# AP ${ }^{\circledR}$ STATISTICS SECTION I <br> Time --- 1 hour and 30 minutes <br> Number of questions --- 40 <br> Percent of total grade --- 50 

Directions: Solve each of the following problems, using the available space for scratch work. Decide which is the best of the choices given and fill in the corresponding box on the answer sheet. Do not spend too much time on any one problem.

1. Academia High School collected heights of students in their intramural basketball league. The boxplot and statistics below provide a summary of the distribution of heights, in inches.


| Mean | Std Dev | Minimum | Q1 | Median | Q3 | Maximum |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 68.2 | 4.21 | 62 | 65 | 67 | 71 | 79 |

Which conclusion about the distribution is most plausible?
(A) $50 \%$ of the students are taller than 68.2 inches.
(B) $75 \%$ of the students are taller than 71 inches.
(C) There are more students between 68.2 inches and 79 inches than are between 62 inches and 68.2 inches.
(D) Less than $25 \%$ of the students have heights between 68.2 and 71 inches.
(E) The height that occurs most frequently is 67 inches.
2. Researchers conducting a study of the effectiveness of Tamiflu for reducing the duration of flu symptoms recruited 50 subjects who agreed to report to the researchers' lab within 24 hours of getting flu symptoms. Half of the subjects were randomly assigned to take a full dosage of Tamiflu and the other half received a placebo. The time (in hours) to alleviation of all flu symptoms for the two groups are compared in the back-to-back stemplot below.


Key: $15 \mid 6=156$ hours
Which of the following statements is true about the shapes of the distributions of recovery times?
(A) The distributions of recovery times for the Tamiflu and placebo groups are both symmetric.
(B) The distributions of recovery times for the Tamiflu and placebo groups are both skewed to the left.
(C) The distributions of recovery times for the Tamiflu and placebo groups are both skewed to the right.
(D) The distribution of recovery times for the Tamiflu group is skewed to the left, and the distribution of recovery times for the placebo group is skewed to the right.
(E) The distribution of recovery times for the Tamiflu group is skewed to the right, and the distribution of recovery times for the placebo group is skewed to the left.
3. Last week, Mrs. Thomas, the AP Statistics teacher at Green Valley High School, gave her students an assessment on material they had recently studied. The students' scores on the assessment followed an approximately normal distribution with mean 83.54 and standard deviation 11.21. Andrew, a student in Mrs. Thomas' class, had a standardized score of $z=0.88$. What was Andrew's score on the test, and in which quarter of the distribution would Andrew's score fall?
(A) 73.68 ; his score would fall in the first quarter because it is below the first quartile
(B) 73.68; his score would fall in the second quarter because it is between the first quartile and the median
(C) 84.42; his score would fall in the fourth quarter because it is above the third quartile
(D) 93.40; his score would fall in the fourth quarter because it is above the third quartile
(E) 93.40; his score would fall in the first quarter because it is below the first quartile
4. A study was conducted to test the effect of product price and display level on supermarket sales. At one supermarket, the price level (regular, reduced price, and at cost to supermarket) and the display level (normal display space, normal display space plus end of aisle display, and twice the normal display space) were varied to determine if they had any effect on the weekly sales of a particular supermarket product. Each combination of price and display was recorded. Each combination was used three times over the course of the experiment. Identify the treatments used in this experiment.
(A) The three price levels used by the supermarket
(B) The three display levels used by the supermarket
(C) The supermarket
(D) The weekly sales for each of the weeks
(E) The nine combinations of price and display levels used by the supermarket
5. Consider the following residual plot for the least squares regression line of $y$ versus $x$.


Which of the following is an appropriate conclusion based on the residual plot?
(A) There is a pattern in the residual plot, so there will not be a linear trend in the scatterplot of $y$ versus $x$.
(B) There is a strong, positive linear relationship between $x$ and $y$ because there is no pattern in the residual plot.
(C) Because there is no pattern in the residual plot, there will be a linear trend apparent in the scatterplot of $y$ versus $x$.
(D) There will be a linear trend apparent in the scatterplot of $y$ versus $x$, but the data values will tend to fall closer to the regression line for smaller values of $x$ than for larger values of $x$.
(E) Because none of the residuals equal zero, the least squares regression line will not be useful for prediction.
6. Before opening a new restaurant, a chef wants to gather information about the eating habits of the local residents. He randomly selects 1000 households from all households in the area and mails a questionnaire to them. Of the 1000 surveys mailed, he receives 150 back. Which of the following is the most obvious concern with how this information is gathered?
(A) The 1000 selected households are not a simple random sample of households.
(B) Mailing questionnaires instead of conducting in-person interviews produces a convenience sample.
(C) Those who chose to respond to the survey may have different eating habits from those who did not respond.
(D) Only residents from the local area were polled.
(E) The chef must conduct a census in order to avoid all sources of bias.
7. Does the color of cards affect a person's ability to sort the cards? Twelve volunteers were asked to sort two sets of 200 cards into two piles. One set of cards were bright red and bright green. The other set of cards were black and white. The order (red/green first or black/white first) was determined by a coin toss with ample time between the two tasks to lessen the effect of practice. The time (in seconds) to complete each of the two tasks are given in the table below.

