MthS 3190 CT2	Introduction to Proof	Semester TBD
Meeting Times TBD	Course Syllabus	Meeting Location TBD

Course Basics:

Meetings: TBD

Textbook: *Mathematics: A Discrete Introduction* (Third edition) by Edward Scheinerman.

Description: A course which introduces mathematical proofs with topics including proof techniques, elementary logic, induction, sets, functions, and relations.

CT2: This course is a CT^2 seminar in which you will not only study the course material but also develop your critical thinking skills.

Calculator: You may occasionally want to use a calculator for homework. Calculators are not allowed on quizzes or exams.

Prerequisites: MthS 1080: Calculus of One Variable II or MthS 1110: Calculus II for Biologists.

Prerequisite For: MthS 4110: Introduction to Combinatorics, MthS 4550: Topics in Geometry, MthS 4560: Topology. This course is required for the math major or the education major with a mathematics speciality. The department has planned for MthS 3190 to be a prerequisite for more 4000-level courses in the near future.

Attendance: Attendance at every class is expected. If you have an unavoidable conflict with an exam or other in-class assessment, please contact me *before* the class. In the case of a sudden emergency or illness, contact me as soon as possible.

Texting: Texting is not appropriate during class. If I catch you texting during class, I will try to embarrass you.

Online Information: I will communicate with you via email and Canvas for this class.

Lateness: If the instructor is late for the course, class will be cancelled after 15 minutes.

Instructor Information:

Instructor: Dr. Michael Burr

Email: burr2@clemson.edu (This is the best way to contact me.)

Office: Martin Hall, O-19.

Office Hours:

- TBD
- TBD "recitation office hour"
- and by appointment. Please email me to set up an appointment outside of normal office hours.

Office Phone: (864)-656-5220

Homework, Quizzes, and Exams:

Grading: Your final grade will be computed using the following formula:

- 5% Participation and Reactions
- 10% Homework
- 15% Quizzes
- 40% Exams (2 Exams, 20% each)
- 30% Final

Final Grades: Final letter grades (after rounding) will be determined using the standard scale:

- A: ≥ 90
- B: $80 - 89$
- C: $70 - 79$
- D: $60 - 69$
- F: ≤ 59

Recitation Office Hours: On one day a week, I will hold a "recitation office hour" in the afternoon where attendees will practice the course material in small groups. Here, you might work on practice problems or consider a given proof in more depth. Recitation office hours are not mandatory, but are suggested.

Participation and Reactions: You will have daily assignments before and after most lectures. In these assignments, you will prepare for lecture or react to the proofs presented in class. The reactions will be completed online.

Homework: You will have several written homework assignments per week. Expect to work on each week's homework for several hours. Each homework problem must be well-organized and thoroughly explained in words for full credit. Note that a thorough explanation is *necessary* for you to receive full credit for your work. Homeworks turned in one class late will receive 50% credit and will not be accepted beyond that.

Quizzes: There will be weekly (announced) short quizzes (5-10 minutes) at the beginning of class. The quiz questions will be short answer questions which test your recall of the basic course material. Questions on these quizzes may be drawn from any of the class material, not necessarily just the most recent material.

Exams: There will be two in-class exams in this course. The in-class exams will be held on TBD and TBD.

Final Exam: According to the current final exam schedule, the final will be held on TBD. There will be no exemptions from the final.

Group Work: Group work is encouraged on the homework, but you must write up your solutions separately. This means that the techniques and approaches that you use may be the same, but the text or explanation of the solution should be written up *in your own words*. Identical or very similar homeworks *may not be graded*. If you work on the homework together or receive a hint from someone else, you *must write the names of the people with whom you worked at the top of your submission*. By writing the names of the people with whom you worked, you are giving them credit for their contributions, just as you would expect credit for helping someone else.

Topics:

Proof Techniques: The main skill that is developed in MthS 3190 is the ability to write mathematical proofs. The mathematical proof techniques in this course include the following techniques: direct proof, proof by contrapositive, proof by contradiction, quantifier proof, epsilon-delta proof, and proof by induction.

