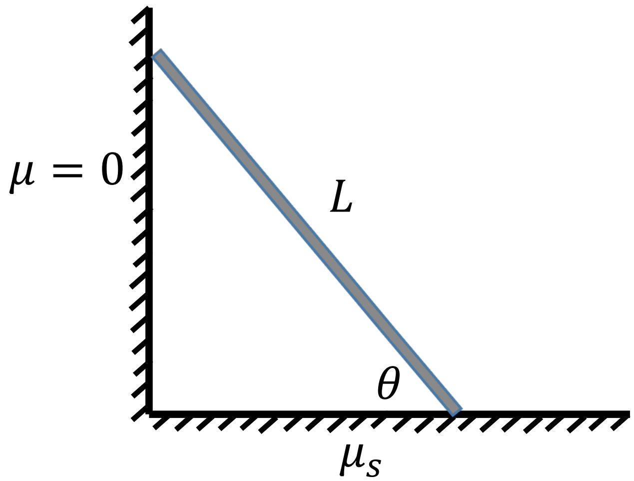
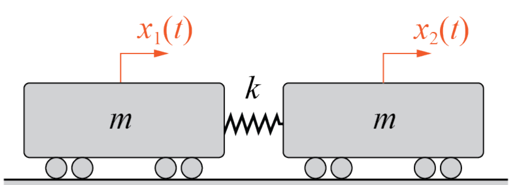
Problem 1. Consider a rocket of mass $m(t)$ moving with velocity $v(t)$ in the absence of any external forces. The mass of the rocket changes at a rate $\dot{m} = dm/dt$ and exits at a constant velocity v_e relative to the rocket. What is the mass of the rocket when the velocity reaches twice the initial velocity $v(0)$?

Problem 2. Falling ladder. A uniform rod of mass m and length L rests on a frictionless wall $\mu = 0$ as shown in Figure 1. The coefficient of friction for the ground is μ_s and the mass moment of inertia of the rod is $I = \frac{1}{12}mL^2$.

i) What is the smallest inclination angle for which the rod will not slide along the floor?
ii) Now assume both the wall and ground are frictionless $\mu = 0$. Derive the equations of motion for the rod in terms of the inclination angle θ. When will the rod leave the wall?



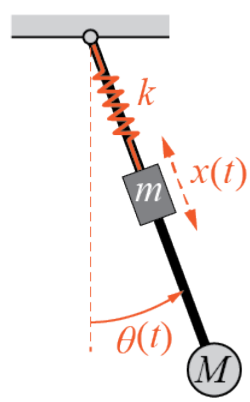
Problem 3. The schematic diagram below shows the equivalent model of a two-cart subway train. Suppose each subway cart weighs m = 4000kg, the coupling between them has a spring stiffness of k = 6×10^4 N/m, and the contact between subway wheels and the track is frictionless.



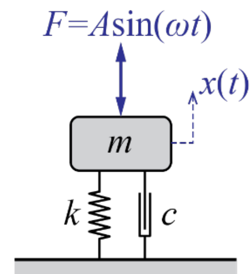
Calculate the natural frequency (or frequencies) and mode shapes of this system. Interpret the physical meaning of these mode shapes.

Problem 4. The diagram on the right shows a complex pendulum system, where a mass (M) can slide along the pendulum rod and is attached to the pivot through a linear spring (K). Let's assume the pendulum rod itself is rigid, massless, frictionless, and its total length is L.

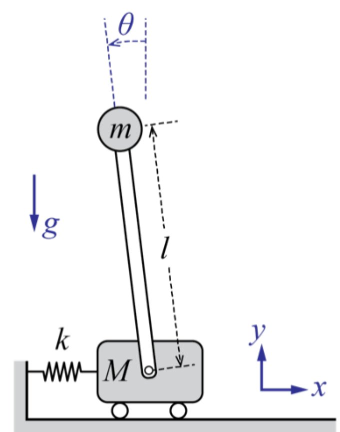
Derive the equation of motion of this system, and then linearize your solution by assuming a small pendulum angle (θ) and small x from static equilibrium.



Problem 5. The figure on the right shows a 1DOF mass-spring oscillator subjected to harmonic excitation. Please 1) calculate the steady-state response of the mass displacement $x_{ss}(t)$, and 2) derive the relationship between $x_{ss}(t)$ and damping coefficient c at resonance.



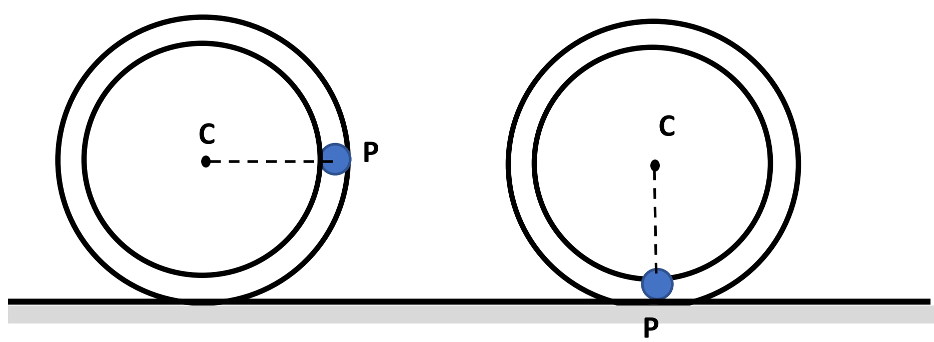
Problem 6. The figure on the right shows an inverted pendulum on a moving cart. Let's assume the pendulum rod is rigid and massless, and all contact surfaces are frictionless. Please derive this system's governing equation of motion.



