catenary approximation, the magnetic field of a single period of a single phase was determined. The equation for the original magnetic field was used as a guideline to derive an equation for the approximation's magnetic field. Employing Biot-Savart's Law, the following is the equation for the magnetic field with the use of the original and more complicated catenary equation:

$$d\mathbf{l} = dx\left(\frac{dy}{dx}\mathbf{a}_y + \mathbf{a}_x\right) = dx\left(\sinh(ax)\mathbf{a}_y + \mathbf{a}_x\right)$$

$$\mathbf{r}_o = (x_o - x)\mathbf{a}_x + \left(y_o - \left(\left(\frac{1}{a}\right)\cosh(ax) - y_d\right)\right)\mathbf{a}_y + z_o\mathbf{a}_z$$

$$\therefore \mathbf{B} = \frac{\mu_o \mathbf{I}}{4\pi}\int_{-\frac{L}{2}}^{\frac{L}{2}}\left[\frac{z_o \sinh(ax)}{d}\mathbf{a}_x - \frac{z_o}{d}\mathbf{a}_y - \frac{\frac{\cosh(ax)}{a} - (y_o + y_d) - (x - x_o)\sinh(ax)}{d}\mathbf{a}_z\right]dx$$

$$d = \left[(x - x_o)^2 + z_o{}^2 + (\cosh(ax)/a - (y_o + y_d))^2\right]^{\frac{3}{2}}$$

This equation for the magnetic field was evaluated using Maple V Integration Software. The results of this were plotted and were used to determine the validity of the approximation equation for the magnetic field. The following values were used for each of these plots:

Table 2: Parameter Values

Term	Value Used	Term	Value Used
L	100 m	Y_d	614.333
H	13 m	a	.0015991
h	11 m	I	100 A
S	2 m		

The approximation equation for the catenaries was then used to derive an equation for the magnetic field. These derivations were based on the original derivation as given above. The following equations show the results of the approximation of the magnetic field:

$$d\mathbf{l} = dx\left(\frac{dy}{dx}\mathbf{a}_y + \mathbf{a}_x\right) = dx\left(\frac{8sx}{L^2}\mathbf{a}_y + \mathbf{a}_x\right)$$

$$\mathbf{r}_o = (x_o - x)\mathbf{a}_x + \left(y_o - \left(\frac{4sx^2}{L^2} + h\right)\right)\mathbf{a}_y + z_o\mathbf{a}_z$$

$$\therefore \mathbf{B} = \frac{\mu_o \mathbf{I}}{4\pi}\int_{-\frac{L}{2}}^{\frac{L}{2}}\left[\frac{\frac{8sx}{L^2}z_o}{d}\mathbf{a}_x - \frac{z_o}{d}\mathbf{a}_y - \frac{y_o - \frac{4sx^2}{L^2} - h - \frac{8sx}{L^2}(x_o - x)}{d}\mathbf{a}_z\right]dx$$

$$d = \left[(x - x_o)^2 + z_o{}^2 + \left(y_o - \frac{4sx^2}{L^2} - h\right)^2\right]^{\frac{3}{2}}$$

This approximation was also plotted with respect to the x-axis and the results were compared with the original equation. With this approximation validated, the equation could then be expanded to accommodate for the multiple sections and phases that effect the single-phase power line.

ii. Multiple Periods, Multiple Phases

With the derivation of the single period, single phase equation for magnetic field, the study was then focused on determining the magnetic field as applied by the entire power line system. This involves the adjacent catenaries of the analyzed section along with the remaining two phases. To account for the effect of the remaining phases and their adjacent catenaries, yet another equation was derived to determine the magnetic field. The derivation is based on Biot-Savart's Law and is given below:

$$y = \frac{4s}{L^2}(x - kL)^2 + h$$

$$d\mathbf{l} = dx\left(\frac{dy}{dx}\mathbf{a}_y + \mathbf{a}_x\right) = dx\left(\left(\frac{8sx}{L^2} - \frac{8sk}{L}\right)\mathbf{a}_y + \mathbf{a}_x\right)$$

$$\mathbf{r}_o = (x_o - x)\mathbf{a}_x + \left(y_o - \left(\frac{4s}{L^2}(x - kL)^2 - h\right)\right)\mathbf{a}_y + (z_o + id)\mathbf{a}_z$$

$$\therefore \mathbf{B} = \sum_{i=-1}^{i=1}\sum_{k=-4}^{k=4}\frac{\mu_o \mathbf{I}}{4\pi}\int_{-\frac{L}{2}+Lk}^{\frac{L}{2}+Lk}\left[\begin{array}{l}\dfrac{\left(\frac{8sx}{L^2} - \frac{8sk}{L}\right)(z_o - id)}{d}\mathbf{a}_x - \dfrac{(z_o - id)}{d}\mathbf{a}_y + \\ \dfrac{y_o - \frac{4s(x-kL)^2}{L^2} - h - \left(\frac{8sx}{L^2} - \frac{8sk}{L}\right)(x_o - x)}{d}\mathbf{a}_z\end{array}\right]dx$$

$$d = \left[(x_o - x)^2 + (z_o - id)^2 + \left(y_o - \frac{4s}{L^2}(x - kL)^2 - h\right)^2\right]^{\frac{3}{2}}$$

In this equation, the term 'k' is used to step through the adjacent sections of each phase. With this term, Biot-Savart's law is applied to the analyzed section and four adjacent sections in either direction. The term 'i' refers to the phase that is being examined. This term is evaluated from -1 to 1 to account for the contribution of all three phases. The term 'd' refers to the distance between each phase. This equation is an adaptation to the previous derivation which allows the z coordinate to be toggled to account for each of the phases and the *y* parameters to be adapted to calculate the field as applied by adjacent sections. These sections are then added and the total system magnetic field is determined via superposition.

This equation was then used to determine the magnetic field at one catenary section due to the entire system. This sum and integration was carried out on Maple V integration software. Each of these results was then plotted to display the distribution of the field along the x-axis. These calculations and plots were then repeated for the remaining two phases.