## Completion time (in seconds)

| Volunteer | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Red/Green | 148 | 198 | 193 | 128 | 206 | 158 | 214 | 167 | 207 | 173 | 190 | 139 |
| Black/White | 136 | 182 | 186 | 133 | 198 | 162 | 196 | 163 | 199 | 161 | 183 | 144 |

Assuming the conditions for inference are met, what statistical test should be used to determine if there is a significant difference in the mean time required to sort the two sets of cards?
(A) A chi-square goodness-of-fit test
(B) A chi-square test of independence
(C) A matched-pairs $t$-test for means
(D) A two-sample $t$-test for means
(E) A linear regression $t$-test
8. Consider a tack like the one pictured below. When dropped on a table, the tack can land either point down or point up.


Suppose we assume that $\mathrm{P}($ tack lands point down $)=0.40$. Which of the following must be true?
(A) If the tack is dropped 50 times and lands point down only 10 times, then it must land point down more than $40 \%$ of the time in the next 50 drops.
(B) If the tack is dropped on a table 10 times, it will land point down 4 times.
(C) If the tack is dropped on a table $1,000,000$ times, it will land point down 400,000 times.
(D) Over many tack drops, the proportion that land point down will be very close to 0.40 .
(E) The tack landed point down twice in a simple random sample of 5 people who dropped it on a table.
9. Let $X$ be a random variable that has a skewed right distribution with mean $\mu=10$ and standard deviation $\sigma=10$. Which of the following histograms could display the distribution of the sample mean $\bar{x}$ from many random samples of size 400 from this population?

(B)


(D)

(E)

10. Beatrice is graduating from college this semester and has received two job offers, one in city A and one in city B. To help her decide where to move, she would like to estimate the difference in average home prices for these two cities. From a random sample of 40 listings in city A the average home price is $\$ 190,000$ and from a random sample of 40 listings in city B the average home price is $\$ 185,000$. She finds that the margin of error for a $95 \%$ confidence interval for the difference in average home price between the two cities $(\mathrm{A}-\mathrm{B})$ is $\$ 3,250$. Which of the following conclusions is correct?
(A) Beatrice can be $95 \%$ confident that the average home price is between $\$ 1,750$ and $\$ 8,250$ more in city A than in city B .
(B) Beatrice can be $95 \%$ confident that the average home price is between $\$ 1,750$ and $\$ 8,250$ less in city A than in city B.
(C) Beatrice can be $95 \%$ confident that the average home price is $\$ 5,000$ less in city A than in city B .
(D) Beatrice can be $95 \%$ confident that the average home price is $\$ 5,000$ more in city A than in city B .
(E) Beatrice can be $95 \%$ confident that there is not a significant difference in the average home price between these two cities.
11. In a manufacturing process, there are strict tolerances on the dimensions of a part produced. If the part is too long, it must be trimmed to the correct length. Suppose $5 \%$ of all parts are too long and $1 \%$ of all parts are too short. Parts of the correct length (either originally or trimmed) can be sold for $\$ 5$ each. However, if the part is too short, it cannot be used at all and is discarded. The cost of manufacturing each part is $\$ 2$ and the cost to trim a part to length is $\$ 1$. The expected profit per part is:
(A) $\$ 1.00$
(B) $\$ 2.05$
(C) $\$ 2.90$
(D) $\$ 3.00$
(E) $\$ 4.95$
12. A 2015 survey of 400 randomly selected drivers reported that 20 percent of the drivers surveyed never use their cruise control. A $95 \%$ confidence interval is given by $(0.161,0.239)$. Which of the following is a correct interpretation of the $95 \%$ confidence level?
(A) We are 95 percent confident that the true proportion of all drivers who never use their cruise control is between 0.161 and 0.239 .
(B) There is a 0.95 probability that the true proportion of all drivers who never use their cruise control is between 0.161 and 0.239 .
(C) 95 percent of all random samples of 400 drivers chosen from the population will result in confidence intervals which contain 0.20 .
(D) 95 percent of all random samples of 400 drivers will result in confidence intervals which contain the true proportion of all drivers who never use their cruise control.
(E) 95 percent of all random samples of 400 drivers chosen from the population will have sample proportions between 0.161 and 0.239 .
13. A survey asked male and female students how many text messages they send in one day. The results are summarized in the boxplots below.


Based on the boxplots, which of the following statements must be true?
(A) There were more females surveyed than males.
(B) Both distributions contain outliers.
(C) The distribution of number of text messages sent by males is approximately normal.
(D) $50 \%$ of the females send more text messages than $75 \%$ of the males.
(E) The male responses vary more than the female responses.
14. The University of Georgia has several football players participating in the NFL draft this season. A few former coaches are helping the players prepare for the NFL tryout by tracking their practice performance. One coach calculates the athlete's speed in feet per second each round and then finds the mean and standard deviation for all rounds. Since the workout tested at the NFL tryout is the 40 yard dash, the coach decides to convert these speeds to yards per second instead of feet per second, where 1 yard is equal to 3 feet. Which of the following best describes what would happen to the summary statistics for the athletes?
(A) The mean would increase, but the standard deviation would remain the same.
(B) Both the mean and standard deviation would remain the same.
(C) Both the mean and standard deviation would decrease.
(D) The mean would decrease, but the standard deviation would remain the same.
(E) The mean would decrease, but the standard deviation would increase.
15. In a study of achievement scores, the class size (number of students in the classroom) and the average student achievement for that class are recorded. Let $x=$ class size and $y=$ average achievement. A scatterplot of $\ln (y)$ versus $x$ shows a strong negative linear relationship. Output from a regression analysis is shown below.

