CLEMSON UNIVERSITY

Department of Mechanical Engineering
Spring 2010 Ph.D. Qualifying Exam

SYSTEMS AND CONTROLS

INSTRUCTIONS:

- The exam time is 2 hours.
- This exam is closed-book and closed-notes.
- Scientific calculators are allowed.
- Please clearly show all the necessary work for partial credit.

HONORS PLEDGE: "*I have neither given nor received aid on this examination.*"

Sign Here: ___________________ (use your assigned identifier number)

Problem	Points	Score
1	40	
2	40	
Total	80	

PROBLEM 1. (40 points) A ball and beam system is shown in Figure 1. The ball can be positioned on the beam by controlled rotation of the beam with a rack and pinion system. The beam is hinged in the middle and rests on a linear spring with stiffness K at one end. The other end is driven up and down by the rack and pinion system.

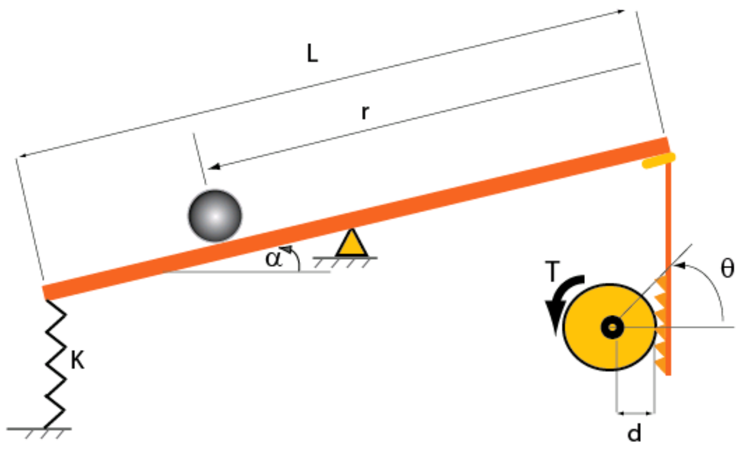

Figure 1. Ball and Beam System.

a. (points 15) First we want to derive the dynamic model relating the torque applied to the pinion T and the angle of the beam α. In this part we NEGLECT the influence of the ball on the dynamics. This is an acceptable assumption because the mass of the ball is much smaller than the rest of the components. The moment of inertia of the beam around its center is J_b, the moment of inertia of the pinion is J_p and mass of the rack is m_r. Friction and viscous damping are negligible. As it is shown in the figure the radius of the pinion is d and the length of the beam is L. No slipping happens between the rack and pinion. In this part assume the angles are small so sin(α) can be replaced by α.
   First draw the free body diagram for the system (excluding the ball) and then obtain the differential equation representing the dynamic relationship between the torque T and the angle of the beam α.
b. (10 points) Write a state-space representation for the dynamic model you obtained in part a.
c. (8 points) It can be shown that the dynamic relation between the angle of the beam α and the position of the ball on the beam r is governed by the following nonlinear differential equation:

$$\left(\frac{J_{ball}}{R_{ball}^2}+m_{ball}\right)\ddot{r}-m_{ball}g\sin(\alpha)+m_{ball}r(\dot{\alpha})^2=0$$

   where J_{ball}, R_{ball}, and m_{ball} are moment of inertia, radius, and mass of the ball respectively and r is the position of the ball on the beam. Linearize this equation around the horizontal position of the beam (around α=0).

d. (7 points) The goal is to control the position of the ball on the beam by manipulating the input torque T applied on the pinion. Sketch a block diagram showing the corresponding closed-loop system. Also explain if the closed-loop can be stabilized with proportional control action only?

PROBLEM 2. (40 points) A Satellite tracking radar is being designed to follow the motion of a satellite as shown in Figure 2.