C. Magnetic Forces

With the determination of the system magnetic field, the project goal was one step closer to being completed. It was then necessary to determine the resulting force applied on the power lines. This information will be useful in preventing the occurrence of a secondary power fault. To calculate the force applied on the system, the following equation was used:

$$\mathbf{F} = \mathbf{I}\int d\mathbf{l} \times \mathbf{B}$$

The integrand of this equation was computed in the determination of the magnetic field. The derivation of this equation must take into account the fact that the magnetic field and the current are each complex numbers and must be multiplied accordingly. The following is the derivation of the equation for the force applied on one catenary section of the system:

$$d\mathbf{l} = dx\left(\frac{dy}{dx}\mathbf{a}_y + \mathbf{a}_x\right) = dx\left(\frac{8sx}{L^2}\mathbf{a}_y + \mathbf{a}_x\right)$$

$$\mathbf{B} = \mathbf{B}_x\mathbf{a}_x + \mathbf{B}_y\mathbf{a}_y$$

$$\mathbf{F} = \mathbf{I}\int_a^b d\mathbf{l} \times \mathbf{B} = \mathbf{I}\mathbf{a}_z\left(\mathbf{B}_x \frac{-8s}{2L^2} x^2\Big|_a^b + \mathbf{B}_y\right)$$

$$\mathrm{Re}\{\mathbf{F}\} = \mathrm{Re}\{\mathbf{I}\}\mathbf{a}_z\left(\mathrm{Re}\{\mathbf{B}_x\}\frac{-8s}{2L^2} x^2\Big|_a^b + \mathrm{Re}\{\mathbf{B}_y\}\right) - \mathrm{Im}\{\mathbf{I}\}\mathbf{a}_z\left(\mathrm{Im}\{\mathbf{B}_x\}\frac{-8s}{2L^2} x^2\Big|_a^b + \mathrm{Im}\{\mathbf{B}_y\}\right)$$

$$\mathrm{Im}\{\mathbf{F}\} = \mathrm{Re}\{\mathbf{I}\}\mathbf{a}_z\left(\mathrm{Im}\{\mathbf{B}_x\}\frac{-8s}{2L^2} x^2\Big|_a^b + \mathrm{Im}\{\mathbf{B}_y\}\right) + \mathrm{Im}\{\mathbf{I}\}\mathbf{a}_z\left(\mathrm{Re}\{\mathbf{B}_x\}\frac{-8s}{2L^2} x^2\Big|_a^b + \mathrm{Re}\{\mathbf{B}_y\}\right)$$

In this equation, the divisions 'a' and 'b' are not the entire catenary section. In the determination of the magnetic field, the values were determined for specific points in two meter sections. Since the magnetic field is a complex integration, it must be assumed that the field is constant for a short area around the evaluated point. For example, if the magnetic field were calculated for x=5.1, it is assumed constant near this point and the force is determined for a=4.1 and b=6.1. This was done for the entire length of the catenary and the sections were summed to determine the total force applied on the catenary section.

In this study, the magnetic force was determined for the entire length of a single period single-phase catenary. This was determined using Maple V and the results were plotted.

VI. Results

A. *Original Catenary and Catenary Approximation*

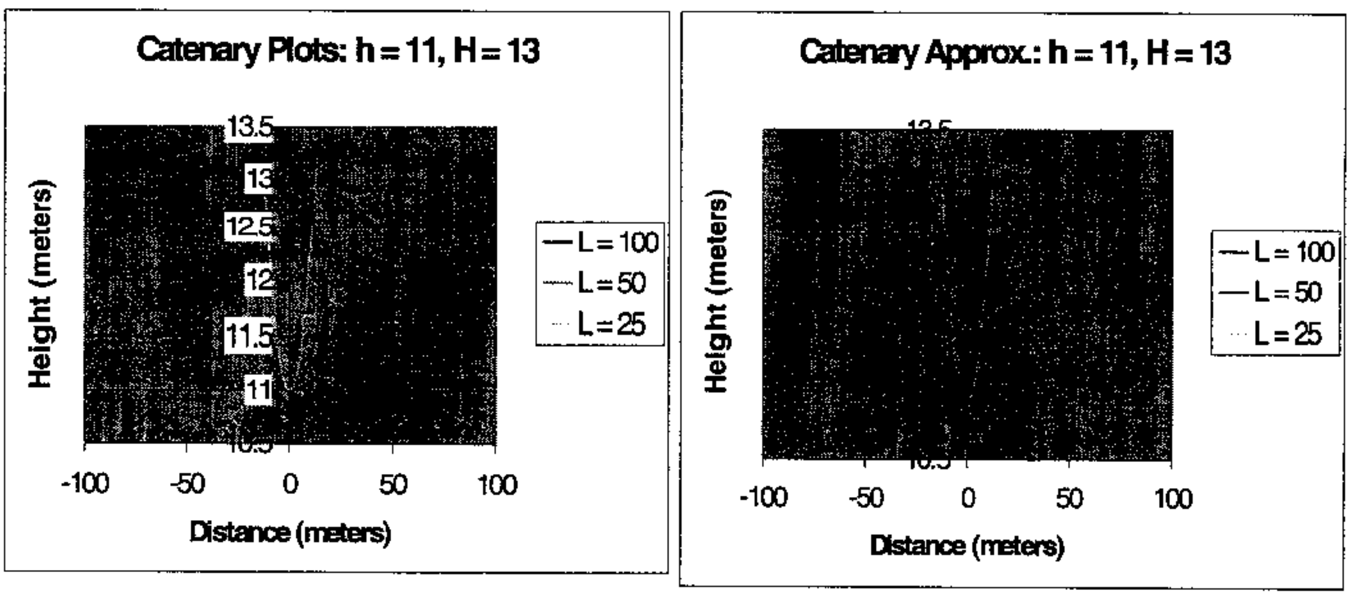
The preliminary step of this study was to validate the approximation for the catenary shape. To accomplish this, the catenary was plotted utilizing the original equation and the approximation equation and the results were compared. To plot the original equation, it was first necessary to determine the value of 'y_d' for the given parameters. This was done by way of iteration on the Matlab software. The following table displays the parameters that were chosen and the corresponding 'y_d' that was found:

Table 3: y_d Results

Term	Value Used	Term	Value Used
L	100 m	Y_d	614.333
H	13 m	a	.0015991
h	11 m	I	100 A
S	2 m		

These values were used to plot the original equation, and the approximation equation was also plotted. The results of these plots can be seen in Figure 2. These plots were created using the same parameters and were expected to produce nearly the same results.

