CLEMSON UNIVERSITY

Department of Mechanical Engineering
Fall 2009 Ph.D. Qualifying Exam

SYSTEMS AND CONTROLS

INSTRUCTIONS:

- The exam time is 2 hours.
- This exam is closed-book and closed-notes.
- Scientific calculators are NOT allowed.
- Please clearly show all the necessary work for partial credit.

HONORS PLEDGE: "*I have neither given nor received aid on this examination.*"

Sign Here: ___________________ (use your assigned identifier number)

Problem	Points	Score
1	30	
2	30	
3	30	
Total	90	

PROBLEM 1. (30 points) Suppose you came across a field-controlled DC motor with unknown values for the field inductance (L) and resistance(R) in your research project. A simple test of the field winding with an initially charged capacitor of a known capacitance should allow you to estimate the values of R and L. Fig.1 shows a schematic of the test arrangement. The test is initiated by closing the switch at time t=0.

a. (5 points) Derive the system differential equation governing the system after the switch is suddenly closed.

b. (10 points) Write a state-space representation for the system with the voltage V12 being the output.

c. (10 points) After the switch is closed, the time history of voltage V12 is displayed on an oscilloscope. An oscillation having a natural frequency of 100 rad/sec and a settling time of 4 seconds is observed. Determine the estimated values of the field winding parameters R and L based on the measured response.

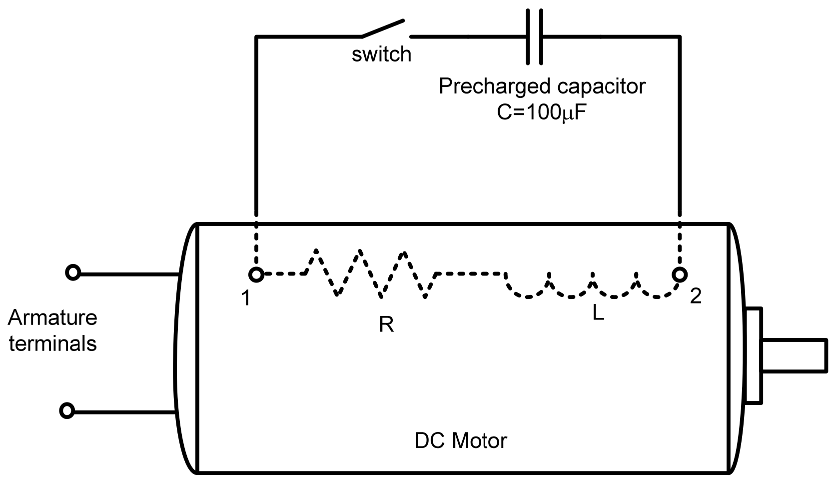


Figure 1.1. Schematic of the DC motor circuit.

d. (5 points) Compute a minimum resistance value that when you add it in series to the field circuit will help you eliminate the observed oscillations in part c above.

PROBLEM 2. (30 points) The bicycle is an interesting dynamical system with a feedback mechanism created by the design of its front fork. A detailed model of a bicycle is complex because the system has many degrees of freedom and the geometry is complicated. However, a great deal of insight can be obtained using simplified models.

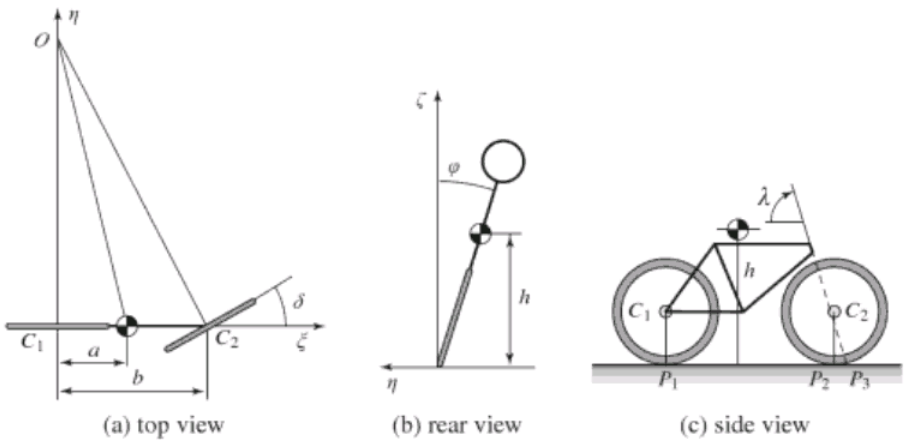


Figure 2.1. Schematic views of a bicycle. The steering angle is δ, and the role angle is φ. The center of mass has height h. the wheel base is b and the trail is c.

