CLEMSON
UNIVERSITY

Department of Mechanical Engineering

Spring 2009 Ph.D. Qualifying Exam

SYSTEMS AND CONTROLS

INSTRUCTIONS:

a) **This is a two hour closed books and notes test.**
b) **The use of scientific calculators is allowed (no laptop or programmable calculators).**
c) **Please show all work in a clear manner.**
d) **Do not use pencil, use black ink pens throughout.**
e) **DO NOT identify yourself in any way on the exam booklet, ALWAYS use your assigned ID number on all the papers.**

HONORS PLEDGE: *"I have neither given nor received aid on this examination."*

Sign Here: ___________________ **(use your assigned identifier number)**

Problem	Points	Score
1	**30**	
2	**30**	
3	**30**	
Total	**90**	

Problem 1: [30 Pts.]

Consider the mechanical system shown below which features two suspended masses (m_1 and m_2), three elastic springs (k_1, k_2, and k_3), two dashpots (c_1 and c_2) and applied force, F(t). In this problem, you will be asked to undertake four tasks **in any order that you choose**. Note that Laplace Transform pairs have been attached for your convenience.

(A) Draw an analogous electric circuit for the mechanical system; please create a table which clearly maps the mechanical elements into electrical components. (5 points)

(B) Write the lumped parameter ordinary differential equations for both the mechanical system and the electrical circuit. (5 points)

(C) Suppose that m_1= 50kg, k_1= 10N/m, c_1= 0Ns/m, m_2= 50kg, k_2= 10N/m, c_2= 0Ns/m, k_3= 0N/m, and F(t)= 0N. Write the mechanical system dynamic equations in state space form with dx/dt=Ax+Bu. (5 points)

(D) If the spring k_2 may be considered to be sufficiently stiff, then it may be neglected and a lumped mass introduced as $m^*=m_1+m_2$. Further, the initial conditions are $x_1(0)$= 1m and $dx_1(0)/dt$= 10m/s. What is the natural frequency of the system? What is the analytical solution for the displacement, $x_1(t)$? Please construct a detailed graph of x1(t) versus time. (15 points)