Figure 2: Catenary and Approximation Plots



B. Magnetic Field

i. Single Section, Single Phase

With the derivation of the approximation equation for the magnetic field, the results were determined for a number of points near the catenary. These integrals were performed on the Maple V integration software and were plotted against the x-axis. The resulting plots can be seen in Figure 3. These plots represent the magnetic field along the line as produced by the single catenary section. As can be seen, the plots resulted in a symmetric field as expected. The field calculations were carried out for a number of sag values to determine how the field as applied by one section of wire will vary as the tension in the line is adjusted.

ii. Multiple Section, Multiple Phase

The final step in determining the magnetic field is to consider the effects of the entire system. To examine this, the equation that was derived for the entire system was evaluated for a number of points near the examined catenary. The results of these evaluations were plotted against the x-axis and can be seen in Figure 4. Figure 4 shows the x-directed fields that resulted from considering the field along Phase A ($I=100A \angle 120°$) at time $t=0$. This was carried out for the x and y-directed components of the field for all three phases. The additional results from these calculations can be seen in Appendix A. It should be noted that for this step only the x and y directed components of the fields were evaluated. This is due to the nature of the secondary fault. With the given orientation of the system, the phases are separated with respect to the z axis. To study the forces between these phases, the z directed force is desired, which corresponds to the x and y directed components of the magnetic field as given by the right-hand rule.

C. Magnetic Force

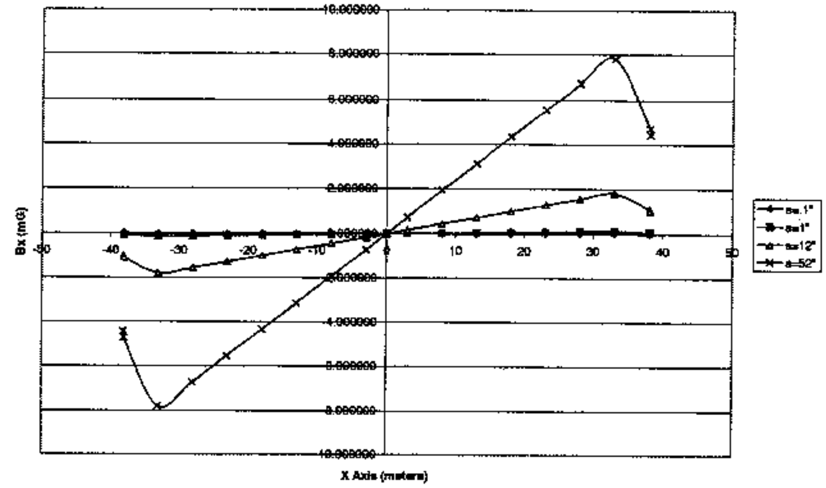
After the determination of the magnetic field, the magnetic force was then examined. The equation that was derived for the force was evaluated for intervals along the entire catenary section. These values were plotted to examine the results. This plot can be seen in Figure 5. This figure is used to determine the force distribution along the section. The values that were plotted are the magnetic forces as applied by the entire system on a two meter section of one phase at time $t=0$. The forces on each two meter section were also summed to determine the total force applied on the catenary section by the system:

Table 4: Total Magnetic Force (N)

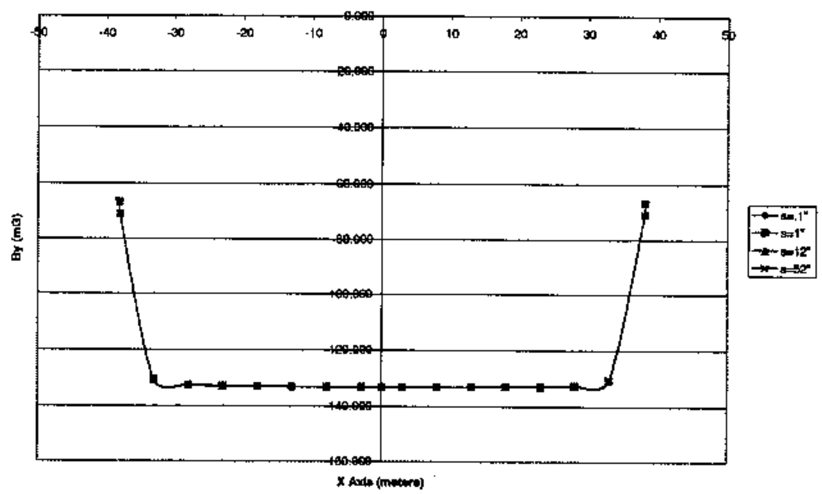
Fz - Phase A	Fz - Phase B	Fz - Phase C
0.033826784	0.052644394	0.086471178

Figure 3: Magnetic Field (Single Section, Single Phase)

Single Section, Single Phase: L=76.2 m, I=100 A, H=10.668 m, Zo=1.5 m



Single Section, Single Phase: L=76.2 m, I=100A, H=10.668 m, Zo=1.5 m



Single Section, Single Phase: L=76.2 m, I=100A, H=10.668 m, Zo=1.5 m

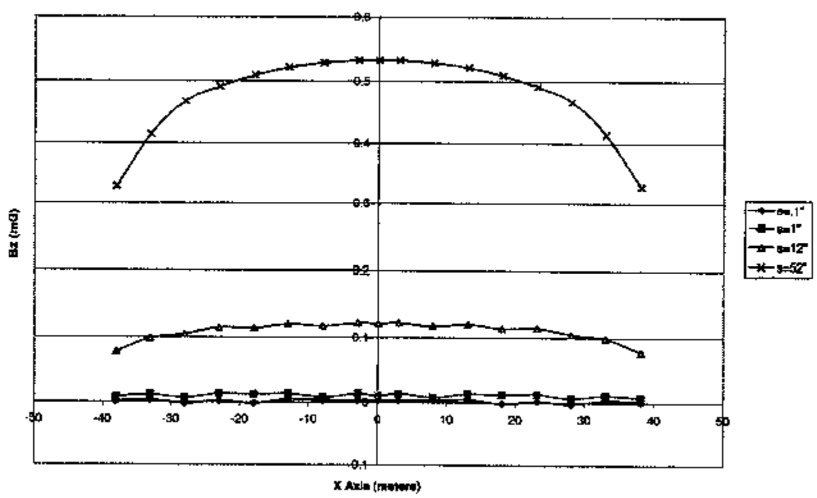
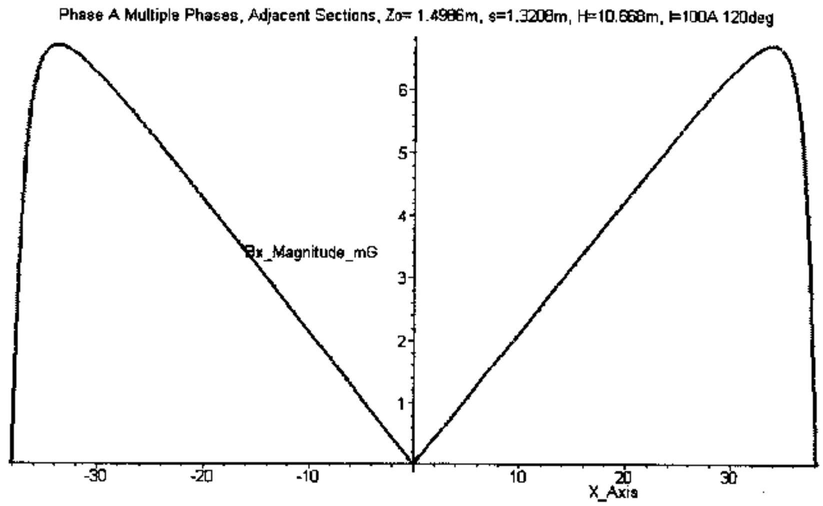