Problem 7. A particle of mass $\boldsymbol{m_1}$ can slide without friction along the inside of a circular tube of mass $\boldsymbol{m_2}$ and radius $\boldsymbol{r}$. The particle-tube system is released from rest in the configuration shown in figure (a). *The friction between the tube and the ground is sufficient to prevent any slipping*. Assume that the difference between the inner and outer radius of the tube is negligible.

(i) Find the velocity of the tube when the particle reaches the vertically downward position as shown in fig. (b).

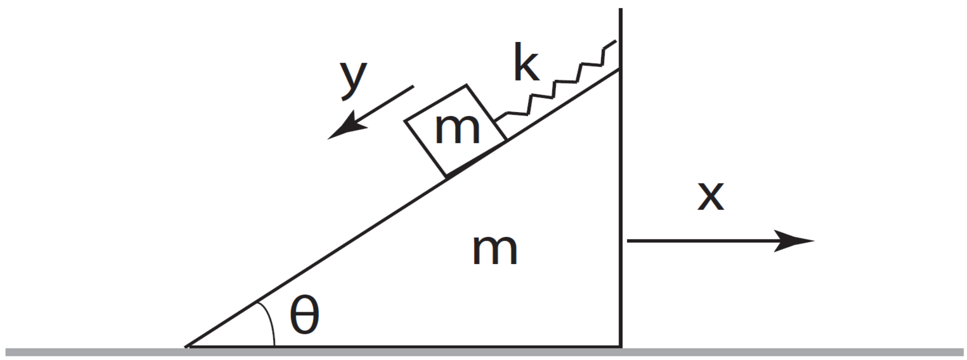
(ii) Find the normal reaction on the hoop from the ground when the particle reaches the vertically downward position shown in fig. (b)



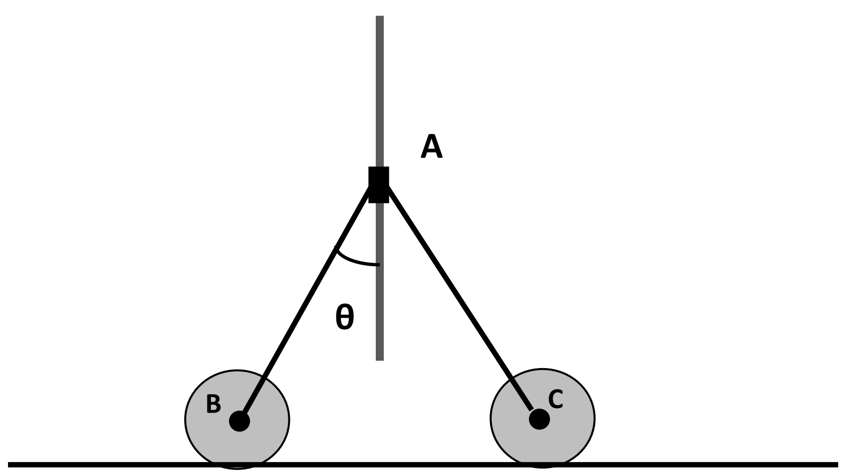
(a) **(b)**

Problem 8. A block of mass m rests on another triangular block also of mass m. The smaller block is attached to the larger block via a spring of stiffness k. Both the blocks are initially at rest and the spring is undeformed. The smaller block is then released from rest. Assume that all surfaces are frictionless.

(a) State if there are any conserved quantities for the system.
(b) When the block on the inclined plane has slid down by a distance y, what is the velocity of the triangular block?



Problem 9. A mechanism consists of two rods of equal mass m and length L hinged at a massless collar at point A. The collar is free to slide on a vertical frictionless rod. Each of the rods are connected to a disk of radius R=0.25L and mass m at their respective centers, B and C. The friction between the disks and the ground is sufficient to prevent any slipping. Assume that both the disks and rods have uniformly distributed mass. The mechanism is released from rest when $\theta = 30^0$. Find the angular velocity of the disks, when the angle θ is 60^o.



Problem 10. A rod of length **L** and mass $\boldsymbol{m_1}$ is pinned to a collar of mass $\boldsymbol{m_2}$ that can slide on a horizontal rail with negligible friction. The rod is released from rest the from the horizontal position.

(a) Determine the ignorable coordinates and the associated conserved quantity.

(b) Find the velocity of the collar and its displacement from its initial position when the rod reaches the vertically downward position.

(c) Find a single second order differential equation as the equation of motion of the system.