# Engineering Materials

## Problem #1:

The net potential energy $U$ between two adjacent ions is sometimes approximated by the expression

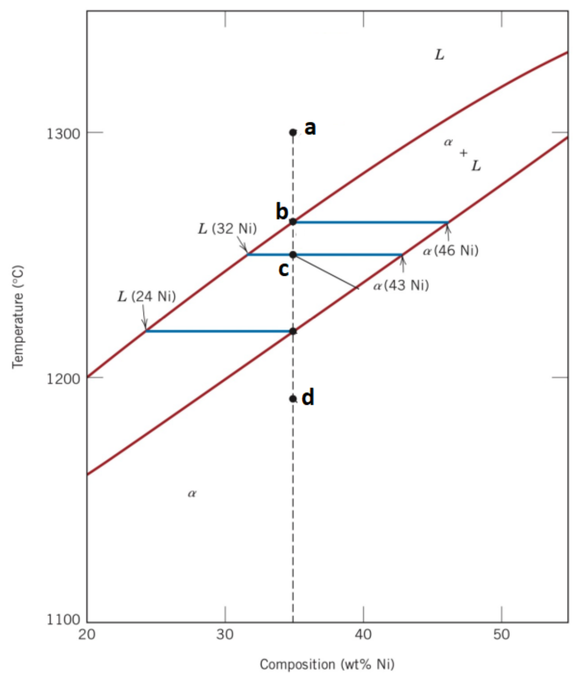
$$U(r) = -\frac{C}{r} + D\, e^{-r/\rho},$$

In which $r$ is the interionic separation and $C$, $D$, and $\rho$ are constants whose values depend on the specific material. If $U_o$ and $r_o$ are the bonding energy and equilibrium separation. Derive an expression for $U_o$ in terms of $r_o$, $D$, and $\rho$.

## Problem #2:

a) For a binary alloy of composition $C_o$ in a two-phase region (i.e., phases $\alpha$ and $\beta$), the lever rule describes the mass fraction of each of the phases in terms of the phase compositions. Letting $C_\alpha$ and $C_\beta$ be the phase compositions and $W_\alpha$ and $W_\beta$ be the corresponding phase fractions, derive the expression for the lever rule [Hint: think about conservation of mass].

b) A section of the Cu-Ni phase diagram is shown in the Figure to the right. Starting with an alloy with Ni composition of 35wt.% that is cooled from the liquid state at point ***a*** through the sequence ***a-b-c-d*** [see Figure to the right], draw schematics of representative microstructures at points ***b***, ***c***, and ***d***. Assume equilibrium cooling.




c) Using the Figure to the right, for the tie line that passes through point ***c***, what is the mass fractions of the liquid (L) and solid ($\alpha$) phases?

## Problem #3:

a) Define the fracture toughness.
b) An aircraft component is fabricated from an aluminum alloy that has a plane strain fracture toughness of 35 MPa $\sqrt{m}$. It has been determined that fracture results at a stress of 250 MPa when the maximum (or critical) internal crack length is 2.0 mm. For this same component, alloy, and loading configuration, will fracture occur at a stress level of 325 MPa when the maximum internal crack length is 1.0 mm? Show your work. Some relevant equations: $K_{IC} = Y\sigma\sqrt{\pi a}$ ; $\sigma_c = \left(\frac{2E\gamma_s}{\pi a}\right)^{1/2}$; and $\dot{\epsilon}_s = K\sigma^n$.

## Problem #4:

a) Describe Fick's first and second laws.
b) For a steel alloy, it has been determined that a carburizing heat treatment (10 h-duration) at a temperature T=600 K will raise the carbon concentration to 0.4 wt% at a distance 1 mm from the surface. Estimate the time (in h) necessary to achieve the same concentration at a distance 3 mm for an identical steel at a temperature T= 700 K. The activation energy is Q=0.8 eV. Reminder: The solution of the diffusion equation for these boundary conditions is given by:

$$\frac{C_x - C_0}{C_s - C_0} = 1 - erf\left[\frac{x}{2\sqrt{Dt}}\right]$$

$$D = D_0 exp\left(-\frac{Q}{kT}\right)$$

## Problem #5:

a) Explain the difference between thermoplastics, thermoset, and elastomers, and provide an example polymer for each.
b) When plastics undergo long-term loading, they exhibit two important properties, creep and stress relaxation. Explain these phenomena and propose a test to measure each.
c) What is ductile-to-brittle transition in metallic materials? What is its cause? What metals/alloys are prone to this transition?

## Problem #6:

Draw an A-B binary phase diagram containing one eutectic transformation and two (limited-solutbility) terminal solutions, $\alpha$ and $\beta$. Make sure the drawing is large enough so that it could be fully labeled.

a) Label the axes and the melting points for A and B.
b) Label all phase fields and identify the phases.
c) Label the eutectic temperature and the eutectic composition.
d) Label the point at which the solubility of B in A is maximal.

## Problem #7:

The behavior of a material is described by

$$\sigma = K(\varepsilon + 0.02)^n \ [MPA]$$

and

$$\varepsilon_f \text{ is the fracture strain.}$$

a) Determine Young's modulus for this material? $\left[\text{Hint: } E = \lim_{\varepsilon \to 0} \left(\frac{d\sigma}{d\varepsilon}\right)\right]$
b) Derive an expression for the material toughness.

## Problem #8:

a) Clearly define the concepts of a "slip plane" and a "slip direction".
b) Briefly describe the main crystallographic characteristics of the slip planes and the slip directions.
c) Draw an FCC and a BCC unit cell; identify one of the slip planes in each structure, and one of the slip directions in each of the chosen slip planes.
d) Explain how the number of slip planes in a material affects its ductility.

## Problem #9:

Unidirectional and continuous glass fibers reinforce a nylon matrix.

a) Draw the stress-strain diagram for the composite; assume loading is parallel to the fiber direction (defined as *0° orientation*), and strain extends beyond the point where the matrix deforms, causing the reinforcement to carry the entire load. Label the matrix deformation point.
b) Assuming the load is applied in this direction and fibers are rigidly bonded to the matrix (no relative slip), the composite strain $\varepsilon_c$, the fiber strain $\varepsilon_f$, and the matrix strain $\varepsilon_m$ can be considered equal. Derive an expression for the composite modulus of elasticity $E_{c,0°}$ as a function of the individual fiber and matrix moduli $E_f$ and $E_m$ and their respective volume fractions $f_f$ and $f_m$, where $f_f + f_m = 1$. Begin by expressing the force in the composite as a sum of loads carried by the fiber and matrix: $F_c = F_f + F_m$
c) Now consider the case where the composite is loaded perpendicular to the fiber direction (defined as *90° orientation*). Here it can be assumed that the stresses in the composite, fiber and matrix are equal ($\sigma_c = \sigma_f = \sigma_m$). Derive an expression for the composite modulus of elasticity $E_{c,90°}$ as a function of the individual fiber and matrix moduli $E_f$ and $E_m$ and their respective volume fractions $f_f$ and $f_m$, where $f_f + f_m = 1$.