Figure 4: *X*-Directed Magnetic Field of Phase A (Multiple Sections, All Phases)



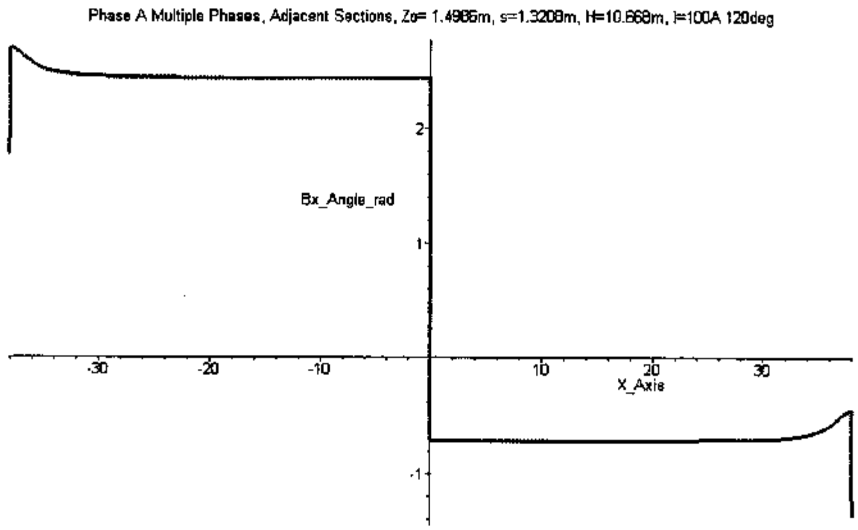
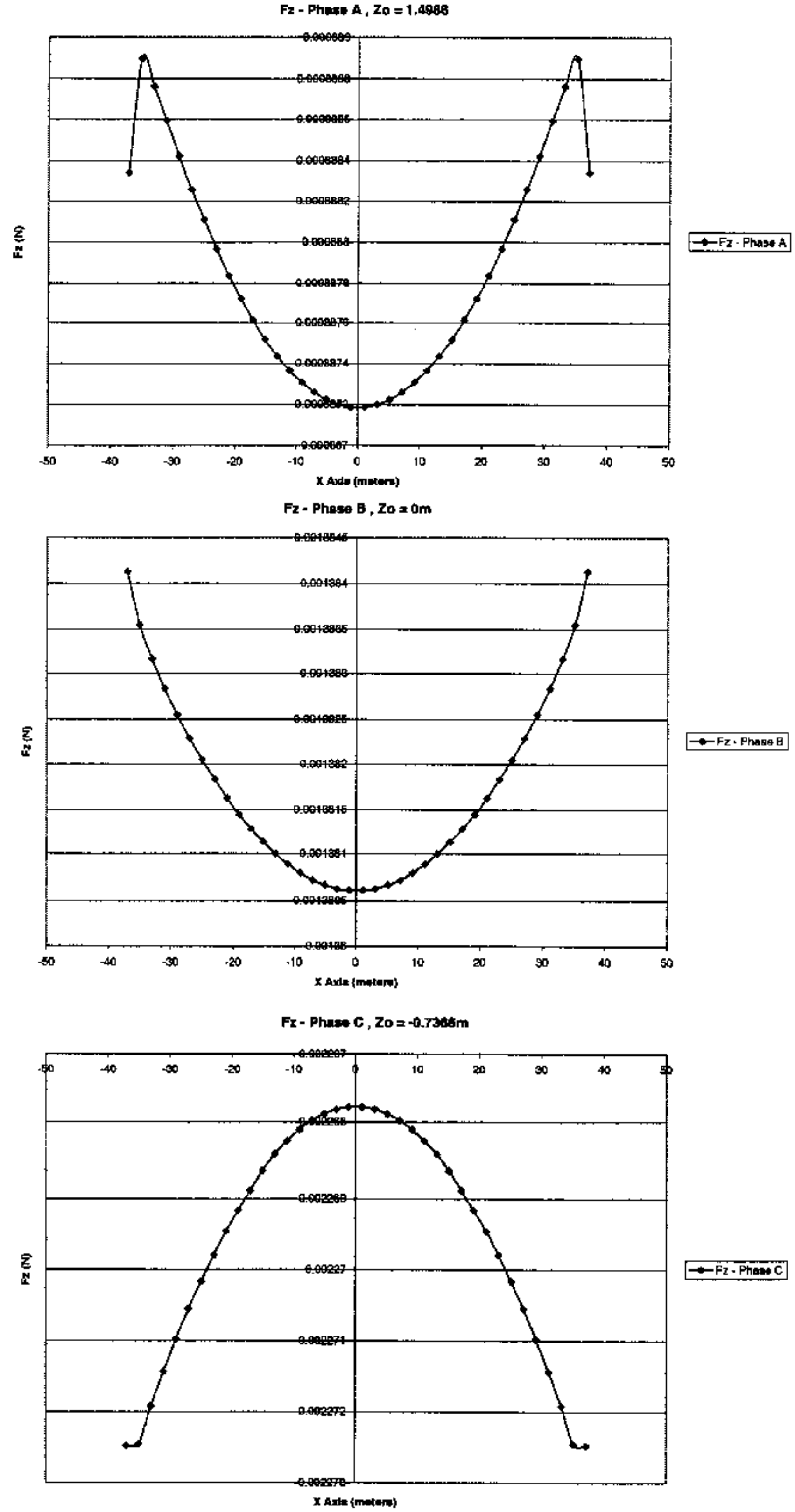