Mathematical Objects: This class will also introduce you to the language of mathematics and basic definitions that are used in most 4000-level courses. In particular, the basic mathematical objects discussed in this class include quantifiers, sets, functions, relations, and equivalence relations.

Critical Thinking: Since this course is a CT^2 seminar, we will also explicitly discuss the role of critical thinking in mathematics. In particular, we will not only discuss the role of critical thinking in the development of proofs, but we will also discuss why mathematics is done in the way it is, what makes good mathematics, and why mathematicians have chosen a particular language for mathematics.

Selected Topics: If there is time at the end of the semester, we will cover selected topics from combinatorics, elementary number theory, and graph theory.

Learning Objectives:

Upon successful completion of this course, students will be able to

- Define basic mathematical objects.
- Describe the standard mathematical proof techniques.
- Identify the key steps of a given proof.
- Summarize and analyze the key steps of a given proof.
- Apply basic proof techniques to prove a variety of theorems.
- Compute the basic data of mathematical objects using their properties.
- Give examples and counterexamples for mathematical statements
- Classify proofs by technique and identify similarities and differences between them.
- Inspect and characterize the data and questions in a statement.
- Assemble basic proof techniques into a coherent multi-step plan to prove complex statements.
- Assess the applicability of various proof techniques for a given problem.
- Produce written mathematical proofs that express complex and technical arguments clearly.
- Critique given proofs for correctness and completeness.
- Discuss the differences between computations, algorithms, and proofs.
- Monitor and assess one's own thoughts and arguments for clarity, precision, correctness, and logicalness.
- Evaluate several proofs of the same statement for their clarity and mathematical style.

Academic Integrity:

Class Policy: Your are expected to follow the university's policy on academic integrity:

"As members of the Clemson University community, we have inherited Thomas Green Clemson's vision of this institution as a "high seminary of learning." Fundamental to this vision is a mutual commitment to truthfullness, honor, and responsibility, without which we cannot earn the trust and respect of others. Furthermore, we recognize that academic dishonesty detracts from the value of a Clemson degree. Therefore, we shall not tolerate lying, cheating, or stealing in any form."

In particular, in this course, if you use references outside of the course materials, e.g., the internet, a different book, or discuss the problems with anyone, you are expected to provide a short citation. On homeworks, you are also required to write the names of the people with whom you worked at the top of your submission.

Disability Services:

Class Policy: It is University policy to provide, on a flexible and individualized basis, reasonable accommodations to students who have disabilities. Students are encouraged to contact Student Disability Services to discuss their individual needs for accommodation. The Student Disability Services department is located in Redfern Health Center.

In addition, students with disabilities who need accommodations should make an appointment with me to discuss specific needs within the first month of classes (I must receive the letter at least one week before an exam in order to provide accommodations). Students should present a Faculty Accommodation Letter from Student Disability Services when we meet.

Disclaimer:

Syllabus Changes: This syllabus is subject to change, if necessary, for reasons including those related to unforeseen conflicts, course needs, or educational needs. I will contact you whenever a change must be made, and an up-to-date version of the syllabus will always appear on the course website.

What is This Course Not Like?

- In some courses, you are presented with problems in class and asked to solve *very similar problems* on an exam. For example, consider MthS 1060: Calculus of One Variable I. In class, you are given examples of finding a tangent line to a curve, and, on an exam, you are asked to find the tangent line to a curve (for a different curve). In this case, the question from class and the exam question follow the exact same procedure. MthS 3190 is *not* this type of course. In fact, you can expect none of the exam questions to repeat questions from class.
- In some courses, you can pass the course by *memorizing* all of the important data. Classes that are sometimes approached this way by students (even though this method is not the most effective) include organic chemistry and art history. MthS 3190 is *not* this type of course. You will need to memorize some definitions and theorems in this course (this material is tested on weekly quizzes), but memorization alone will not allow you to pass the exams. Memorization only allows you to repeat steps that have been taken before, and it does not help you solve new problems. If this course were just about memorization, mathematicians could just look up everything that is needed in a database and mathematicians would not be necessary. Fortunately, memorizing facts alone is not mathematics. Instead, to succeed in this course, you must learn how to interpret and apply your knowledge.