In this problem we consider a simple linearized model of the bike which relates its tilt angle φ to its steering angle δ :

$$\frac{d^2\varphi}{dt^2} - \frac{mgh}{J}\varphi = \frac{mv_0^2 h}{bJ}\delta + \frac{Dv_0}{bJ}\frac{d\delta}{dt}$$

where v_0 is the velocity of the bicycle, m its mass, J its moment of inertia with respect to ξ axis, h is the height of its center of mass, b is its wheel base, and $D = mha$. These parameters are all constants. Assume the following values:

g= 10 m/s^2 m=100 kg a=0.2m b=1 m h=1 m J=10 kg.m^2 D=20 kg.m^2

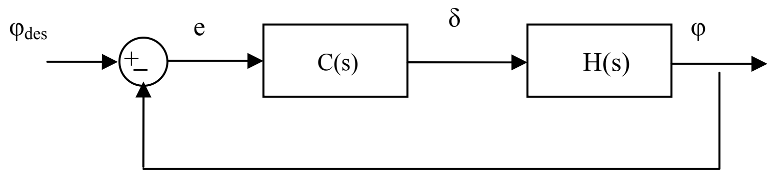
But lets keep the velocity as a parameter v_0, so we can later see what happens at different speeds. Therefore the transfer function relating the input (steering angle δ) to the output (tilt angle φ) is,

$$H(s) = \frac{2v_0 s + 10v_0^2}{s^2 - 100}$$

Using this linearized and simplified model answer the following simple questions.

a. (5 points) Determine the pole(s), zero(s), and order of the system. Also determine if the system is causal. Is the linearized bicycle model stable?

b. (10 points) We want to study a feedback control system that by controlling the steering angle δ keeps the bicycle at desired upright position. Schematic of the feedback system is shown below:



We try a proportional controller: $C(s)=k_p$. Determine the range of values of k_p that stabilizes the system. Your answer is going to be a function of v_0. Is it easier to keep the bike upright at lower or higher speeds?

c. (10 points) To get a better qualitative understanding of the behavior of the closed-loop system at different bicycle speeds, sketch a "quick" root locus of the system as the proportional gain is varied. Sketch two root locus plots corresponding to the two different speeds: $v_0 = 1$ m/s and $v_0 = 4$ m/s. Looking at these root locus plots, explain if it is possible for closed-loop response to be oscillatory for some gain value at either speed?

d. (5 points) You are now looking at a *rear-wheel steered* bicycle made at University of California, Santa Barbara. The transfer function of the bicycle is similar to that of a regular one with the only difference that its zero now is in the right-half plane:

$$H(s) = \frac{-2v_0 s + 10v_0^2}{s^2 - 100}$$

Can this bicycle be stabilized with proportional control? Show the derivations that lead to your answer.

PROBLEM 3. (30 points) The amplifier filter depicted in Figure 3.1 is used in the closed loop system in Figure 3.2.

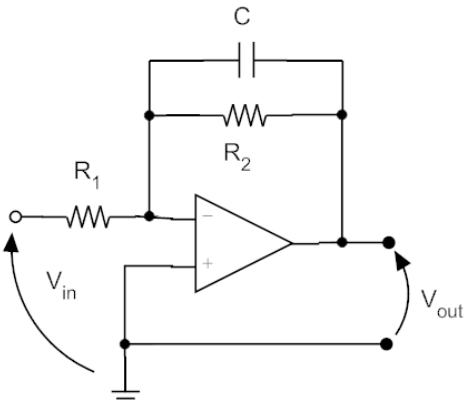


Figure 3.1. Amplifier filter**.**

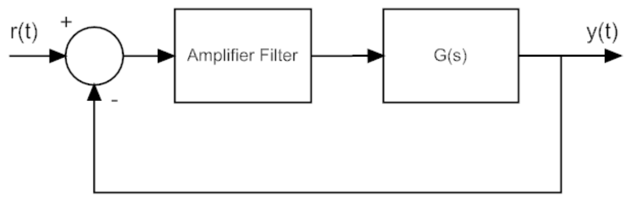


Figure 3.2. Closed-loop system.

The system G(s) is described by

$$G(s) = -\frac{1}{s^2 + 10s}$$

While the differential equation describing the relationship between the input and the output of the amplifier is

$$CR_2\dot{v}_{out} + v_{out} = -\frac{R_2}{R_1}v_{in}$$

a. (10 point) Draw the Bode plots and compute the phase and gain margin assuming $CR_2 = 1$ and
   a. $\frac{R_2}{R_1} = 20$
   b. $\frac{R_2}{R_1} = 2000$

b. (10 points) Determine the upper limit on the ratio $\frac{R_2}{R_1}$ for the system to be stable.
   a. Which is the limiting factor: the phase margin or the gain margin?

c. (5 points) What happens to the Bode plot and stability of the system when the capacitor is removed?

d. (5 points) What happens to the stability of the closed loop system when $R_2 \to \infty$?