Figure 5: Force Applied on Single Catenary Section



VII. Discussion and Analysis

A. Catenary Approximation

The parabolic approximation that was used in this study was proven to be accurate. As seen in Figure 2, the approximation is nearly indistinguishable from the original catenary equation. For the six catenaries that were tested, it was found that the percent error of the approximation was never exceeding .035%. This verifies the approximation and allows the approximation to be used for the calculation of the magnetic field. This is a very positive improvement in that the original equation was particularly difficult to use for the field calculations. When evaluated on the Maple V integration software, it was found that the approximation equation for the magnetic field took approximately one third of the computing time that was needed to compute the field using the original equation. With the graph that appears in Figure 2, the catenary approximation is validated and the simpler equation is deemed an accurate representation.

B. Magnetic Fields

The graphs produced for the magnetic fields were as expected for each instance. Each plot was checked with the expected value with respect to the right-hand-rule of cross products. As more of the system was considered (more sections, more phases), the relationships stayed relatively stable and the values slightly increased. This would suggest that the adjacent catenaries and other phases have a considerable effect on the catenary, all of which is directed similarly to the field produced by the section that is being examined.

C. Magnetic Force

The calculation of the magnetic force applied on the system yielded a symmetric result as expected. In Figure 5, it can be seen that the force is applied in a parabolic shape with respect to the x-axis. This also shows that the greatest force on the catenary will be applied in the center of Phase C for this evaluation at time $t=0$. This will prove to be the point that should be studied when considering the secondary fault of the power system. This center point bears the largest force and also has the greatest freedom of motion along the catenary. This force distribution must be evaluated for a full time cycle of the three-phase current present in the system to determine the most volatile point with respect to the line orientation and the current values.

VIII. Future Calculations

The results that have been presented in this report will be utilized to determine the forces present within the three-phase system throughout an entire time cycle of the AC current. The techniques and computer programs that are developed through this activity will then be used to determine the forces that exist in the presence of a primary fault. The parameters that were used to produce the presented results will be altered to model a number of systems that have been subjected to a primary fault to evaluate the probability of a secondary fault. New equations will also be developed to describe the angular acceleration and velocity of these lines that are pulled into contact with one another.

IX. Conclusion

It can be concluded that this study has been a success. The project objectives that have been discussed have been completed and each of the results appears to agree with application of the right-hand rule. The following conclusions have been made about the research that has been presented in this report:

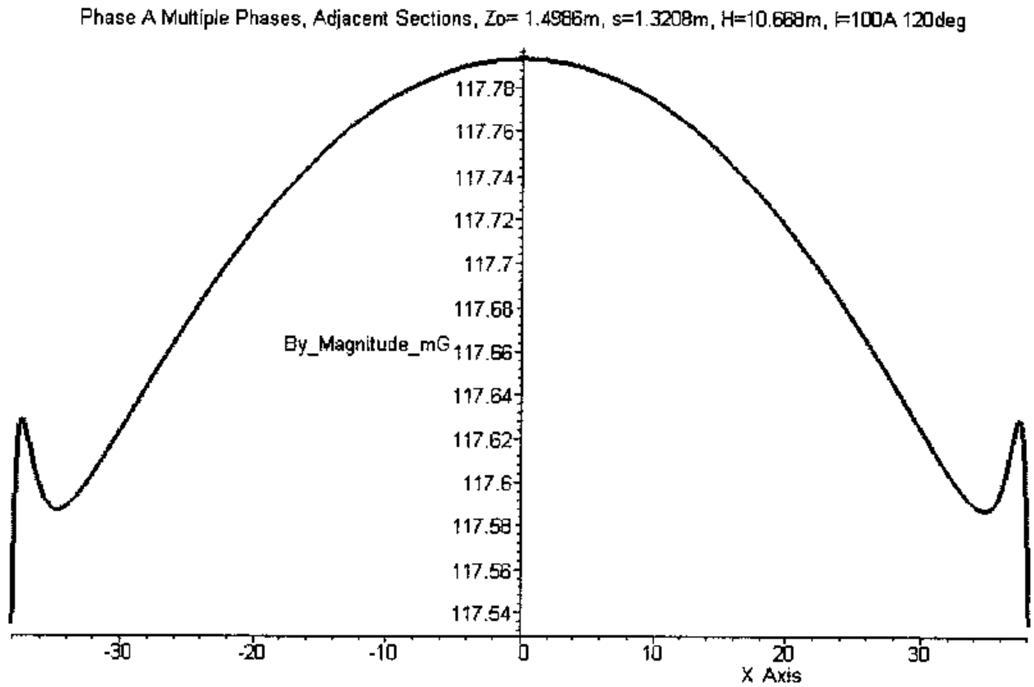
- The approximation for the catenary equation is an accurate representation of the catenary power line system.
- The adjacent periodic sections and the other phases of the power line system have a considerable effect on the magnetic field experienced by a given section of the line.
- The magnetic force applied on any phase of the three-phase system is applied in a parabolic shape with respect to the x-axis. This force is greatest in the center of Phase C (100A $\angle$-120°) of the catenary at time $t=0$ and is a result of the x and y directed components of the magnetic field.

