MATH 3190-CT2

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MATH 3190

- Required for all math majors and minors.
- Transition from calculus sequence to upper level mathematics.
- Students learn mathematical proof.
- Course for late Sophomore year.

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Mathematical Proofs

- Structured arguments.
- Explain why a statement is true (not computational).
- Follow the rules of logic.
- Require precision and clarity.
- Require a new way of thinking.

Syllabus Preparation

- University of Louisville: Ideas to Action: Faculty Exemplars.
- `http://louisville.edu/ideastoaction/resources/exemplars/faculty`
- Before and after examples.

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University of Louisville
College of Arts & Sciences

ANTH 336: War and Society
Jennie Burnet, Ph.D.

Text in Red
Indicates the infusion of the **Elements of Reasoning**
Text in Blue
Indicates the infusion of the **Intellectual Standards**
Text in Green
Indicates the infusion of the **Intellectual Traits**
[Bracketed Text]
Indicates the indirect use of critical thinking ***[Elements]***, ***[Standards]***, or ***[Traits]***

View more about the Paul-Elder Framework of Critical Thinking

Dr. Burnet's **Original** Learning Outcomes:
This course introduces a critical approach to war and its relationship to society. It is designed with three main learning goals in mind:

(1) to introduce the methods anthropologists use to understand social institutions such as the military;
(2) to learn how to engage in critical social analysis,
(3) to introduce the idea that war and war preparation cannot be understood as simply the result of the motivations of combatants or the strategies of political and military elites, and
(4) to study the processes of militarization.

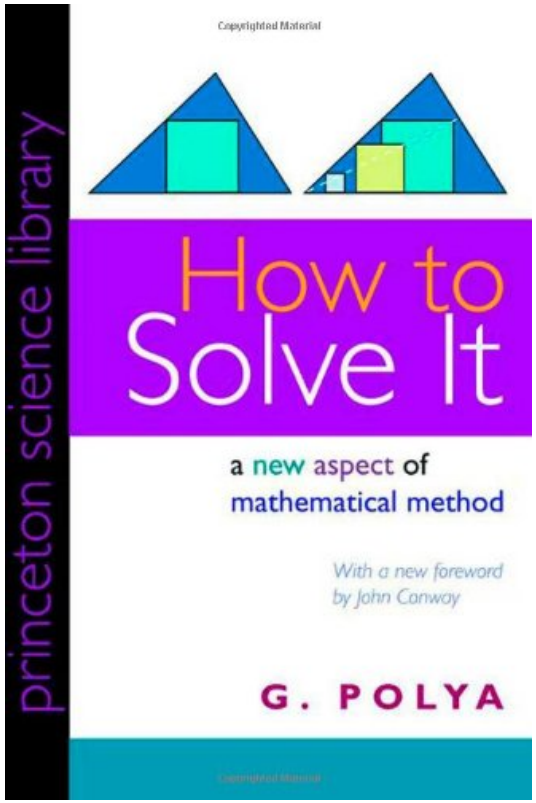
Dr. Burnet's **Revised** Learning Outcomes After:
This course introduces an anthropological perspective on war (and peace) and its relationship to society. The course will explore, in depth, the methods anthropologists use to analyze culture and social institutions.

Upon successful completing of the course, students will be able to:

(1) explain and apply fundamental concepts such as war, peace, militarization, genocide, gender, political economy, and terrorism.
(2) analyze, interpret and evaluate a wide variety of information (scholarly publications, eyewitness testimony, ethnographic films, etc.), assumptions, and arguments about war and peace.
(3) demonstrate their mastery of these concepts by undertaking autonomous research and writing a clear, precise, and accurate paper on a relevant question about war (or peace) and society or culture.