| Regression |  |  |  | Equation: $\ln ($ Achievement $)=5.779-0.025($ ClassSize $)$ |
| :--- | :---: | :---: | :---: | :---: |
| Predictor | Coef | SE Coef | T | P |
| Constant | 5.779 | 0.2948 | 19.60 | 0.000 |
| ClassSize | -0.025 | 0.0030 | -8.33 | 0.000 |
| $\mathrm{~S}=0.4871$ | $\mathrm{R}-\mathrm{Sq}=83.4 \%$ | R-Sq $(\mathrm{adj})=80.8 \%$ |  |  |

Based on the information provided above, which of the following statements must be true?
(A) Large class sizes cause average student achievement to decrease.
(B) The residual plot of the relationship between class size and average student achievement would show a non-random pattern.
(C) The correlation between class size and $\ln$ (achievement) is $r=0.913$.
(D) There is also a strong negative linear relationship between average student achievement and class size.
(E) Larger values of class size are associated with larger values of $\ln$ (achievement).
16. The histograms below display the ages of all American males who have been given the names William and Zachary in the last 50 years. Assume that the two graphs have the same vertical scale.


Zachary (Distribution Z)


Which group has the smaller standard deviation: males named William or males named Zachary?
(A) Distribution W has the smaller standard deviation because the people are almost evenly distributed across the age range.
(B) Distribution W has the smaller standard deviation because it shows a weak, positive association.
(C) Distribution Z has the smaller standard deviation because the distribution is slightly skewed to the left.
(D) Distribution Z has the smaller standard deviation because there are more ages clustered near the center of the distribution.
(E) The two distributions have the same standard deviation because the range of ages is the same.
17. A 2017 survey of 1077 registered voters found that 83 percent of the voters surveyed oppose the government's plan to repeal its net neutrality rules for internet providers. The poll's margin of error was $\pm 3$ percentage points at a $99 \%$ confidence level. Which of the following best describes what is meant by the poll having a margin of error of $\pm 3$ percentage points?
(A) Three percent of those surveyed were unavailable to participate in the poll.
(B) It would not be unexpected for $3 \%$ of the population to readily agree to the net neutrality rules.
(C) Between 862 and 926 of the 1077 voters surveyed responded that they would oppose the government's plan to repeal its net neutrality rules for internet providers.
(D) The poll used a method that gets an answer within $3 \%$ of the truth about the population approximately $99 \%$ of the time.
(E) Between $80 \%$ and $86 \%$ of all registered voters oppose the government's plan to repeal its net neutrality rules for internet providers.
18. A group of students at a local university were asked about their status (undergraduate, graduate, or non-degree seeking) and how they get to campus (walk/bike or car/bus/etc.). The results are summarized in the table below.

|  | How do you get to Campus? |  |  |
| :---: | :---: | :---: | :---: |
|  | Walk/Bike | Car/Bus/etc. | Total |
| Undergraduate | 80 | 20 | 100 |
| Graduate | 40 | 10 | 50 |
| Non-degree Seeking | 10 | 40 | 50 |
| Total | 130 | 70 | 200 |

Suppose we choose a person at random from this sample. Which of the following statements is true?
(A) If the person uses a car/bus/etc. to get to campus, then he or she is more likely to be a nondegree seeking student than a degree seeking student (undergraduate or graduate).
(B) If the person is an undergraduate, then he or she is more likely to use a car/bus/etc. to get to campus than to walk/bike to campus.
(C) The person is more likely be a graduate student if he or she uses a car/bus/etc. to get to campus than if he or she walks/bikes to campus.
(D) The person is more likely to use a car/bus/etc. to get to campus than to walk/bike to campus.
(E) The person is more likely to be a graduate student than to be an undergraduate student.
19. Researchers are testing the breaking strength of a new brand of rope using a large sample of ropes. In a test of $H_{0}: p=0.80$ versus $H_{a}: p>0.80$, where $p$ is the true proportion of all ropes of this brand that would break when subjected to a weight of 1000 pounds, the test statistic is $z=1.45$. Which of the following is true?
(A) $H_{0}$ is rejected at the $5 \%$ level, but not at the $10 \%$ level of significance.
(B) $H_{0}$ is rejected at the $10 \%$ level, but not at the $5 \%$ level of significance.
(C) $H_{0}$ is rejected at both the $5 \%$ and $10 \%$ level of significance.
(D) $H_{0}$ is not rejected at both the $5 \%$ and $10 \%$ level of significance.
(E) $H_{0}$ is not rejected at any level of significance.
20. Max would like to know whether there is community interest in having local musicians perform music in the park in the evenings during the summer. Max goes to the park for several evenings in a row and asks people entering the park whether they would like to hear music in the evening. Out of the 200 people he surveys, $58 \%$ respond favorably. This scenario is describing what type of sampling?
(A) Simple random sampling
(B) Volunteer sampling
(C) Convenience sampling
(D) Cluster sampling
(E) Stratified sampling
21. Last football season the starting quarterback of a professional football team completed just $55 \%$ of his attempted passes. The owner of this team has given the quarterback the ultimatum that he must improve his completion percentage during the off season or he will be fired. The owner plans to use a random sample of 50 passing attempts at the end of spring training to test the hypotheses