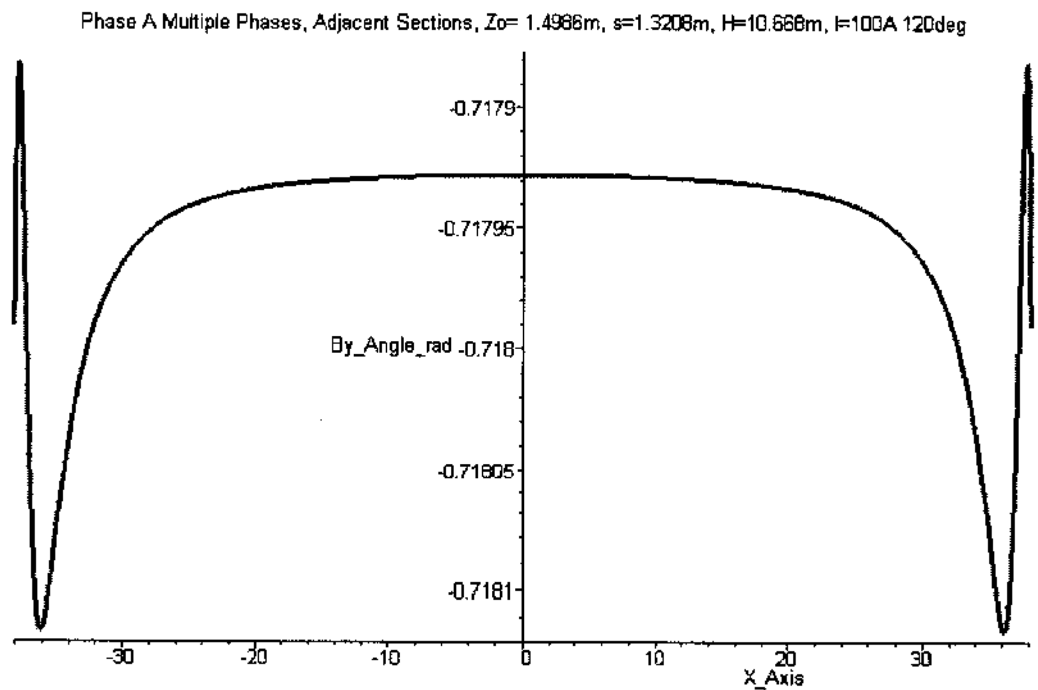
With the results and conclusions in this study, the power system can now be examined more closely to prevent the secondary fault within the system. With the knowledge of the force applied on the section, the effects of a change in current magnitude or direction can be examined.

X. References

[1] A.V. Mamishev and R.D. Nevels, "Effects of Conductor Sag on Spatial Distribution of Power Line Magnetic Field", IEEE Transactions on Power Delivery, Vol. II, No. 3, pp1571-1576, July 1996.

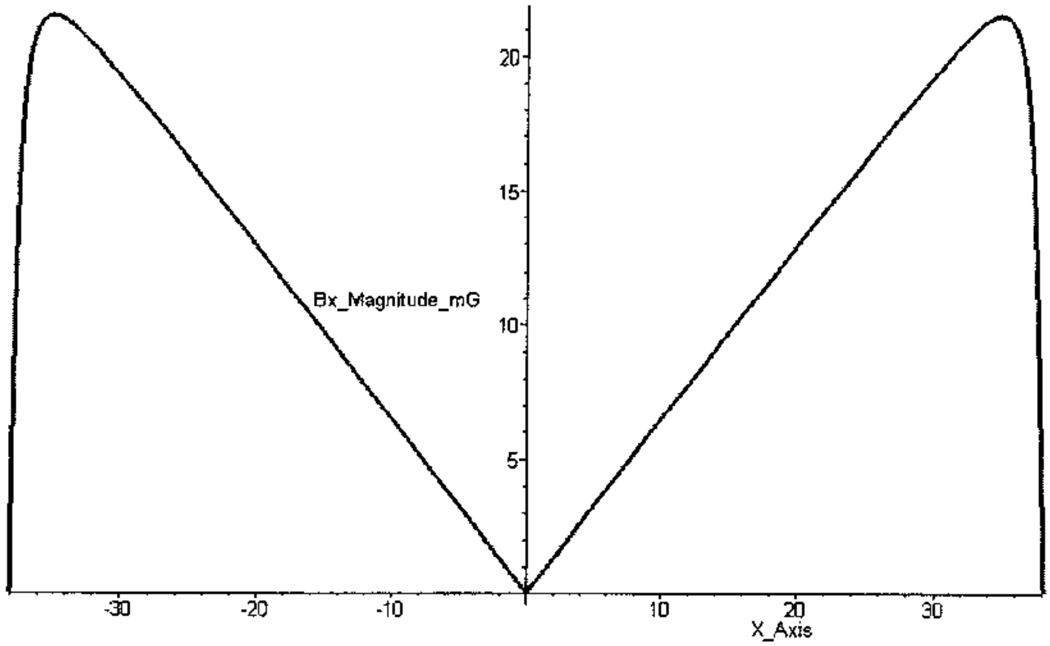
Appendix A: Magnetic Field (Multiple Sections, All Phases)