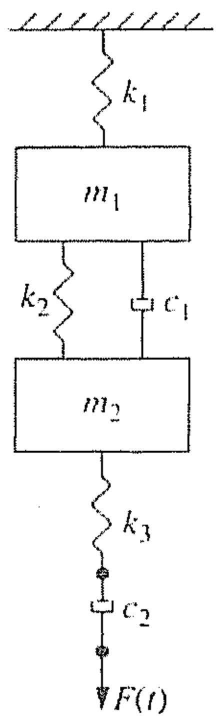


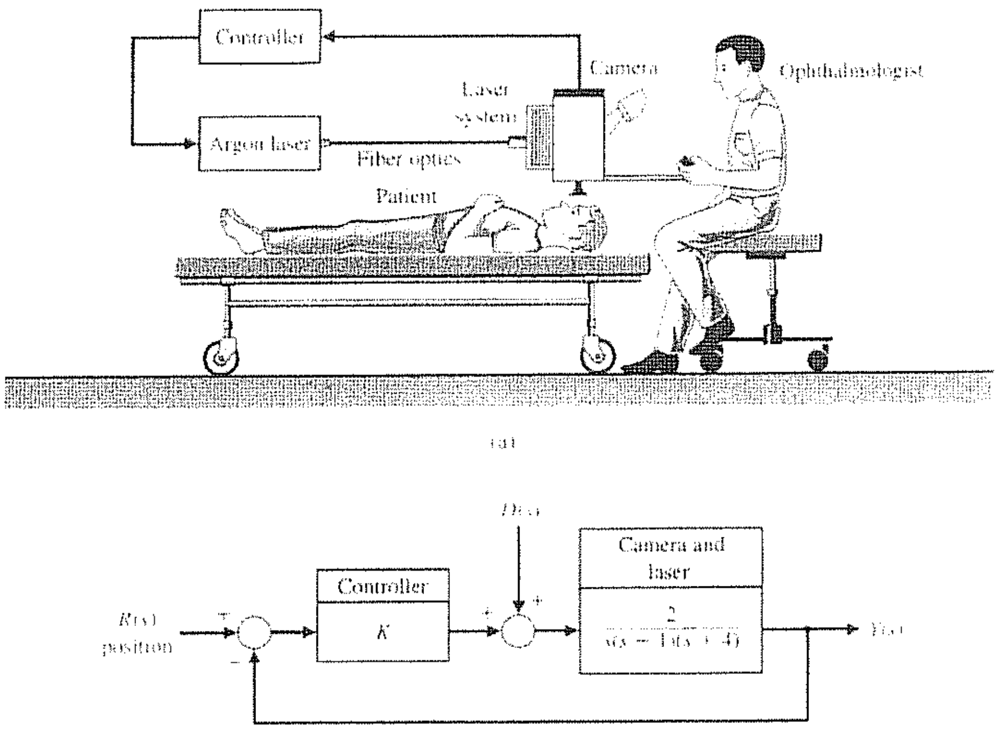
TABLE 5.1 LAPLACE TRANSFORM PAIRS

	$f(t)$	$F(s) = \mathcal{L}\{f(t)\}$
1	e^{at}	$\frac{1}{s-a}$
2	$\sin(\omega t)$	$\frac{\omega}{s^2+\omega^2}$
3	$\cos(\omega t)$	$\frac{s}{s^2+\omega^2}$
4	t	$\frac{1}{s^2}$
5	t^2	$\frac{2}{s^3}$
6	t^n	$\frac{n!}{s^{n+1}}$
7	$\delta(t-a)$	e^{-as}
8	$u(t-a)$	$\frac{e^{-as}}{s}$
9	$\sinh(\omega t)$	$\frac{\omega}{s^2-\omega^2}$
10	$\cosh(\omega t)$	$\frac{s}{s^2-\omega^2}$
11	te^{at}	$\frac{1}{(s-a)^2}$
12	$e^{-\zeta\omega_n t}\sin(\omega_n\sqrt{1-\zeta^2}t)$	$\frac{\omega_n\sqrt{1-\zeta^2}}{s^2+2\zeta\omega_n s+\omega_n^2}$
13	$e^{-\zeta\omega_n t}\cos(\omega_n\sqrt{1-\zeta^2}t)$	$\frac{s+\zeta\omega_n}{s^2+2\zeta\omega_n s+}$
14	$t\sin(\omega t)$	$\frac{2s\omega}{(s^2+\omega^2)^2}$
15	$t\cos(\omega t)$	$\frac{s^2-\omega^2}{(s^2+\omega^2)^2}$
16	$e^{-at}-e^{-bt}$	$\frac{(b-a)}{(s+a)(s+b)}$
17	$1-e^{-at}$	$\frac{a}{s(s+a)}$
18	$ae^{-at}-be^{-bt}$	$\frac{(a-b)s}{(s+a)(s+b)}$

Problem 2: [30 Pts.]

Lasers have been used in eye surgery for more than 25 years. A laser can be used to "weld" a detached retina into its proper place on inner surface of the back of the eye. This is a very precise procedure and the laser should be very accurately pointed to the right place on the retina.

Automated control of position enables the ophthalmologist to indicate to the controller where lesions should be inserted. The controller then monitors the retina and controls the laser's position so that each lesion is placed at the proper location. A wide-angle video camera is required to monitor the movement of the retina, as shown in Figure 3a. If the eye moves during the irradiation, the laser will be adjusted automatically. The position control system is shown in Figure below. The camera and laser are modeled as a linear 3[rd] order system with transfer function given in the block diagram. In this problem our goal is to design the controller gain such that Y tracks R as closely as possible.



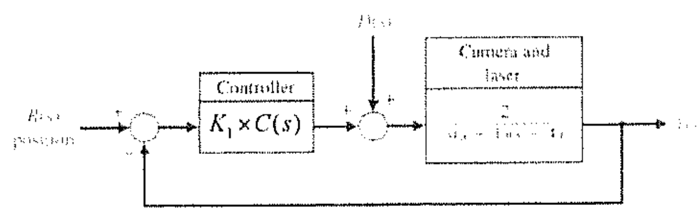
Laser eye surgery system

(A) Obtain the transfer functions from reference R to Y and from the disturbance D to Y. (5 points)

(B) Sketch the root-locus plot of the system as the gain K varies from 0 to +∞. Determine the angle of the asymptotes and their crossing point with the real axis. (No need to find other specific values.) (10 points)

(C) Find the range of gains K for which the closed-loop system is stable. You need to show your calculations! (5 points)

(D) We want to add a compensator $C(s)$ to the forward path above such that the "dominant" closed-loop poles are at $s = -1 \pm 1j$. It has been determined that a proportional controller will not satisfy this requirement so we decide to use a PD controller.