What is This Course Like?

- The most important goal of this course is to teach proof-writing. A proof is a formal and logical argument, in essay form, in which you argue for the truth of a statement. Many 4000-level mathematics courses assume competence in proof-writing and require proof-writing as a major component of the class. This course is a gateway course to advanced mathematics. After you have taken this course, you will be prepared for a wide and exciting range of mathematics.
- Your first step in this course will be to
  (1) memorize and understand the basic mathematical definitions and theorems in the course, and
  (2) study and internalize the basic structures for proof writing.
- The heart of this course lies in proof writing. Proofs require you to take your previously learned mathematical knowledge (definitions, theorems, and structures) and find a way to combine that knowledge into a coherent argument to justify the truth of a statement beyond question.
- Students find proof writing to be hard because solving a problem is not just mimicking the steps of a problem from class, but requires developing your own steps. In addition, you may find the amount of information daunting, and, to successfully complete a proof, you must decide which knowledge is most relevant to your problem.

- This course is a CT^2 critical thinking seminar. Therefore, in addition to covering the material of MthS 3190, this course has a critical thinking component, which is designed to improve your critical thinking skills. Critical thinking is more than memorization. Since improvement in your critical thinking is one of the goals of this course, we will spend time in this course analyzing your own thinking and discussing how to apply critical thinking to writing proofs. In particular, some of the critical thinking steps that we will discuss include the following:
  (1) When given a problem, you must make assessments of what of the given information is most important to the problem.
  (2) When given a problem, you must make assessments of what mathematical knowledge will be most applicable to this problem.
  (3) After determining the appropriate mathematical knowledge, you must plan out how to combine that knowledge into a coherent argument.
  (4) After planning out your argument, you must carry out your plan and turn it into a well-communicated short essay that expresses how to combine the appropriate information.
  (5) After finishing a proof, you will be expected to reflect upon it and study what made this plan successful and how to apply or generalize the results.

  Note that these steps follow the spirit of "The List" in *How to Solve It* by George Polya.

What is Expected of You?

- Learning how to write proofs is a challenging and very personal process. Proof writing cannot be learned passively by listening to lecturers; instead, you can only learn proof writing by practice. The best that I can do for you is to guide you as you make this transition.
- Some of the most important work that you do in this class will be outside of the classroom, when you practice and study on your own. Some of your assignments will guide you in how to practice and understand the course material successfully.
- To get the most out of each class, you should prepare for class and review the material after class. In order to encourage you to do this, there will be assignments before and after each class.
  (1) Before each class, the theorems that we will cover in that class will be posted online, and you will be required to study the given information and try to connect the problems to your previous knowledge. You will be required to think about the problem and try to give a suggestion on how to solve it.
  (2) After each class, you will need to answer, on an online form, a few questions about the proofs discussed in class which are designed to make you understand the mathematics, techniques, and approaches in a deeper way.
- It is very important that you are continuously studying the material in this class. If you only memorize the facts just before the exam, you will not know how to apply them when you face a problem you have never seen before. See the hints for how to best approach studying.

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This course is a CT^2 seminar on critical thinking. Therefore, your critical thinking skills will be evaluated, and explicit attempts will be made during the semester to develop your critical thinking skills.

Classroom Strategies for Critical Thinking:

Modeling: During the course of the semester, we will be doing many proofs in class. While completing these proofs, I will model my thought process for each problem. I will focus my discussion on what choices I'm making and why I'm making them.

"The List"/Socratic Method: At a few points in class and always during office hours, I will use "The List" from *How to Solve It* by George Polya, which is a specialized form of the Socratic Method to help you prove various statements. "The List" will be handed out part-way through the semester and consists of a collection of questions which you can ask yourself to help organize your knowledge and choose the proper steps.

Analyzing Work: Throughout the semester, we will frequently analyze proofs. To analyze a proof means to study it and to investigate its components. In particular, we will ask questions about why certain choices were made, understand how all of the conditions in the theorem were used, and consider generalizations or applications. After each lecture, you will have a short reaction assignment that asks you to analyze the proofs discussed in class. A few times in the semester we will also more thorougly analyze proofs in class or on homework.