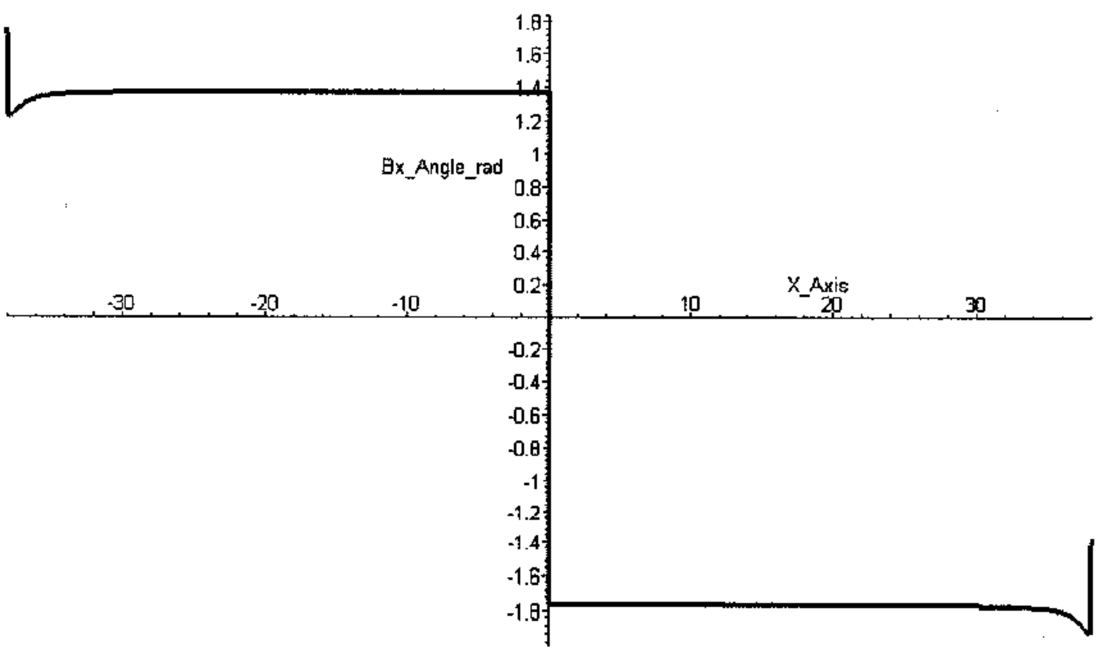


Appendix A: **Magnetic Field (Multiple Sections, All Phases)**

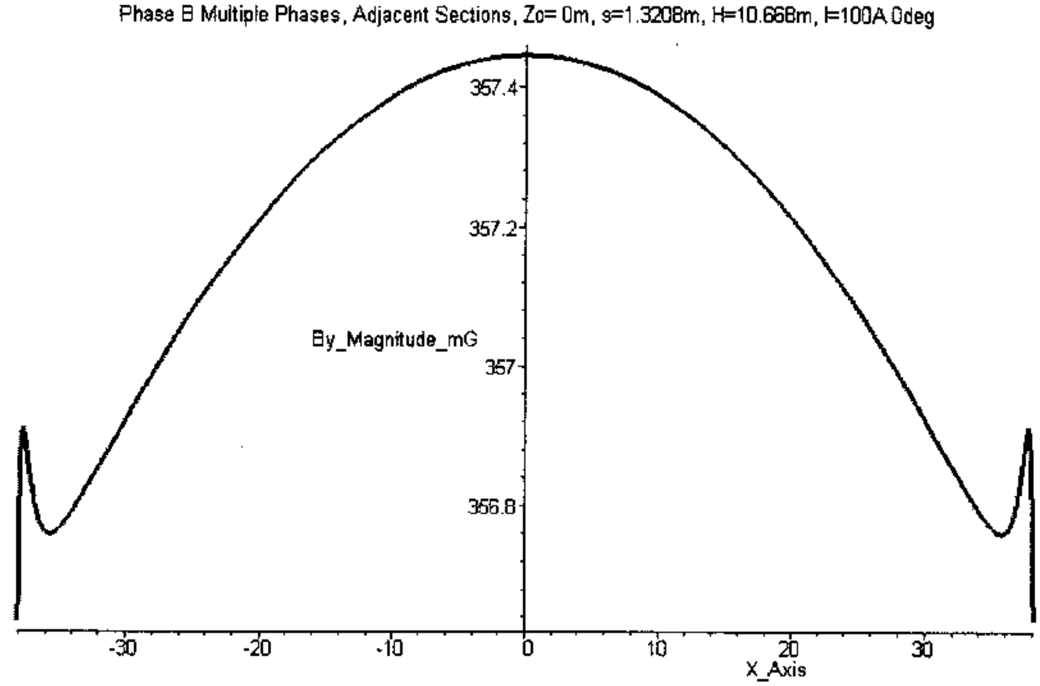
Phase B Multiple Phases, Adjacent Sections, Zo= 0m, s=1.3208m, H=10.668m, I=100A 0deg

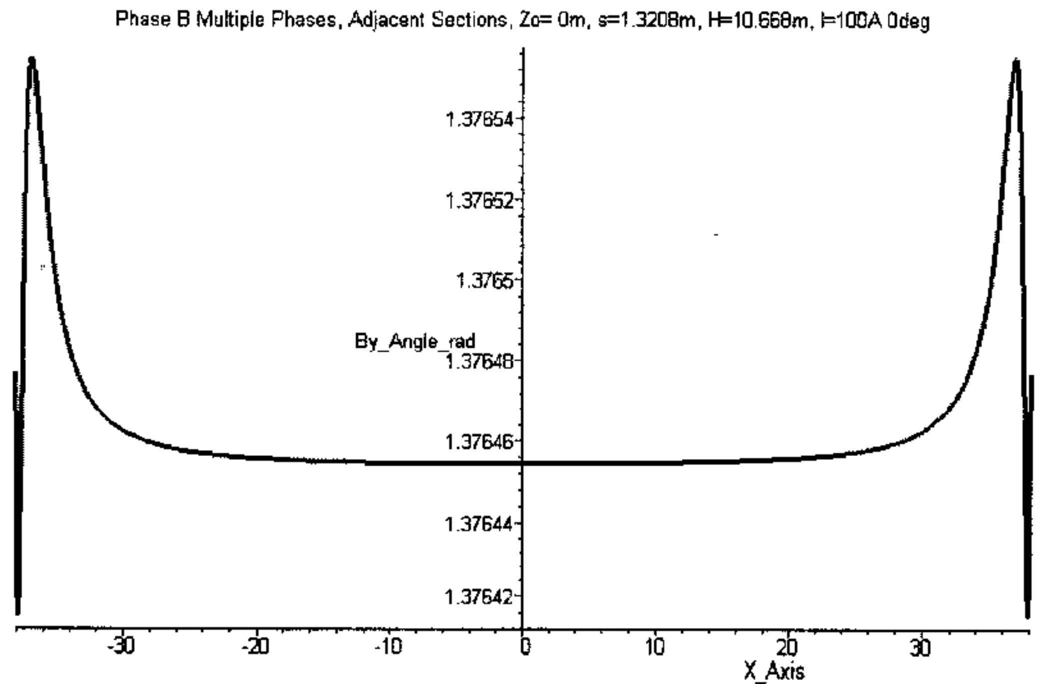


Phase B Multiple Phases, Adjacent Sections, Zo= 0m, s=1.3208m, H=10.668m, I=100A 0deg

