

Learning Objectives

- **Explain** how position, velocity and acceleration are defined for curvilinear motion.
- **Solve** projectile motion problems.

Curvilinear Motion

- Particle moves along a curved path.
- The particle's location is measured by a position vector, \mathbf{r} .
- The velocity is $\mathbf{v} = \frac{d\mathbf{r}}{dt}$, perpendicular to the path.
- Speed is $v = \frac{ds}{dt}$, where s is the curve length.
- Acceleration is $\mathbf{a} = \frac{d\mathbf{v}}{dt}$. Since the velocity is changing direction, the derivative \mathbf{a} is perpendicular to change in \mathbf{v} , so it isn't perpendicular to the path.
- In a fixed frame of reference, $\mathbf{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$, so $\mathbf{v} = \frac{d\mathbf{r}}{dt} = v_x\mathbf{i} + v_y\mathbf{j} + v_z\mathbf{k}$, where $v_x = x'$, $v_y = y'$ and $v_z = z'$, and $\mathbf{a} = \frac{d\mathbf{v}}{dt} = a_x\mathbf{i} + a_y\mathbf{j} + a_z\mathbf{k}$, where $a_x = v_x' = x''$, $a_y = v_y' = y''$ and $a_z = v_z' = z''$; $a^2 = a_x^2 + a_y^2 + a_z^2$.

Projectile Motion

- The most common type of curvilinear motion.
- Gravity accelerates particles in the y -direction, so using constant acceleration equations:

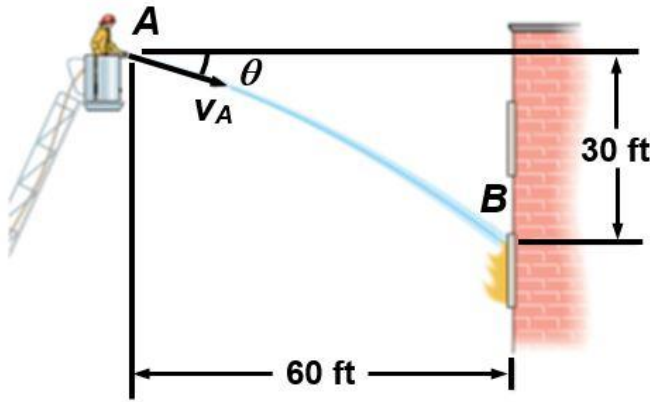
$$\mathbf{v} = \mathbf{v}_0 + \mathbf{a}t$$

$$s = s_0 + v_0t + \frac{1}{2}at^2$$

$$v^2 = v_0^2 + 2a(s - s_0)$$

(only 2 of the 3 are independent of each other)

- No force accelerates the particle horizontally, so the horizontal velocity remains constant. $(v_0)_x = v_x$



Ex #1 The fireman standing on the ladder directs the flow of water from his hose to the fire at B. Determine the velocity of the water at A if it is observed that the hose is held at $\theta = 20^\circ$.

Ex #2 Small packages traveling on the conveyor belt fall off into a 1-m-long loading car. If the conveyor is running at a constant speed of $v_c = 2$ m/s, determine the smallest and largest distance R at which the end A of the car may be placed from the conveyor so that the packages enter the car.