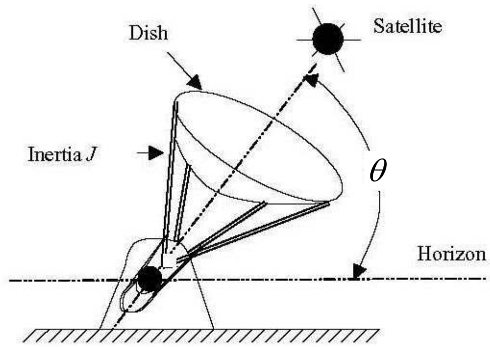

Figure 2. Satellite Tracking Antenna

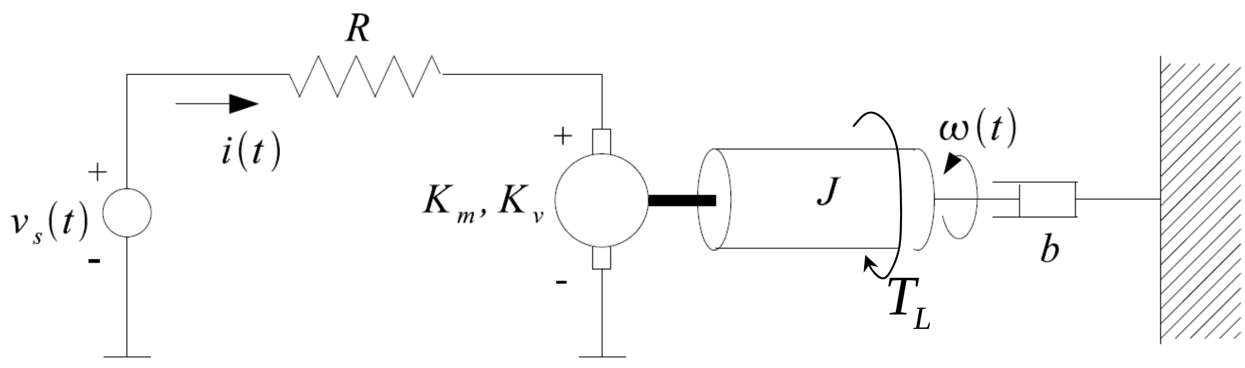

Figure 3. Servo control system for tracking antenna

An automatic control system is desired to position the elevation angle θ of the radar dish in proportion to a referenced input *r*. A servomotor is available that is represented by the system depicted in Figure 3, in which *J* represent the dish inertia, v_s is the voltage applied to the motor, and T_L is the wind torque. Additional data for the DC motor subsystem are: motor resistance R = 2Ω; the torque constant K_m =1 [Nm/A] and back–emf constant K_v = 1 [Vs/rad]; the load shaft inertia J = 1 [kg m^2] and viscous friction coefficient *b* =0.5 [Nms/rad]. The motor's own inertia and inductance are negligible.
It is proposed to measure θ by an electrical device such that

$$Y(s) = \frac{K_1}{s+100}\Theta(s)$$

And to let v_s be proportional to the error (*r* - *y*), i.e.

$$v_s = K(r-y)$$

a. (points 10) Derive the open loop transfer functions $\frac{\Theta(s)}{V_s(s)}$ and $\frac{\Theta(s)}{T_L(s)}$ for the servomotor and draw the block diagram for the entire system.

b. (Points 10) Assume T_L =0 and K_1 =100, sketch the root locus of the system and determine for what values of K the system is stable.

c. (Point 10) Assume T_L =0 and K_1 =100 as before and K=10, sketch the Bode plot and estimate gain and phase margin.

d. (Point 5) By keeping the same gain K=10, design a lead or lag compensator such that the closed loop system is stable and gain margin is equal to 30dB.

e. (Point 5) For a constant wind torque , *r*=0 and K_1 =100, find the steady state error as function of K. Clearly, K is a proportional controller. If we want to achieve zero steady state error in presence of constant wind torque, what type of controller should we use? Justify your answer.
Suppose now the reference *r* is a ramp. If we want to have zero tracking error, what type of controller should we use?