Jennie Burnet

- Define basic mathematical objects.
- Describe the standard mathematical proof techniques.
- Identify the key steps of a given proof.
- Summarize and analyze the key steps of a given proof.
- Apply basic proof techniques to prove a variety of theorems.
- Compute the basic data of mathematical objects using their properties.
- Give examples and counterexamples for mathematical statements
- Classify proofs by technique and identify similarities and differences between them.
- Inspect and characterize the data and questions in a statement.
- Assemble basic proof techniques into a coherent multi-step plan to prove complex statements.
- Assess the applicability of various proof techniques for a given problem.
- Produce written mathematical proofs that express complex and technical arguments clearly.
- Critique given proofs for correctness and completeness.
- Discuss the differences between computations, algorithms, and proofs.
- Monitor and assess one's own thoughts and arguments for clarity, precision, correctness, and logicalness.
- Evaluate several proofs of the same statement for their clarity and mathematical style.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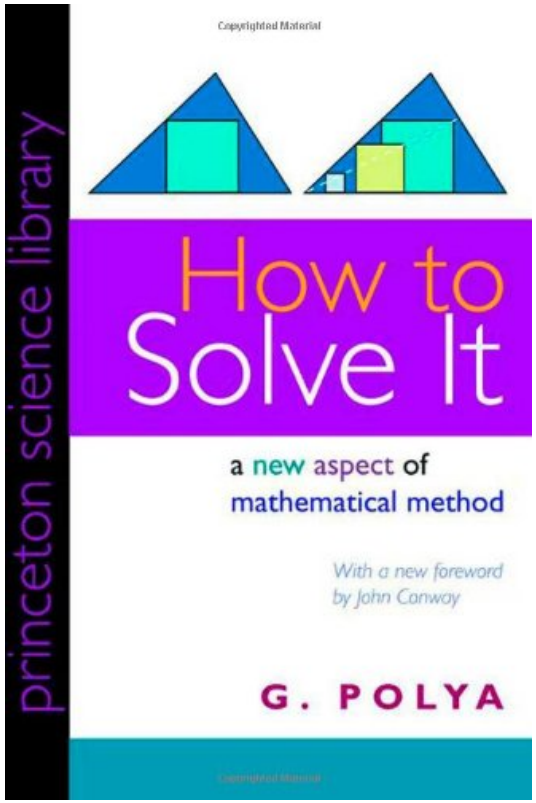
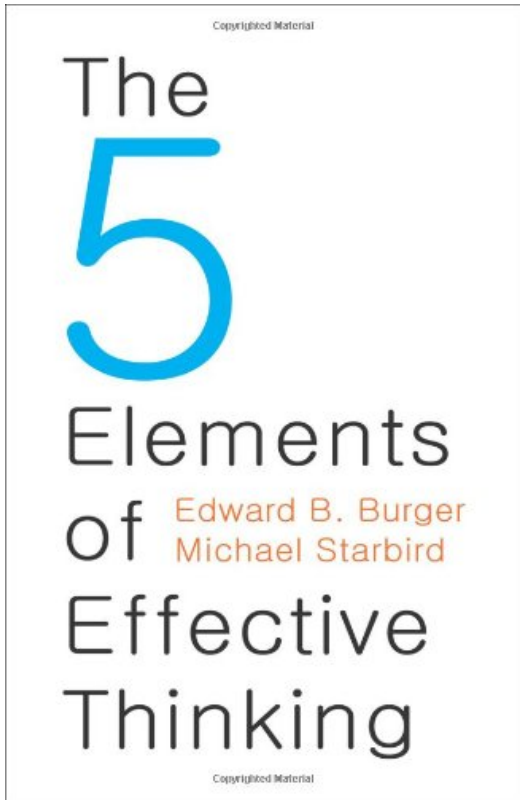
How to Solve It
a new aspect of mathematical method
With a new foreword by John Conway
G. POLYA

G. Polya, 1945



G. Polya, 1945



E. Burger, M. Starbird, 2012

HOW TO SOLVE IT

UNDERSTANDING THE PROBLEM

First.

You have to *understand* the problem.

What is the unknown? What are the data? What is the condition? Is it possible to satisfy the condition? Is the condition sufficient to determine the unknown? Or is it insufficient? Or redundant? Or contradictory?

Draw a figure. Introduce suitable notation.

Separate the various parts of the condition. Can you write them down?

DEVISING A PLAN

Second.

Find the connection between the data and the unknown. You may be obliged to consider auxiliary problems if an immediate connection cannot be found. You should obtain eventually a *plan* of the solution.

Have you seen it before? Or have you seen the same problem in a slightly different form?

Do you know a related problem? Do you know a theorem that could be useful?

Look at the unknown! And try to think of a familiar problem having the same or a similar unknown.

Here is a problem related to yours and solved before. Could you use it? Could you use its result? Could you use its method? Should you introduce some auxiliary element in order to make its use possible?

Could you restate the problem? Could you restate it still differently? Go back to definitions.

If you cannot solve the proposed problem try to solve first some related problem. Could you imagine a more accessible related problem? A more general problem? A more special problem? An analogous problem? Could you solve a part of the problem? Keep only a part of the condition, drop the other part; how far is the unknown then determined, how can it vary? Could you derive something useful from the data? Could you think of other data appropriate to determine the unknown? Could you change the unknown or the data, or both if necessary, so that the new unknown and the new data are nearer to each other?