$$
\begin{aligned}
& H_{0}: p=0.55 \\
& H_{a}: p>0.55
\end{aligned}
$$

where $p$ represents the true proportion of passes this quarterback can complete after training during the off season. If the owner wants to minimize the chance of firing the quarterback when he really has improved, should he use a significance level of 0.01 or 0.10 ?
(A) The owner should use $\alpha=0.01$ because this will minimize the chances of a Type I error.
(B) The owner should use $\alpha=0.01$ because this will minimize the chances of a Type II error.
(C) The owner should use $\alpha=0.10$ because this will minimize the chances of a Type I error.
(D) The owner should use $\alpha=0.10$ because this will minimize the chances of a Type II error.
(E) The significance level used will not affect the probability the owner mistakenly fires the quarterback when he really has improved.
22. The major television networks are interested in knowing what percentage of Super Bowl viewers plan to also stick around to watch the show that airs after it. Which of the following will give the minimum sample size that needs to be surveyed to be 99 percent confident of the true proportion to within $\pm 4$ percent?
(A) $2.326 \sqrt{\frac{(0.04)(0.96)}{n}} \leq 0.04$
(B) $2.326 \frac{0.5}{\sqrt{n}} \leq 0.04$
(C) $2.576 \sqrt{\frac{(0.04)(0.96)}{n}} \leq 0.04$
(D) $2.576 \frac{\sqrt{n}}{0.5} \leq 0.04$
(E) $2.576 \frac{0.5}{\sqrt{n}} \leq 0.04$
23. Trisomy 18 (T18) is a rare genetic disorder that severely disrupts a baby's development prior to birth. Many die before birth and most die before their first birthday. T18 occurs in only 1 in 2500 pregnancies in the U.S. A genetic test on the mother's blood can be done to test for T18 in her baby. The overall probability of a positive test result is 0.010384 . The probability of a positive test result for a baby with T18 is 0.97 . The probability of a negative test result for a baby without T18 is 0.99 .

A mother's blood tests positive for T18. What is the probability that her baby has T18?
(A) 0.0004
(B) 0.0374
(C) 0.4902
(D) 0.97
(E) 0.99
24. Porsche Cars North America keeps detailed records on all current Porsche owners. The marketing director takes a random sample of 9 customer records and finds that the average age of a Porsche owner is 52.8 years with a standard deviation of 6.1 years. Assuming all conditions of inference were met, what is a $95 \%$ confidence interval estimate for the average age of all Porsche owners?
(A) $52.8 \pm 2.306 \times \frac{6.1}{3}$
(B) $52.8 \pm 2.306 \times \frac{6.1}{9}$
(C) $52.8 \pm 2.262 \times \frac{6.1}{3}$
(D) $52.8 \pm 2.262 \times \frac{6.1}{9}$
(E) $52.8 \pm 1.96 \times \frac{6.1}{3}$
25. A roll of Life Savers candies contains 14 pieces. Suppose the individual candies have a mean weight of 0.09 ounces and standard deviation of 0.01 ounces. Find the mean and standard deviation of the total weight of the roll of candies, ignoring the weight of any packaging material.
(A) mean $=1.26$ ounces, standard deviation $=0.0014$ ounces
(B) mean $=1.26$ ounces, standard deviation $=0.037$ ounces
(C) mean $=1.26$ ounces, standard deviation $=0.14$ ounces
(D) mean $=0.09$ ounces, standard deviation $=0.037$ ounces
(E) mean $=0.09$ ounces, standard deviation $=0.14$ ounces
26. The time that an Olympic skier takes on a downhill course has a normal distribution with a mean of 85.5 seconds and standard deviation of 0.8 seconds. The probability that on a random run the skier takes between 85.1 and 85.9 seconds is:
(A) 0.1915
(B) 0.3085
(C) 0.3830
(D) 0.6170
(E) 0.6826
27. Reading and understanding nutrition labels on foods may be an important precursor to dietary change. Researchers randomly selected four medical clinics in Missouri. A total of 885 patients were asked to complete a survey while waiting to see their physicians. The study found that those who read labels tended to have better diets. Can the researchers conclude that reading nutrition labels on foods causes patients to have better diets?
(A) No, because the 885 respondents do not represent a simple random sample.
(B) No, this was an observational study so the researchers can only claim an association.
(C) Yes, because the use of randomization in the study allows for cause and effect conclusions.
(D) Yes, because the sample size of 885 is sufficiently large.
(E) The researchers can only claim a cause and effect relationship among individuals similar to the patients who completed the survey.
28. A research specialist for a large seafood company plans to investigate bacterial growth on oysters subjected to two different storage temperatures, 0 and 10 degrees Celsius. There are 18 refrigerators available for the experiment, six each of three different brands (Brand A, Brand B and Brand C). Each refrigerator has been stocked with a random selection of fresh oysters harvested from the same body of water. For each brand, the researcher randomly selects three refrigerators to set at 0 degrees Celsius and the other three are set at 10 degrees Celsius. At the end of the storage period, the bacterial count will be determined on all the oysters and the average count for the two temperatures will be compared within refrigerator brand. For this experiment, identify the blocking factor, if any.
(A) There are no blocks in this experiment
(B) The type of seafood
(C) The temperature level
(D) The refrigerator brand
(E) The bacteria count
29. Powerball is a national lottery with drawings held twice a week. Players buy a ticket for $\$ 2$ for a chance to win monetary prizes ranging from $\$ 4$ to the jackpot which can be in the hundreds of millions of dollars. Let $X$ be a player's winnings (in dollars) on one ticket. For a cash jackpot of $\$ 100,000,000, \mathrm{E}(X)=0.66$. Interpret the value of $\mathrm{E}(X)$.
(A) Each player will win $\$ 0.66$ per ticket.
(B) The probability of winning any monetary prize is 0.66 .
(C) The lottery can expect to profit $\$ 0.66$ per ticket purchased.
(D) The lottery can expect to pay out $66 \%$ of the jackpot in prize money.
(E) Over many players, the average winnings per ticket will be close to $\$ 0.66$.
30. A linear regression analysis reveals a strong, negative linear relationship between $x$ and $y$. Which of the following could possibly be the results from this analysis?
(A) $\hat{y}=13.1-27.4 x, r=0.85$
(B) $\hat{y}=27.4+13.1 x, r=-0.95$
(C) $\hat{y}=13.1+27.4 x, r=0.95$
(D) $\hat{y}=542-385 x, r=-0.15$
(E) $\hat{y}=0.85-0.25 x, r=-0.85$
31. A survey of 120 randomly selected married couples asked each partner to rate how often they dine out as a couple. The results are presented below with expected counts in parentheses.