Side-by-side: Several times in the semester, I will give you several different completed proofs for the same theorem. In class and for homework, the task will be to compare them side-by-side and discuss the advantages and disadvantages of each technique.

Incorrect Proofs: I have a collection of incorrect proofs. Often throughout the semester, we will break into small groups and your task will be to study these proofs. You will be asked to look at these proofs line-by-line, check that each statement follows logically from the previous steps, and attempt to find any errors. Finally, you will give each proof a grade.

Analyze the Work of Others: Periodically throughout the semester, I plan on collecting your proofs and selecting a few of them for you to analyze. I will retype the work so that your identities are hidden and you will be asked, not to grade, but to comment on the good and bad parts of the work.

Reflect on Work: A few homework assignments throughout the semester will go into metacognition in a little more depth. You will be asked to describe your thought processes. You will be asked about both good and bad ideas and how you were able to make them work.

Critical Thinking Portfolio

Critical Thinking Portfolio: Over the course of this semester, we will be creating a critical thinking portfolio. This portfolio will consist of a selection of your work which is designed to exhibit the growth of your critical thinking skills over the course of the semester. These artifacts will be designed to exhibit how much you've learned this semester. These artifacts *may be* acceptable for either the mathematics or the critical thinking proficiencies.

Although the artifacts in this portfolio are likely to *not* be acceptable for your e-portfolio, it will show you how much you've learned this semester.

Artifacts: The artifacts in your critical thinking portfolio will come from approximately three assignments over the course of the semester. These homework assignments will be identified as critical thinking artifacts, and they will be more substantial than typical homework problems. These artifacts will include reflective questions and may involve several drafts.

Rubrics: The artifacts will be evaluated using the "Problem Solving VALUE Rubric" to assess your development over the course of the semester. The artifacts will be graded, however, according to the proof grading rubric.

Critical Thinking Testing

California Critical Thinking Skills Test: In this course, we will use the California Critical Thinking Skills Test to evaluate and monitor your growth in critical thinking. You will receive a grade for the completion of this test.

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Meeting Times TBD	Proof Grading Rubric	Meeting Location TBD

A main component of this course is proof writing. Proof writing can be challenging to learn, but competence in writing proofs is required in most 4000-level courses. The following rubric is the proof grading rubric (based on a 10-point question) which will be used in this class as a *guide* to the grading of proof-based questions.

Statement of the Problem:

0-1: The statement or setup of the problem is incorrect. The solution may misinterpret what is given or what must be shown. The solution to an if-and-only-if proof may be missing one or two directions. The solution to a proof by contradiction or proof by contrapositive may assume the wrong statement. The stated approach for the type of proof is not the type of proof that is actually used.

2: Correct, but incomplete statement or setup of the problem. The setup may be missing any of the following: clearly stated assumptions, clearly stated goals, or clearly stated type of proof. Also, solutions where you write that you do not know how to solve a problem, but provide a clear description of what you'd like to try (a wish list).

3: Correct statement of the problem. Informs the reader of what type of proof is being used (or an explanation of the logical argument), what is being assumed, and what is to be shown.

Correctness of proof:

0-1: The solution includes many incorrect computations or improperly deduced results. The proof may be based on largely incorrect statements or contain several improperly justified statements. There is little or no sense of how to prove the result. The proof may assume what is trying to be shown or may be circular.

2-3: Unconnected, mostly true statements properly deduced from the given. Also, solutions where you write that you do not know how to solve a problem, but provide an interesting example of the problem and a list of facts (with justification for their use) that might prove useful in solving the problem without understanding how to connect the facts.

4-6: A correct approach to proving the theorem is attempted. Some statements may not be completely justified, but the errors are minor and can be fixed. The proof should have no incorrect statements. These errors may include small gaps which are easily fixed.

7: A correct and complete proof is given. This includes a thorough explanation of each and every step with no logical gaps.

What to do if You Need Help?