Determine a PD control in the form C(s)=s+z which satisfies this design requirement. (You can use the root-locus design approach; you don't have to plot the root-locus; but a quick sketch may help you visualize things better.) (5 points)

(E) Is the current control system capable of rejecting the influence of constant disturbances on the output? If not what solution do you recommend? You do NOT need to calculate new control gains. Just explain your solution. (5 points)

Problem 3: [30 Pts.]

For parts A-C, please use the Bode plot in Figure 1.

(A) What is the transfer function of the systems represented in Figure? Show all calculations and make sure that you describe how you arrived at your answer. (5 Points)

(B) Clearly, in Figure there is a roll off on the magnitude plot of 20 dB/dec. In fact, for any linear system, the slope of the magnitude plot is always a multiple of 20 dB/dec. Why is this so? Please limit your discussion and mathematical proof to one page (a single side). (5 Points)

(C) Clearly, in Figure there is a high frequency phase shift of 90° In fact, for any linear system, the high frequency phase shift is always a multiple of 90°. Why is this so? Please limit your discussion and mathematical proof to one page (a single side). (5 Points)

(D) Sketch the Bode plot for the following transfer function:

$$\frac{Y(s)}{U(s)} = \frac{10(s+10)}{(s+1)(s+100)} \tag{1}$$

Please use units of radians/s, degrees and dB for all of your plots. Make sure to provide all details on your plots including any breakpoints, asymptote information, DC gain, etc. (10 Points)

(E) Given the transfer function shown in equation (1), approximate (within 10%) the time domain function y(t) if u(t) is given by: (5 Points)

$$u(t) = 10 + 3\sin(0.1t) + 0.5\sin(5t) + 2\sin(30t) + 10\sin(200t) + 20\sin(1000t)$$

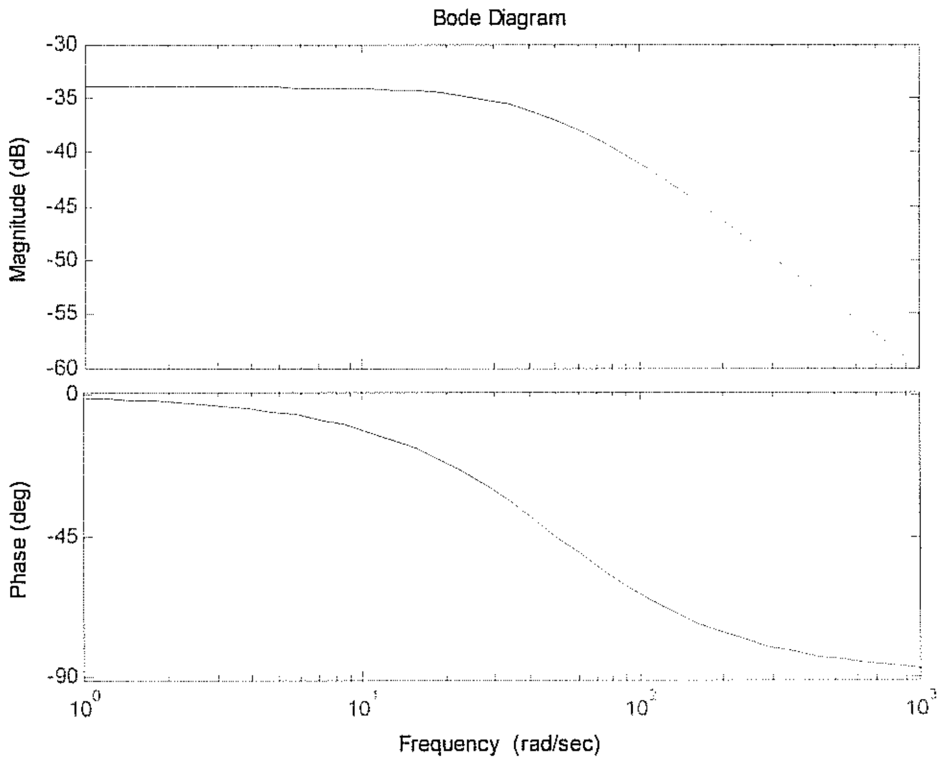


Figure 1 - The First Bode Plot