| Wife's Response |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Never or <br> Occasionally | Fairly Often | Very Often | Almost <br> Always |
|  | Never or <br> occasionally | $14(8.78)$ | $12(9.04)$ | $2(7.75)$ | $3(5.43)$ |
| Husband's <br> Response | Fairly Often | $7(8.50)$ | $10(8.75)$ | $6(7.50)$ | $7(5.25)$ |
|  | Very Often | $9(8.22)$ | $4(8.46)$ | $10(7.25)$ | $6(5.08)$ |
|  | Almost <br> Always | $4(8.50)$ | $9(8.75)$ | $12(7.50)$ | $5(5.25)$ |

In testing to see if there is evidence of an association between the ratings of husbands and wives, a chi-square test statistic of 19.48 was calculated. Which of the following statements is correct?
(A) The data prove the couples' ratings are associated.
(B) The data prove the couples' ratings are not associated.
(C) There is sufficient evidence to suggest the couples' ratings are associated at the $5 \%$ significance level but not at the $1 \%$ significance level.
(D) There is sufficient evidence to suggest the couples' ratings are associated at the $1 \%$ significance level.
(E) The chi-square test should not have been used because four of the observed counts are less than 5 .
32. A local university surveyed a large random sample of students at the end of their freshmen year in 2017 and asked them to report their level of satisfaction with the university. To which of the following populations can the results of this survey be safely generalized?
(A) All university students
(B) All students at this local university
(C) All freshmen university students
(D) Only students who were freshmen at this local university in 2017
(E) Only the freshmen students at this local university in 2017 who were in the survey
33. A group of chemical engineers are trying to determine the cost of refining a mixture based on the concentration of a particular chemical. After refining 50 samples of the mixture and recording the cost they fit a simple linear regression model to the data. Partial computer output is given below.

|  |  |  |  |  |
| :--- | :--- | :---: | :---: | :---: |
| Predictor | Coef | SE Coef | T | P |
| Constant | 2.51 | 0.563 | 4.45 | $<0.0001$ |
| Concentration | 2.55 | 0.980 | 2.61 | 0.0122 |

A scatterplot of cost versus concentration had a moderately, strong curved relationship. Based on these results, what conclusion can be drawn from this analysis at the $5 \%$ significance level?
(A) There is evidence of a significant positive linear relationship between concentration and cost.
(B) There is evidence of a significant negative linear relationship between concentration and cost.
(C) There is evidence of a significant linear relationship between concentration and cost but the direction is unknown.
(D) There is evidence of a significant non-linear relationship between concentration and cost.
(E) The analysis is inconclusive because the assumptions for the linear regression $t$-test are not met.
34. Earthworms of the species Lumbricus terrestris often surface after a large rain storm. It is unclear why they surface in such abundance after large amounts of rain. The lengths of this species of earthworm are normally distributed with an average length of 180 mm with a standard deviation of 15.5 mm . What length would we expect to find at the 30th percentile?
(A) 22.54
(B) 78.89
(C) 109.89
(D) 171.87
(E) 188.13
35. In a random sample of 400 private high school students, 320 said they brought a smartphone to school that day, while in a random sample of 400 public high school students, 288 said they brought a smartphone to school that day. Which of the following represents a 98 percent confidence interval estimate for the difference (private high school minus public high school) between the proportions of all private high school and public high school students who bring a smartphone to school?
(A) $(0.8-0.72) \pm 1.96 \sqrt{\frac{(0.8)(0.2)}{400}+\frac{(0.72)(0.28)}{400}}$
(B) $(0.8-0.72) \pm 2.054 \sqrt{\frac{(0.8)(0.2)}{400}+\frac{(0.72)(0.28)}{400}}$
(C) $(0.8-0.72) \pm 2.326 \sqrt{\frac{(0.8)(0.2)}{400}+\frac{(0.72)(0.28)}{400}}$
(D) $(0.8-0.72) \pm 2.054 \sqrt{(0.76)(0.24)\left(\frac{1}{400}+\frac{1}{400}\right)}$
(E) $(0.8-0.72) \pm 2.326 \sqrt{(0.76)(0.24)\left(\frac{1}{400}+\frac{1}{400}\right)}$
36. The following computer output contains the results from fitting a simple linear regression model to predict the weight (in pounds) of a vehicle from its city mileage (in miles/gallon) based on a random sample of 25 cars.