Did you use all the data? Did you use the whole condition? Have you taken into account all essential notions involved in the problem?

CARRYING OUT THE PLAN

Third.
Carry out your plan.

Carrying out your plan of the solution, *check each step*. Can you see clearly that the step is correct? Can you prove that it is correct?

LOOKING BACK

Fourth.
Examine the solution obtained.

Can you *check the result?* Can you check the argument?

Can you derive the result differently? Can you see it at a glance?

Can you use the result, or the method, for some other problem?

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- I found that these questions helped get students started.
- They had been trained to ask the wrong questions.

Statement of the Problem:

0-1 The statement or setup of the problem is incorrect. The solution may misinterpret what is given or what must be shown. The solution to an if-and-only-if proof may be missing one or two directions. The solution to a proof by contradiction or proof by contrapositive may assume the wrong statement. The stated approach for the type of proof is not the type of proof that is actually used.

2 Correct, but incomplete statement or setup of the problem. The setup may be missing any of the following: clearly stated assumptions, clearly stated goals, or clearly stated type of proof. Also, solutions where you write that you do not know how to solve a problem, but provide a clear description of what you'd like to try (a wish list).

3 Correct statement of the problem. Informs the reader of what type of proof is being used (or an explanation of the logical argument), what is being assumed, and what is to be shown.

Correctness of proof:

0-1 The solution includes many incorrect computations or improperly deduced results. The proof may be based on largely incorrect statements or contain several improperly justified statements. There is little or no sense of how to prove the result. The proof may assume what is trying to be shown or may be circular.

2-3 Unconnected, mostly true statements properly deduced from the given. Also, solutions where you write that you do not know how to solve a problem, but provide an interesting example of the problem and a list of facts (with justification for their use) that might prove useful in solving the problem without understanding how to connect the facts.

4-6 A correct approach to proving the theorem is attempted. Some statements may not be completely justified, but the errors are minor and can be fixed. The proof should have no incorrect statements. These errors may include small gaps which are easily fixed.

- Students study a collection of supposed proofs.
- Students work in groups.
- Check for correctness of the statement
- Check the validity of the argument
- Provide suggestions for improvement.
- Provide grades.

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  A (correct) if the claim and proof are correct, even if the proof is not the simplest or the proof that you would have given.

  C (partially correct) if the claim is correct and the proof is largely correct. The proof may contain one or two incorrect statements or justifications, but the errors are easily correctible.

  F (failure) if the claim is incorrect, or the main idea of the proof is incorrect, or there are too many errors.

B) Suppose a, b and c are integers. If a divides b and a divides c, then a divides b+c

Suppose a ~~divides~~ b and a ~~divides~~ c.

$a \mid b$ $a \mid c$

Then for some integer q, $b = aq$, and then for some integer ~~q~~ k, $c = a\cancel{q}\,k$ (dont use same variable)

Then $b + c = aq + a\cancel{q}\,k = \cancel{2aq = a(2q)}$, $= aq + ak = a(q+k)$, so a ~~divides~~ $a \mid b+c$ b + c.

C

Final artifact

- Students evaluated proofs individually.
- The students considered 8 supposed proofs.
- Answers were more thorough than in class.
- I marked and returned the analyses.
- The students were given a chance to revise.

8. **Claim:** If the relations R and S are transitive, then $R \cap S$ is transitive.
"Proof." Suppose $(x, y) \in R \cap S$ and $(y, z) \in R \cap S$. Then $(x, y) \in R$ and $(y, z) \in S$. Therefore, $(x, z) \in R \cap S$.

This proof uses the right idea, but it is missing a few parts in the middle. The assumption is good, and it is probably the best way to start the proof. However, it must be shown that $(x, z) \in R$ and $(x, z) \in S$ before it can be concluded that $(x, z) \in R \cap S$. This could be done by writing that because $(x, y) \in R \cap S$ and $(y, z) \in R \cap S$, $(x, y) \in R$ and $(y, z) \in R$. That fact could then be used with the assumption that R is transitive to show that $(x, z) \in R$. A similar process could have been used to show that $(x, z) \in S$, then the conclusion that $(x, z) \in R \cap S$ could have been stated because it would have been shown that $(x, z) \in R$ and $(x, z) \in S$. However, this proof is on the right track, and it had some good parts to it, so I would give it a grade of C.

California Critical Thinking Test

Average Improvement:

Overall	2.67
Analysis	0.13
Inference	0.80
Evaluation	1.33
Induction	1.07
Deduction	1.20