Instructor: Whenever you're having difficulty, the first step should be to contact me, your instructor. I want to help you succeed while also challenging you. There is no need to make an appointment, just drop in during office hours. I will not give you answers during office hours, but will try to help you lead yourself to the solution. I will guide you using "The List" of *How to Prove It* by George Polya and by using the Socratic method. Note that these methods can take more time and energy than you expect, so please plan accordingly. Try to ask questions about assignments early because I can do more to help you if you have time to absorb the material.

Academic Success Center: The Academic Success Center is located in the Class of 1956 Academic Success Center building. The center provides many resources including tutoring and academic coaching and counseling. It is a great resource if you are struggling in this class.

Tutoring: The department sometimes keeps a list of private tutors who can help in this course. While it costs to higher these tutors, it may be helpful to have a person dedicated to helping you. More information can be found on the department's website or by talking with the department administrators.

Helpful Hints

(1) This course is unlike most other courses that you've taken in the past. This course cannot be crammed; in particular, if you only study just before the exam, it is likely that you will not do well. You will be learning a new way to think, and it takes a significant amount of time for you to become comfortable with proofs. The time that it takes to become comfortable is different for every person.

(2) Do the homework! To learn mathematics, you must do mathematics. There is no other way, other than experience, to learn the material in this course. You will learn a lot more in this course if we discuss a partially completed solution than if I show you a completed solution.

(3) Begin your homeworks early. In this course, you will be developing a new way to think, and it can take a significant length of time for different ideas to be absorbed into your brain. If you begin your homeworks too close to the due date, you will be stressed, which will prevent you from spending the time to think clearly about the new ideas. This would not be beneficial for your homework.

(4) Don't give up! Sometimes you will need to stare at a problem for several days before you realize how to solve it. Many of the proofs that we work on in this class are well-known and the solutions are available online. If you give up and look for the solution online too early, you'll lose the chance for you to improve your understanding. By not developing the solution yourself, you'll put yourself at a disadvantage for future problems.

(5) If you do not know what to do on a problem, you should be talking to me in office hours. If that is not possible, and you are quite stuck on a problem, do not write down "junk" hoping for partial credit. I can easily spot when you're flailing in this way, and such an approach will cost you points. If you write down the problem clearly, provide some correct ideas and tools that you'd use in the problem, mention some applicable facts, and provide an interesting example, you can earn up to half credit.

(6) Keep two notebooks for this course (or a large notebook). In one notebook, write down the notes from class. In the other notebook, keep an organized list of the definition and theorem statements throughout this course. This list will help you remember what material is available as we build up out knowledge. You can search through this list of facts quickly when you're faced with a new problem.

(7) On homework assignments, write several drafts. The first draft could just be scribbles of figuring out the problem. Don't be afraid to make a mistake, try several examples and see what works and what doesn't. In the second draft, try to write up the solutions - usually, you will find errors at this stage. Rewrite the solutions two or three more times until you don't make a mistake; now it is ready to submit.

(8) After each class, compress your notes. Try to write down the ideas and the smallest amount that you will need to remember in order to reconstruct the notes from class. Write down the ideas of what is discussed, but as few details as necessary. Keep what you find confusing and mention only briefly what you already understand. When you finally understand something, try to write notes to yourself that explain your thinking so that you don't forget your good ideas! After each week, compress your compressed notes from that week following the same pattern. Before each exam, take all your weekly summaries and compress them once again.

(9) When studying for exams, look through old problems and try to write summaries of each problem. Don't try to memorize the solutions in their entireties, but, instead, try to write down the ideas of the proof or a few steps that would allow you to reconstruct the proof. If you remember the key step and notice when key steps of several problems are similar, then you can reconstruct the proof on the spot and can generalize the technique to problems you haven't seen before.

(10) Don't give up in the course! For some people, it can take up to half-way through the semester (or longer) until they're comfortable with proofs. To learn how to write proofs, you must keep working at understanding even though it may be a challenge or struggle in order to understand the material well.

Important University/Class Dates

- TBD: Last day to add a class.
- TBD: Lass day to drop without a W.
- TBD: First in-class exam.
- TBD: No class (university breaks).
- TBD: Last day to withdraw without a final grade.
- TBD: Second in-class exam.
- TBD: Final exam.