|  |  |  |  | P |
| :--- | :---: | :---: | :---: | :---: |
| Predictor | Coef | SE Coef | T | P |
| Constant | 6542.55029 | 332.09991 | 19.70 | $<0.0001$ |
| City MPG | -145.56921 | 16.25832 | -8.95 | $<0.0001$ |

Assuming the conditions for inference are satisfied, which of the following gives a 95 percent confidence interval for the true slope of the regression line based on this model?
(A) $-145.57 \pm(1.96)(16.258)$
(B) $-145.57 \pm(2.069)(3.2516)$
(C) $-145.57 \pm(2.069)(16.258)$
(D) $-145.57 \pm(8.95)(3.2516)$
(E) $-145.57 \pm(8.95)(16.258)$
37. A company that sells quality dishware has two manufacturing plants. In a quality control inspection of a random sample of 200 dishes from plant $\mathrm{A}, 8 \%$ of the dishes had at least one defect. In a random sample of 200 dishes from plant B, $5 \%$ of the dishes had at least one defect. To determine if there is convincing evidence that the true proportion of defective dishes from plant A is more than the true proportion of defective dishes from plant B , you test the hypotheses $H_{0}: p_{\mathrm{A}}-p_{\mathrm{B}}=0$ versus $H_{a}: p_{\mathrm{A}}-p_{\mathrm{B}}>0$ and obtain a $p$-value of 0.112 .
Which of the following is an appropriate interpretation of this $p$-value?
(A) If the true proportion of defective dishes at plant $A$ is more than the true proportion of defective dishes at plant B , there is a 0.112 probability of getting samples with a difference $\hat{p}_{\mathrm{A}}-\hat{p}_{\mathrm{B}}$ equal to 0.03 .
(B) If the true proportion of defective dishes at the two plants are equal, there is a 0.112 probability of getting samples with a difference $\hat{p}_{A}-\hat{p}_{\mathrm{B}}$ equal to 0.03 .
(C) If the true proportion of defective dishes at the two plants are equal, there is a 0.112 probability of getting samples with a difference $\hat{p}_{\mathrm{A}}-\hat{p}_{\mathrm{B}}$ greater than or equal to 0.03 .
(D) The probability of making a Type I error is 0.112 .
(E) The probability that the true proportion of defective dishes at plant A is more than the true proportion of defective dishes at plant B is 0.112 .
38. One of the side effects of flooding a lake in northern Nthacoochie Park (e.g., for a hydro-electric project) is that mercury is leached from the soil, enters the food chain, and eventually contaminates the fish. The concentration in fish will vary among individual fish because of differences in eating patterns, movements around the lake, etc. Suppose that the concentrations of mercury in individual fish follows an approximate normal distribution with a mean of 0.25 ppm and a standard deviation of 0.08 ppm . Fish are safe to eat if the mercury level is below 0.30 ppm . Approximately what percentage of fish are safe to eat?
(A) $23 \%$
(B) $27 \%$
(C) $37 \%$
(D) $63 \%$
(E) $73 \%$
39. The week before the bill to repeal Obamacare came up for consideration in the Senate, one polling agency increased the size of the random sample of U.S. adults they surveyed on this issue from 500 to 1500. What effect would this increase have on the polling agency's estimate of the proportion of U.S. adults in favor of repealing Obamacare, assuming the true proportion in favor remained constant?
(A) A reduction in the variability of the estimate
(B) An increase in the variability of the estimate
(C) A reduction in the bias of the estimate
(D) An increase in the bias of the estimate
(E) A reduction in both the variability and the bias of the estimate
40. A reading specialist at Pleasantville High School suspects that there is a difference in the average reading speed between senior boys and senior girls at the high school. To test her claim, random samples of 25 senior boys and 25 senior girls were selected. The reading speed of each student was measured. The reading specialist uses the data obtained to test the hypotheses $H_{0}: \mu_{\text {boys }}-\mu_{\text {girls }}=0$ versus $H_{a}: \mu_{\text {boys }}-\mu_{\text {girls }} \neq 0$ where $\mu_{\text {boys }}$ and $\mu_{\text {girls }}$ are the true mean reading speed of all seniors boys and girls at Pleasantville High School.
Which of the following statements best describes a Type I error?
(A) The reading specialist finds evidence there is a difference in the average reading speed between senior boys and senior girls at the high school when, in fact, there is a difference in the average reading speed between senior boys and senior girls.
(B) The reading specialist fails to find evidence there is a difference in the average reading speed between senior boys and senior girls at the high school when, in fact, there is a difference in the average reading speed between senior boys and senior girls.
(C) The reading specialist fails to find evidence there is a difference in the average reading speed between senior boys and senior girls at the high school when, in fact, there is no difference in the average reading speed between senior boys and senior girls.
(D) The reading specialist finds evidence there is a difference in the average reading speed between senior boys and senior girls at the high school when, in fact, there is no difference in the average reading speed between senior boys and senior girls.
(E) The reading specialist finds evidence there is a difference in the average reading speed between senior boys and senior girls at the high school.

END OF EXAMINATION

# AP ${ }^{\circledR}$ STATISTICS SECTION II Part A <br> Questions 1-5 <br> Spend about 65 minutes on this part of the exam. Percent of Section II grade ----75 

Directions: Show all your work. Indicate clearly the methods you use, because you will be scored on the correctness of your methods as well as on the accuracy and completeness of your results and explanations.