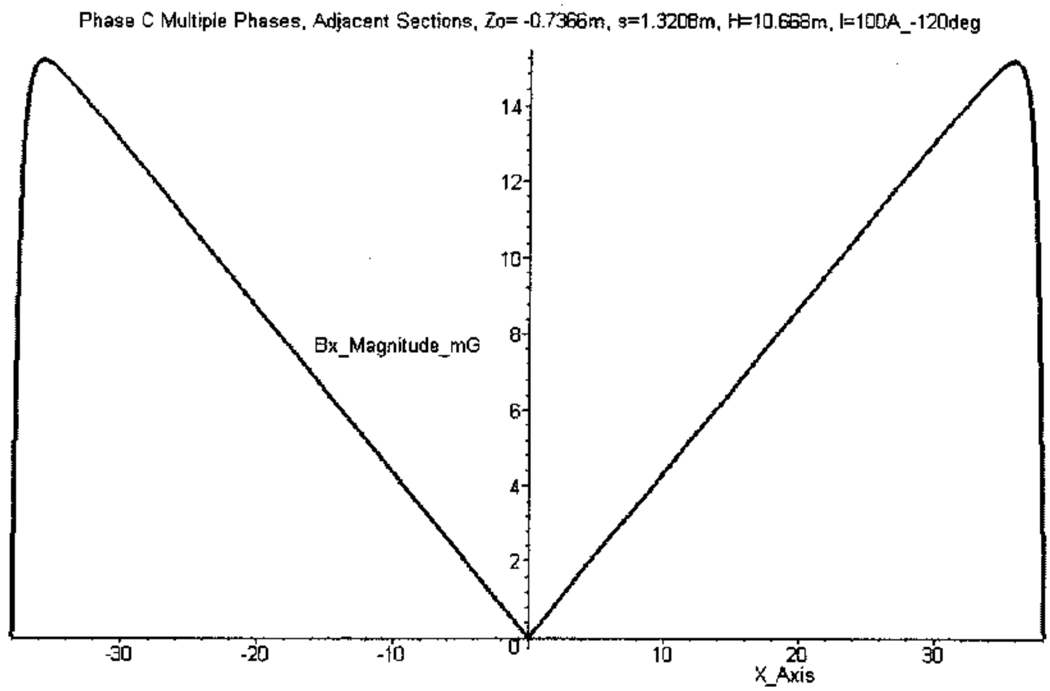


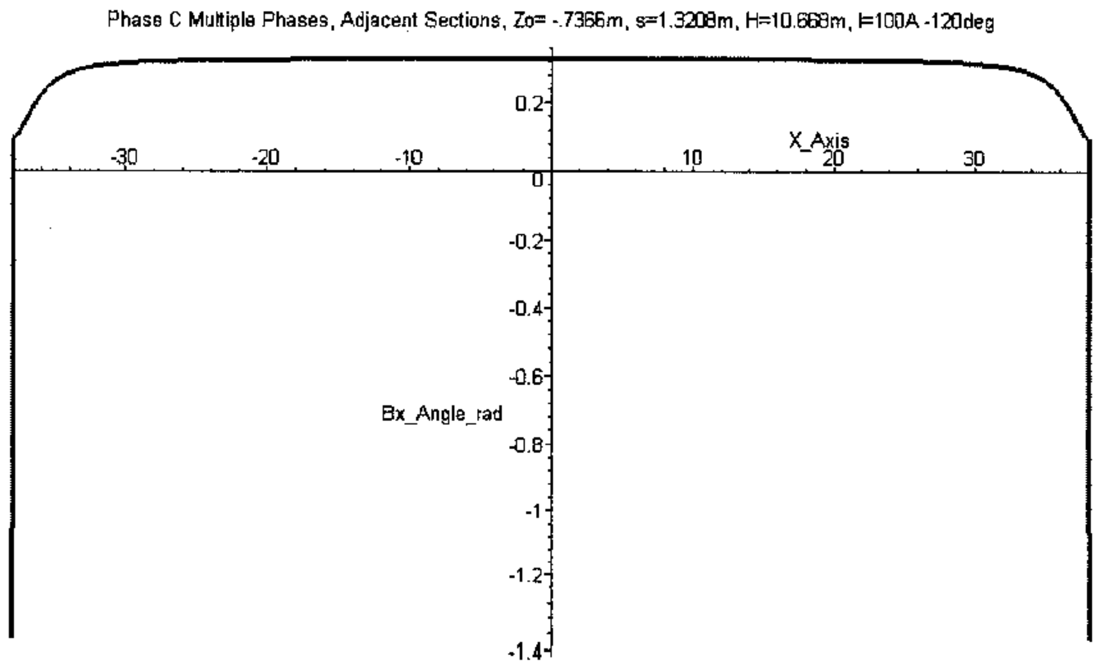
Appendix A: **Magnetic Field (Multiple Sections, All Phases)**





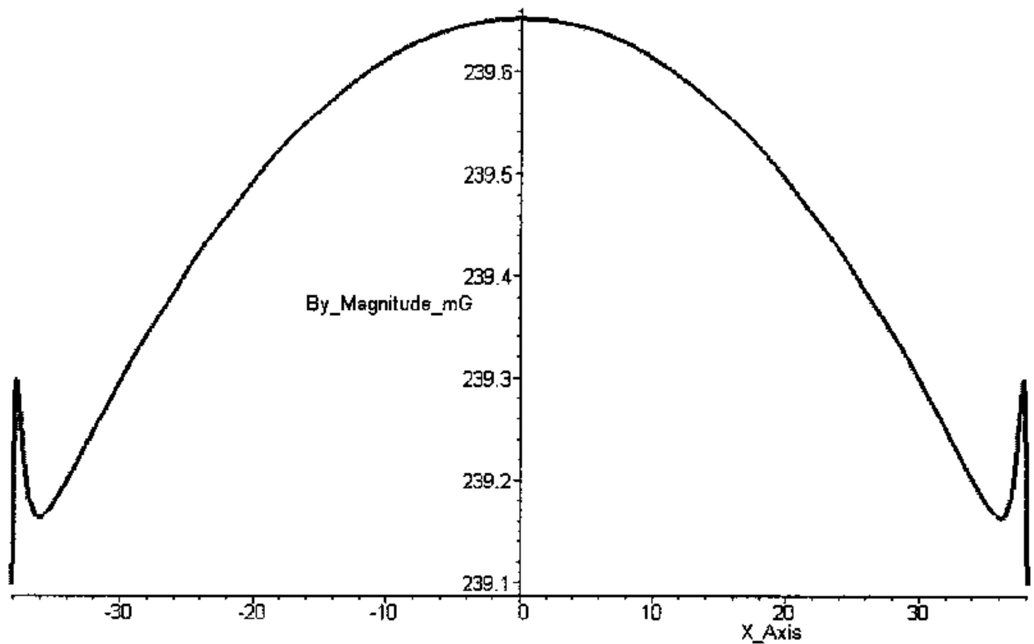
Appendix A: Magnetic Field (Multiple Sections, All Phases)





Appendix A: Magnetic Field (Multiple Sections, All Phases)

Phase C Multiple Phases, Adjacent Sections, Zo= -0.7366m, s=1.3208m, H=10.668m, I=100A_-120deg



Phase C Multiple Phases, Adjacent Sections, Zo= -.7366m, s=1.3208m, H=10.668m, I=100A -120deg