1. At a recent country music awards show, the diversity of the gender and ages of the award recipients was observed. The ages of the first thirty males and the first thirty females to receive an award (including group awards) was recorded. Boxplots of the ages by gender of these award recipients are presented in the figure below.

(a) Write a few sentences to compare the distributions of ages for male and female award recipients.
(b) The male award winner who was 74 years old was identified as a potential outlier. If this award winner had been 54 instead of 74 years old, what effect would this decrease have on the following statistics? Justify your answers.

The interquartile range of male ages:

The standard deviation of male ages:
2. An elementary school principal is interested in determining whether obesity is a common problem among the children who attend the school. Body Mass Index (BMI) is a measure of relative weight based on an individual's mass and height, and is considered a better measure of obesity than weight alone. High values can be used to indicate obesity. The BMI assessment takes some time to complete so it will not be feasible to assess every child. Some local university researchers are willing to help complete the BMI assessments for a sample of children in the school.

The school is organized in wings as depicted below. Each wing contains four classrooms from the same grade level. There are a total of 24 classrooms across grades K through 5. Each classroom contains 20 children.

(a) For convenience, the researchers want to use a cluster sampling method, in which the classrooms are clusters. Their goal is to assess 60 children. Describe a process for randomly selecting classrooms and identifying the sample of children using this method.
(b) One of the researchers suggests that an alternative sampling method would be to select a stratified random sample. Describe a process for randomly selecting 60 children using this sampling strategy and justify your selection of the stratification variable.
(c) In the context of this situation, give one statistical advantage of using a stratified random sampling method as opposed to a cluster sampling method that uses classrooms as clusters.
3. A softball player wants to investigate the number of times she can hit the ball in successive attempts. The player plans to swing at every pitch so that she will either hit the ball or miss it on each attempt. Assume the player hits the ball $25 \%$ of the time and that her attempts are independent.
(a) What is the probability the player gets her third hit on the fourth attempt?

The player continued her inquiry for several weeks. Each day the player was given up to six attempts to get three hits. Let the random variable $X$ represent the number of attempts required for the player to get three hits given that she is successful at hitting the ball three times in her six attempts. The table below gives the observed relative frequencies for each possible value of $X$.

| $x$ | 3 | 4 | 5 | 6 |
| :---: | :---: | :---: | :---: | :---: |
| Observed Probability of $x$ | 0.09 | 0.21 | 0.31 | 0.39 |

(b) Use the given relative frequencies to find the mean and standard deviation of $X$.
(c) Two spectators who are watching the softball player decide to set up a bet between themselves. Spectator A believes that the next time the player is successful at getting three hits in her six attempts that it will only take 3 or 4 attempts, and spectator B believes that it will take 5 or 6 attempts. The spectators have agreed that spectator A will receive $\$ 20$ if she is correct. Based on the observed relative frequencies of $X$, how much should spectator B receive to ensure this is a fair bet? A fair bet is one in which both parties have zero expected winnings.
4. Harpy eagles live in the rain forests of Central and South America and are one of the largest species of eagle. For many birds of prey, females are larger than males. A biologist is interested in investigating whether this phenomenon is also true of the harpy eagle. She selects random samples of 8 female harpy eagles and 9 male harpy eagles. The weights of the eagles, in pounds, were recorded, as shown in the table below.

|  | Weight of Eagle |  |  |  |  |  |  | Mean | Standard <br> Deviation |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Females <br> $\left(n_{\mathrm{F}}=8\right)$ | 13.5 | 14.1 | 15.1 | 16.0 | 17.6 | 14.9 | 14.8 | 13.6 | 14.95 | 1.353 |  |
| Males <br> $\left(n_{\mathrm{M}}=9\right)$ | 13.4 | 13.9 | 13.5 | 12.8 | 14.5 | 12.7 | 13.6 | 13.1 | 12.2 | 13.30 | 0.689 |

Do the data provide convincing evidence that the mean weight of female harpy eagles is greater than the mean weight of male harpy eagles?

If you need more room for your work in question 4, use the space below.
5. The U.S. speed skaters heading to the 2018 Pyeongchang Winter Olympics needed to investigate reasons for the team's poor showing at the 2014 Sochi Winter Olympics. There were many potential reasons presented to explain this poor performance, including that the U.S. speed skating team trained in Collalbo, Italy at elevation $3,792 \mathrm{ft}$ but Sochi, Russia is at sea level (elevation 0 ft ).
(a) In the months leading up to the Sochi Olympics, the U.S. speed skaters competed in many international skating competitions to prepare for the Olympic Games. One competition was located in Salt Lake City, Utah (elevation 4,330 ft) in November 2013 and the other in Berlin, Germany (elevation 164 ft ) in December 2013. For these two competitions the times for 14 athletes in the men's 1500 m were recorded in both races. The difference in finish times was calculated for each athlete and summary statistics are shown below.

|  | Mean | Standard Deviation |
| :---: | :---: | :---: |
| Salt Lake City - Berlin | -3.37 seconds | 0.87 seconds |

Assuming that all conditions for inference have been met, construct a $95 \%$ confidence interval for the mean difference in 1500 m times between the Salt Lake City and Berlin competitions. Does this interval suggest that there is a difference in mean finish times when two competitions are held at different elevations? Explain.
(b) The completion times for 15 athletes who competed in both the Berlin event and in the 1500 m final at the Sochi Olympics (two cities with similar elevation) were also compared and the difference in finish times (Berlin - Sochi) was calculated for each athlete. From this data, a 95\% confidence interval for the mean difference in 1500 m times between the Berlin and Sochi competitions is $0.16 \pm 0.68$ seconds. Assuming that all conditions for inference have been met, does this interval suggest that there is a difference in mean finish times when two competitions are held at similar elevations? Explain.

A scatterplot showing the finishing times by country for the 10 athletes who competed in all three events are shown below. Shani Davis, the U.S. speed skater whose position in the plot is circled, won silver medals in the men's 1500 m during both the 2006 and 2010 Olympics but finished $11^{\text {th }}$ in Sochi. He is also set to compete in the 2018 Pyeongchang Winter Olympics.

(c) Use the scatterplot to compare Shani's finish times across all three competitions (Salt Lake at elevation $4,330 \mathrm{ft}$, Berlin at elevation 164 ft , and Sochi at elevation 0 ft ) and comment on the difference in Shani's performance when competing in cities with different elevations compared to his performance when competing in cities with similar elevations.
(d) Using all the information in this question, what recommendation would you make to the U.S. Speed Skating Association in regards to the elevations of the cities where the team trains for future Olympics?

# AP ${ }^{\circledR}$ STATISTICS SECTION II Part B <br> Question 6 <br> Spend about 25 minutes on this part of the exam. Percent of Section II grade ----25 

Directions: Show all your work. Indicate clearly the methods you use, because you will be scored on the correctness of your methods as well as on the accuracy and completeness of your results and explanations.
6. A researcher is trying to estimate the unknown proportion $p$ of individuals with a rare genetic trait in a particular population. The researcher will take a sample of $n$ individuals from this population and count the number of individuals, $X$, in the sample that possess this genetic trait.

Suppose that a sample consists of $n=20$ trials from this binomial process with success probability $p$. In other words, let the random variable $X$, the number who possess this genetic trait, have a binomial probability distribution with parameters $n=20$ and unknown success probability $p$.
(a) Suppose the population proportion of individuals who possess this genetic trait is 0.03 . A simulation was conducted in which 1,000 random samples of size $n=20$ were taken from this population and the point estimate $\hat{p}=\frac{X}{n}$ calculated. The histogram below displays the distribution of the 1,000 simulated sample statistics, $\hat{p}$.


Summary Statistics
Mean of $\hat{p}$ values: 0.030
Std Dev of $\hat{p}$ values: 0.038

Based on these simulation results, does it appear that $\hat{p}$ is an unbiased estimator of the population proportion $p$ of individuals who possess this genetic trait? Explain.
(b) For each of the 1,000 sample proportions obtained in part (a), a $95 \%$ confidence interval for the population proportion $p$ of individuals who possess this genetic trait was constructed using the usual one-proportion z-interval formula. Of the 1,000 intervals, 469 or $46.9 \%$ succeeded in capturing the population proportion $p$ of individuals who possess this genetic trait within the endpoints of the interval. Explain why the proportion of intervals in the simulation that succeeded in capturing the parameter $p$ was much less than $95 \%$.

Another estimator for $p$ that can provide a statistical advantage over the conventional estimator $\hat{p}$ in certain situations is the following:

$$
\tilde{p}=\frac{X+2}{n+4} .
$$

(c) Carry out the calculations below to investigate the relationship between $\hat{p}$ and $\tilde{p}$.
(i) Suppose that the sample results in 5 individuals who possess this genetic trait among the 20 trials. Determine the values of $\hat{p}$ and $\tilde{p}$.
(ii) Suppose now that the sample results in 12 successes among the 20 trials. Determine the values of $\hat{p}$ and $\tilde{p}$.
(iii) Are there any sample results for which the values of $\hat{p}$ and $\tilde{p}$ would be the same? Justify your answer.
(d) A simulation was conducted in which 1,000 random samples of size $n=20$ were taken from this population and the point estimate $\tilde{p}=\frac{X+2}{n+4}$ calculated. The histogram below displays the distribution of the 1,000 simulated sample statistics, $\tilde{p}$.


Summary Statistics
Mean of $\tilde{p}$ values: 0.107
Std Dev of $\tilde{p}$ values: 0.031

For each of the 1,000 sample proportions obtained, a $95 \%$ confidence interval for the population proportion $p$ of individuals who possess this genetic trait was constructed using $\tilde{p}$ in place of $\hat{p}$ in the usual one-proportion $z$-interval formula and $n+4$ in place of $n$. Of the 1,000 intervals, 976 or $97.6 \%$ succeeded in capturing the population proportion $p$ of individuals who possess this genetic trait within the endpoints of the interval.

Based on these simulation results, does it appear that $\tilde{p}$ is an unbiased estimator of the population proportion $p$ of individuals who possess this genetic trait? Explain.
(e) Based on comparing the summary statistics for the simulation results in parts (a) and (d), state a statistical advantage of the estimator $\hat{p}$.
(f) Based on comparing the summary statistics for the simulation results in parts (a) and (d), state a statistical advantage of the estimator $\tilde{p}$. Explain why this statistical advantage makes sense, given that the new statistic $\tilde{p}$ is calculated by adding 2 to the numerator and 4 to the denominator of $\hat{p}$.

